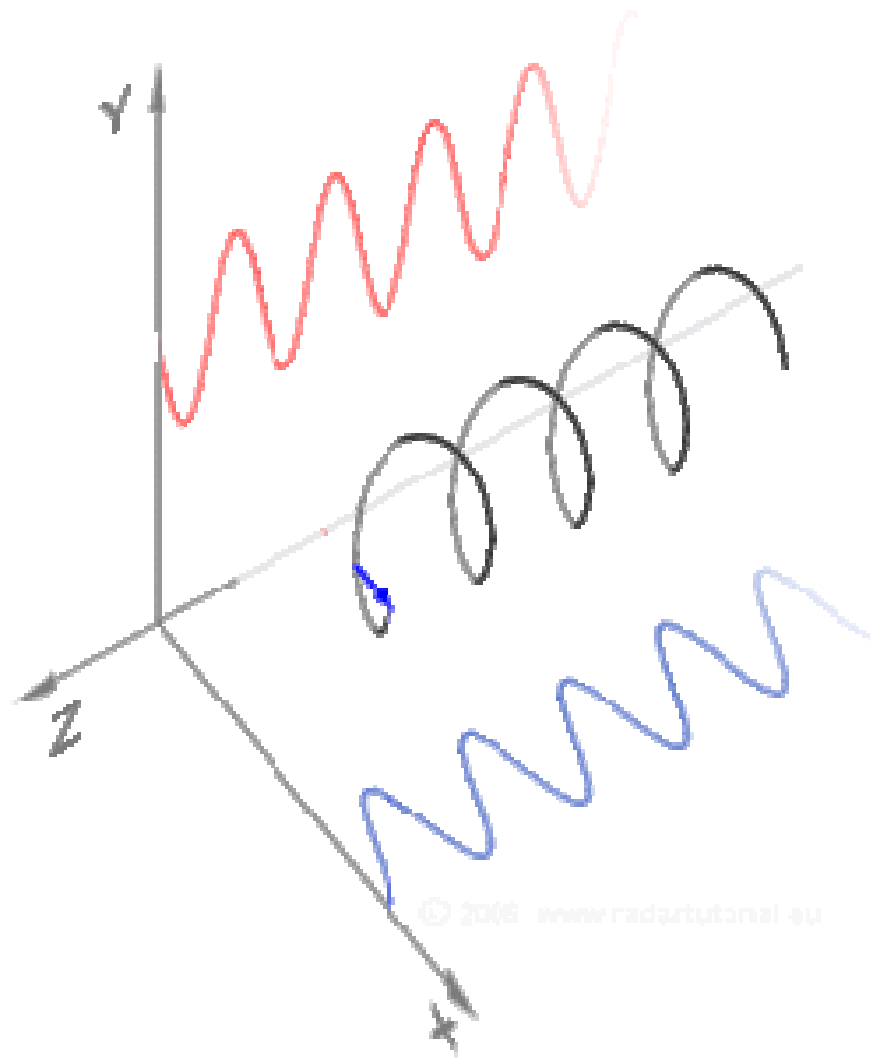# Polarization

M. Creech-Eakman

Radiation and Optics

Basics of Polarization

- Only possible for transverse waves
- Conventionally refers to the E part of the EM wave
- Some materials are isotropic and leave EM waves unchanged; others do not and are called optically active
- A polarizer is a special type of filter that only transmits one polarization
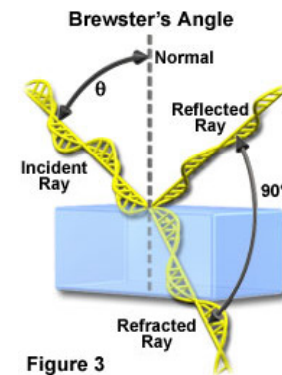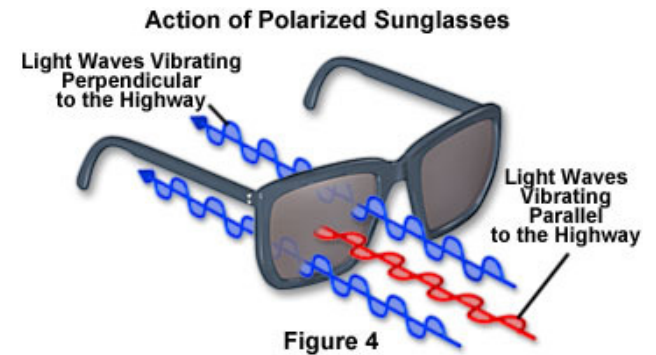
More Background on Lingo

- For use with optics, polarization is often referred to as "s" or "p" type (like in earthquakes)
  - "s" – German *senkrecht* for perpendicular – also TE
  - "p" – E field parallel to the plane of incidence – also TM
- Polarization is commonly referred to as linear, circular, or elliptical (more later)

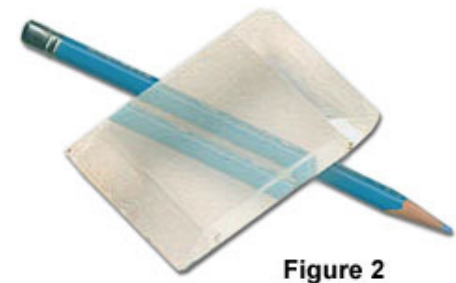Circularly polarized wave split into linear components

Ways to Polarize Light

Action of Polarized Sunglasses
Light Waves Vibrating Perpendicular to the Highway
Light Waves Vibrating Parallel to the Highway
Figure 4

- Transmission – picket fence analogy

- Reflection – Brewster's angle

$$n = \sin(\theta_i)/\sin(\theta_r) = \sin(\theta_i)/\sin(\theta_{90-i}) = \tan(\theta_i)$$

Brewster's Angle
Normal
θ
Reflected Ray
Incident Ray
90°
Refracted Ray
Figure 3

- Refraction – Birefringence

Bi-Refraction in Calcite Crystals
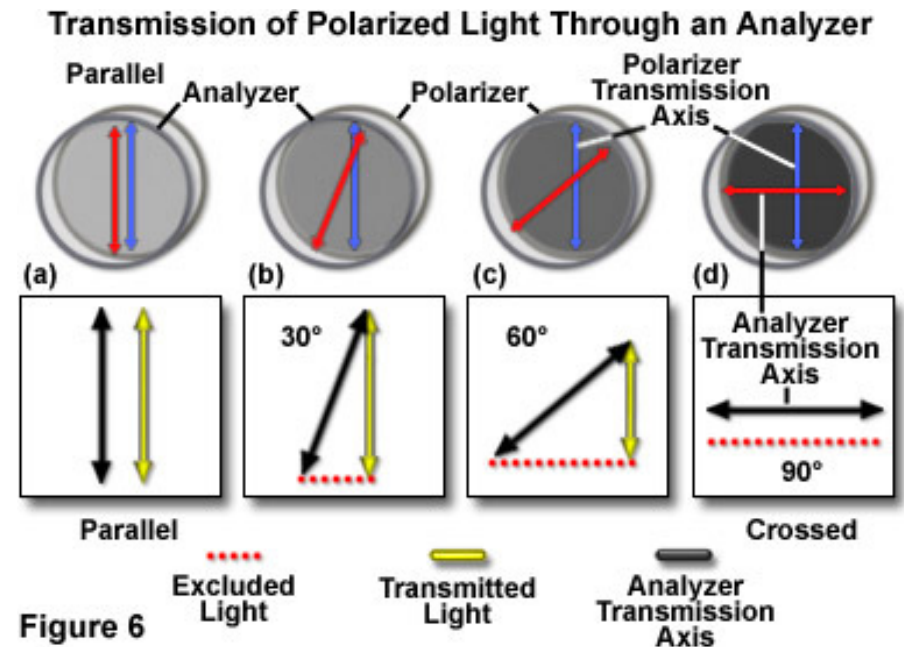Figure 2

- Scattering – blue sky example

Law of Malus

- Developed by Etienne Malus using dichroic materials
- Observation that completely unpolarized light passing through a polarizer, intensity drops by half

$$I = I_o \cos^2 \theta$$

- Theta is angle between polarization direction of polarized light and transmission axis of polarizer

Polarizer-Analyzer Pairs

- Using Malus' Law you can determine the polarization of light by using polarizers in succession

- In this process the second polarizer is called the analyzer

- Need a way to measure the final intensity

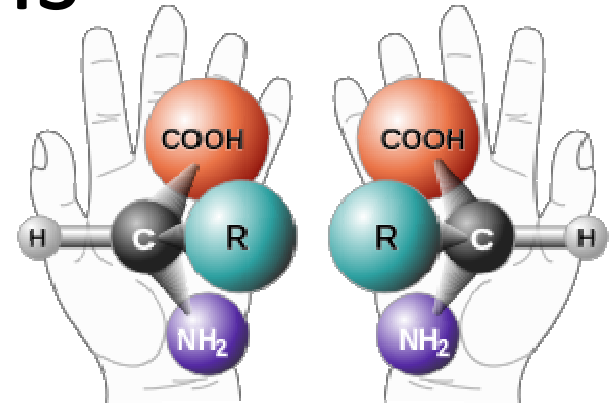Transmission of Polarized Light Through an Analyzer

Figure 6

Some Formalism

- Way to write EM equations for polarized transverse waves (we've been doing this all along)
- Matrix notation is handy and useful in lab settings
  - Jones vectors/matrices
  - Stokes vectors - astrophysics
- Adopted from Colby Jurgenson's lecture from a few years back – show quickly in a bit

Applications

- Chirality in living systems
  - Dextro (R) or Levo (L)

- Pockel's Cell
  - Use E field to change n of crystalline materials resulting in a change in polarization of light going through the cell – linear effect

- Kerr Cell
  - Same as Pockels with a quadratic effect
  - Sometimes called QEO
  - Often seen in liquids

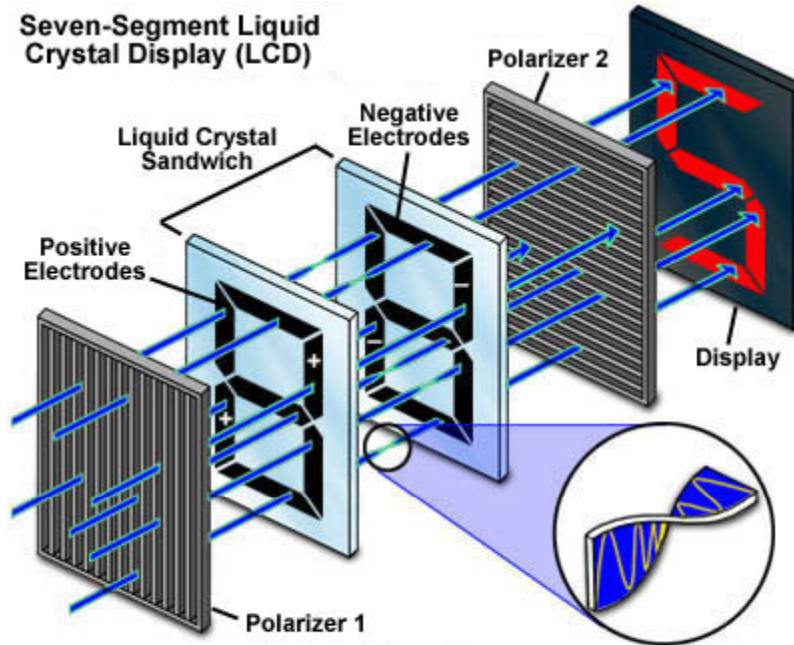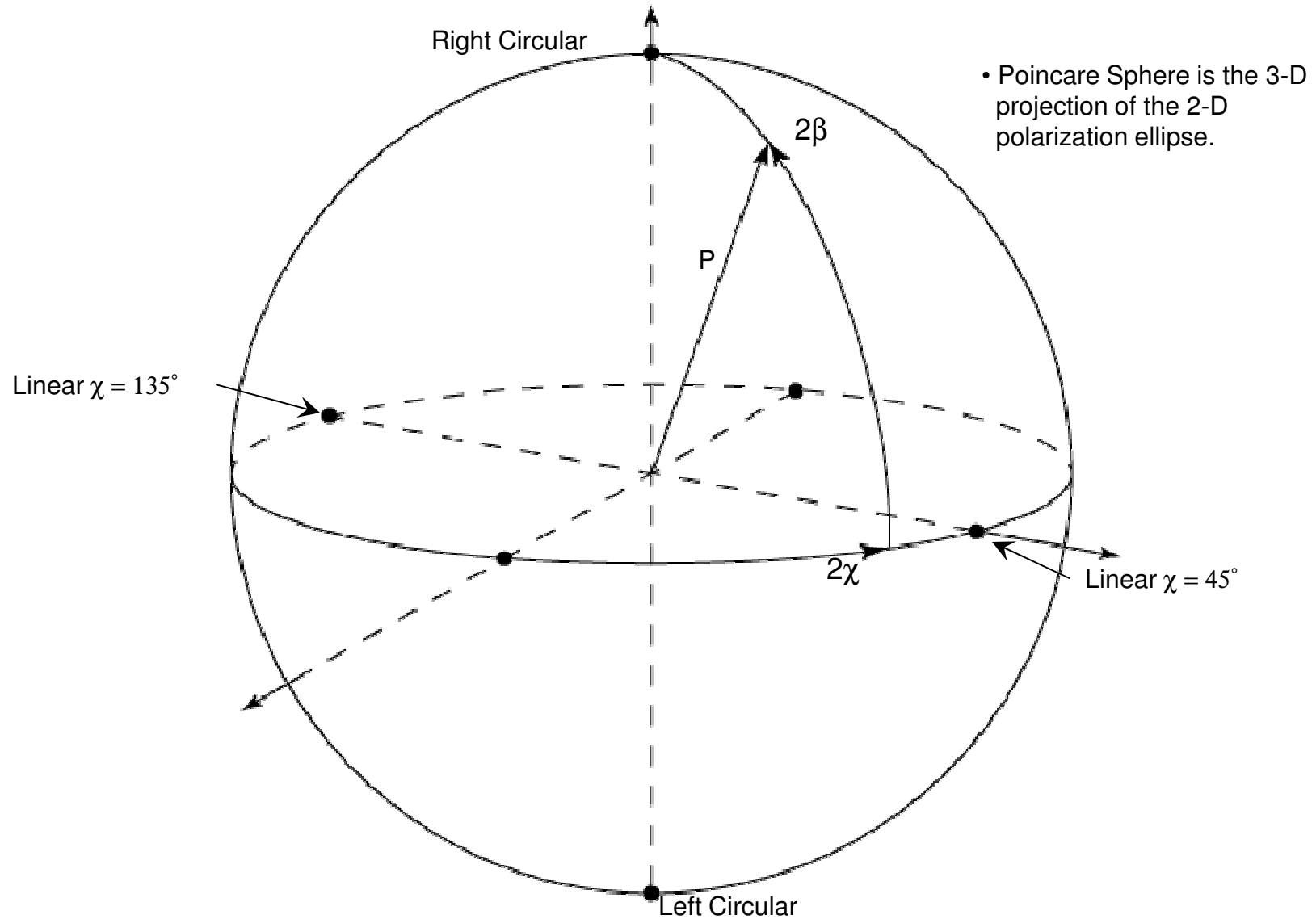# Applications continued

- LCDs

- Movie 3D glasses

- Astronomy – Stokes parameters – I, Q, U & V

Seven-Segment Liquid Crystal Display (LCD)

Liquid Crystal Sandwich

Negative Electrodes

Positive Electrodes

Polarizer 2

Display

Polarizer 1

Figure 3

Polarized Light

Right Circular

2β

P

• Poincare Sphere is the 3-D projection of the 2-D polarization ellipse.

Linear $\chi = 135°$

2χ

Linear $\chi = 45°$

Left Circular

"The Universe is written in mathematical language, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word…"
Galileo

Outline

I.   Polarization Ellipse & its Degenerate Forms
II. The Jones Matrix Calculus
III. Something Cool, Something Silly, and
     Something to Learn How to Do.

➤ I'm going to present this a bit differently than your textbook does.

➤ The main difference is that I will present the polarization ellipse first, derive all it's parameters, and then analyze the Jones vectors with respect to it.

➤ If you think that this discussion might be helpful as a supplement to your textbook, email me and I will send you the presentation.
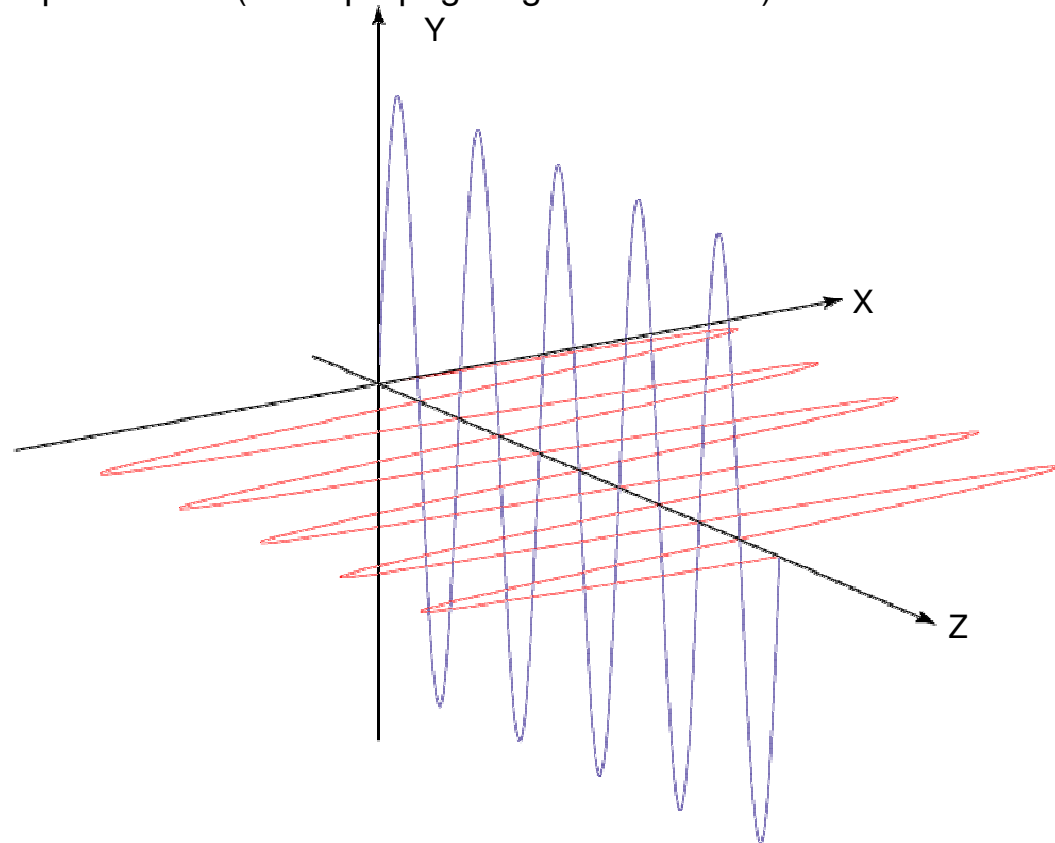
cjurgenson@mro.nmt.edu

Part I: Polarization Ellipse & Its Degenerate Forms

- Mathematical Description of Electromagnetic Wave
- The Polarization Ellipse
- Linear, Circular, and Elliptical Polarization

- Circa1670 Christian Huygens first suggested the vectoral nature of light to explain its behavior propagating through crystals.

- In the words of Newton (~1672), it appeared as though light had "sides…"

- In 1818 Fresnel and Arago were able to explain Young's interference experiment with a with a light wave that consists of two transverse components (oscillating perpendicular to the direction of propagation), and no longitudinal component.

- Initially seen as a "defect" in Fresnel's theory, solving Maxwell's equations in free space verified only transverse components arise.

- These transverse components can be expressed as (wave propagating in z direction):

$$E_x(z,t) = E_{0x} \cos(\tau + \delta_x)$$

$$E_y(z,t) = E_{0y} \cos(\tau + \delta_y)$$

14

➢ The propagation of E_x and E_y give rise to a vector describing a locus of points in space that generates a curve whose form can be derived using the previous equations.

$$E_x(z,t) = E_{0x} \cos(\tau + \delta_x) \quad and \quad E_y(z,t) = E_{0y} \cos(\tau + \delta_y)$$

➢ We start by dividing out by the amplitude and applying the double angle formulae to the argument of the cosine function:

$$\frac{E_x}{E_{0x}} = \cos(\tau)\cos(\delta_x) - \sin(\tau)\sin(\delta_y) \quad and \quad \frac{E_y}{E_{0y}} = \cos(\tau)\cos(\delta_y) - \sin(\tau)\sin(\delta_y)$$

➢ Re-arranging terms, subtracting, and using the double angle formula again, the following two expressions can be written:

$$\frac{E_x}{E_{0x}}\sin(\delta_y) - \frac{E_y}{E_{0y}}\sin(\delta_x) = \cos(\tau)\sin(\delta_y - \delta_x) \qquad \frac{E_x}{E_{0x}}\cos(\delta_y) - \frac{E_y}{E_{0y}}\cos(\delta_x) = \sin(\tau)\sin(\delta_y - \delta_x)$$
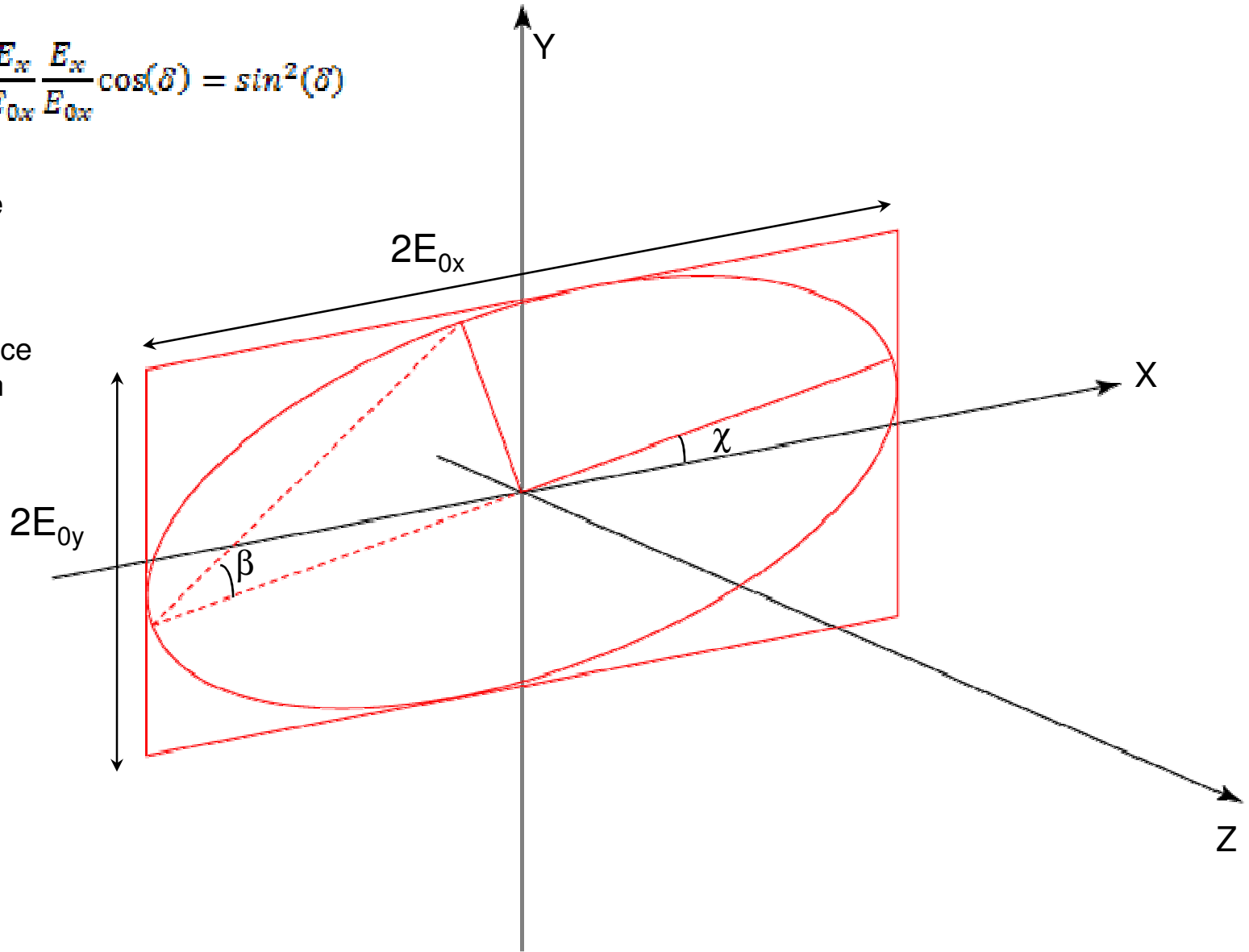
➢ Squaring the above two expressions, and adding them together yields ($\delta = \delta_y - \delta_x$):

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = \sin^2(\delta)$$

➤ This is the equation for an ellipse. What it shows is that at any instant of time the locus of points described by the propagation of E_x and E_y will trace out this curve.

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = \sin^2(\delta)$$
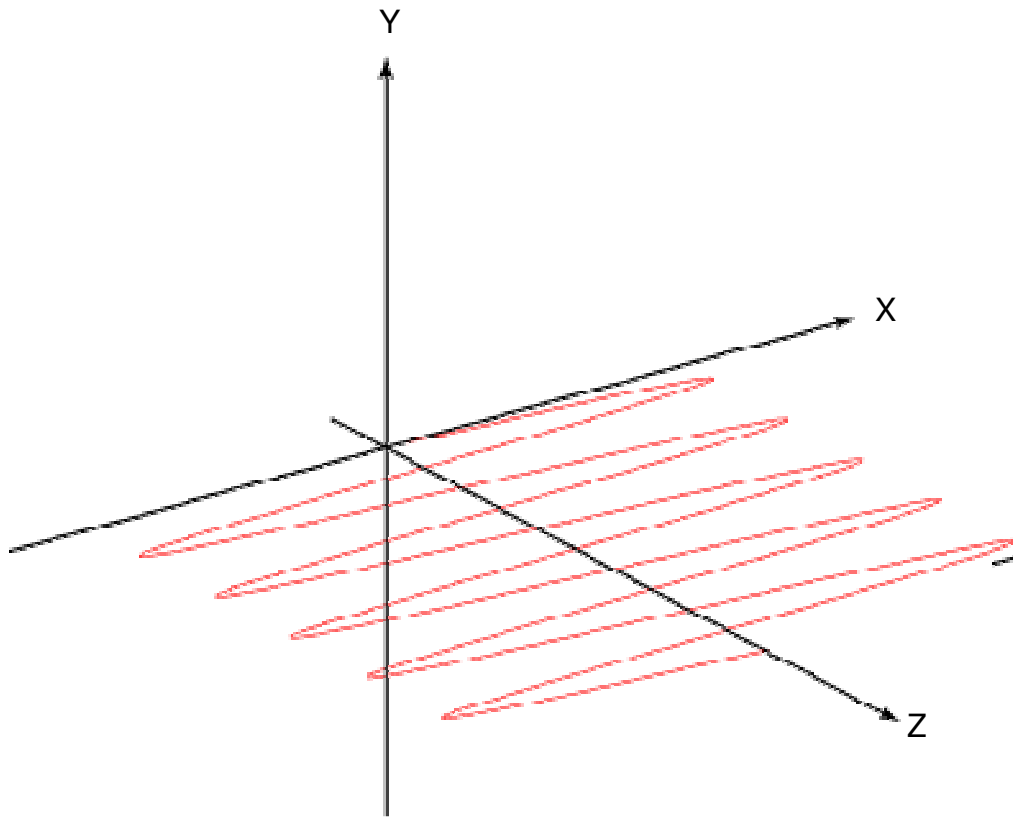
• β is measure of the of the ellipticity

• χ is rotation of the ellipse (consequence of the cross term in above equation)
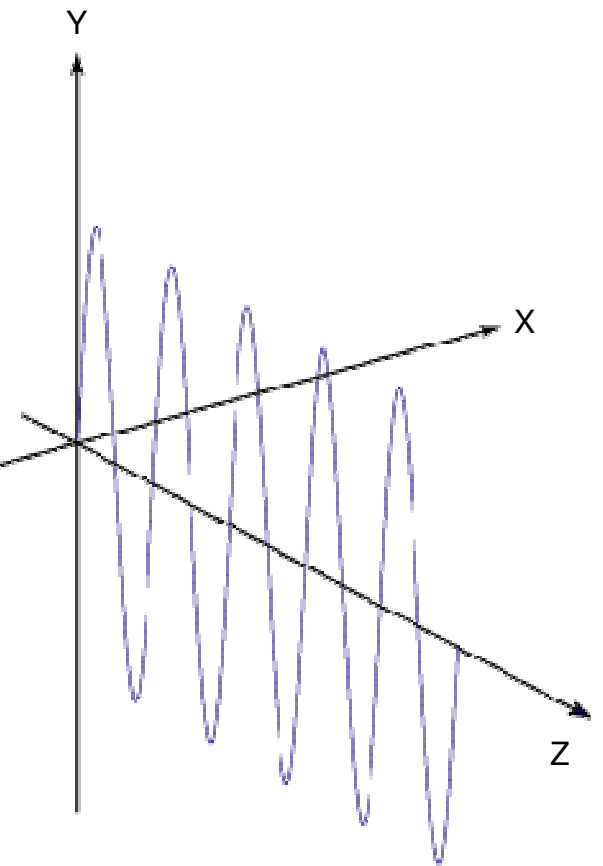
➢ For specific values of E_{0x}, E_{0y}, and δ, the polarization ellipse reduces to 4 specialized forms:

➢ Case I: Linear horizontal, or linear vertical polarization
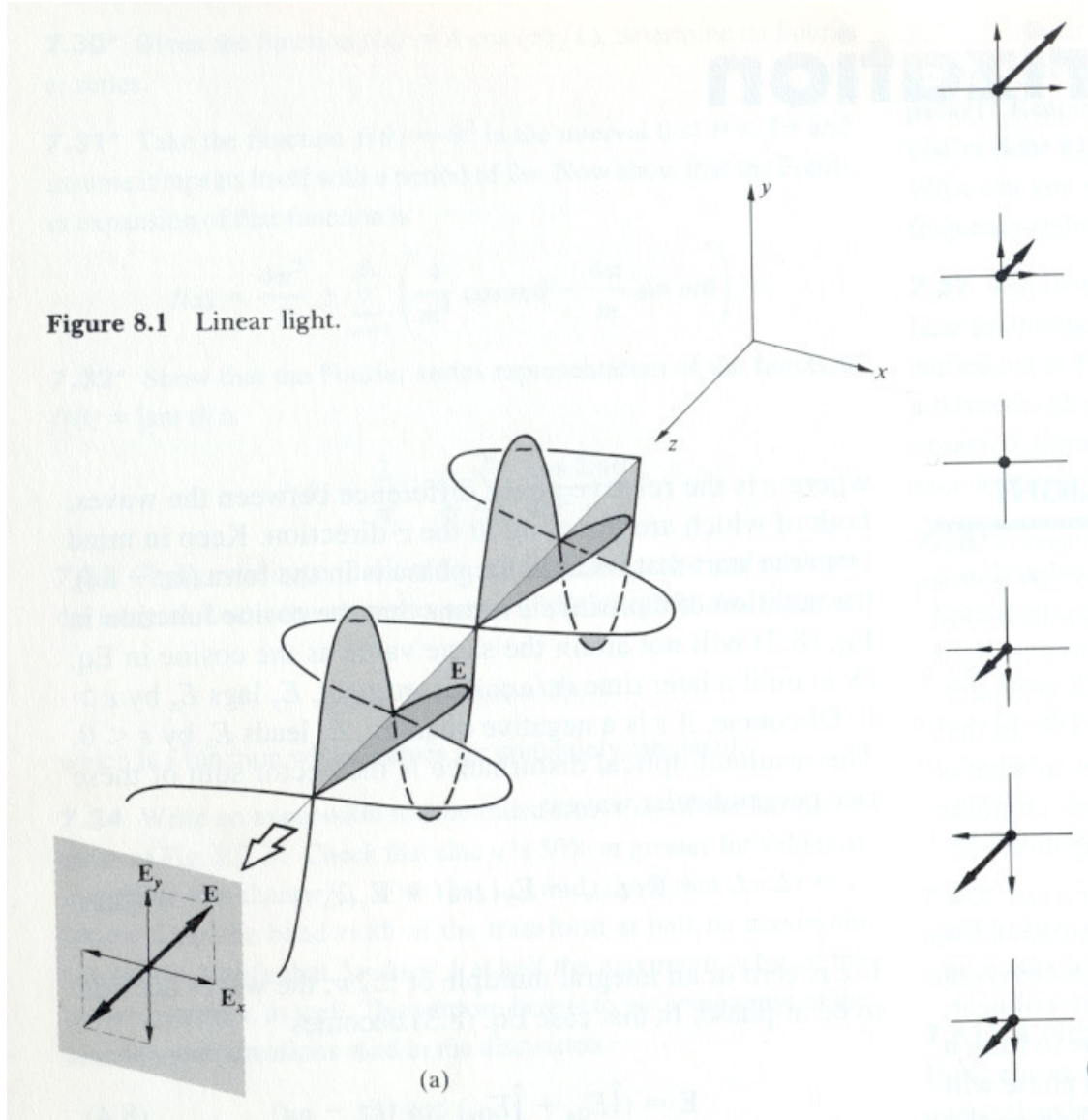
$E_{0y} = 0$ (Linear Horizontal)

$E_{0x} = 0$ (Linear Vertical)

➢ Case II: Linear ±45° polarization (δ = 0 or π, and E_{0x} = E_{0y})

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = \sin^2(\delta) \rightarrow \left(\frac{E_x}{E_{0x}} \pm \frac{E_y}{E_{0y}}\right)^2 = 0 \rightleftharpoons E_y = \pm E_x$$

• The slope of the resultant vector is either positive or negative depending upon the relative phase shift between the two waves.

Figure 8.1   Linear light.

(a)

18

➢ Case III: Right or Left Circular Polarization ($\delta = \pi/2$ or $3\pi/2$, and $E_{0x} = E_{0y} = E_0$)
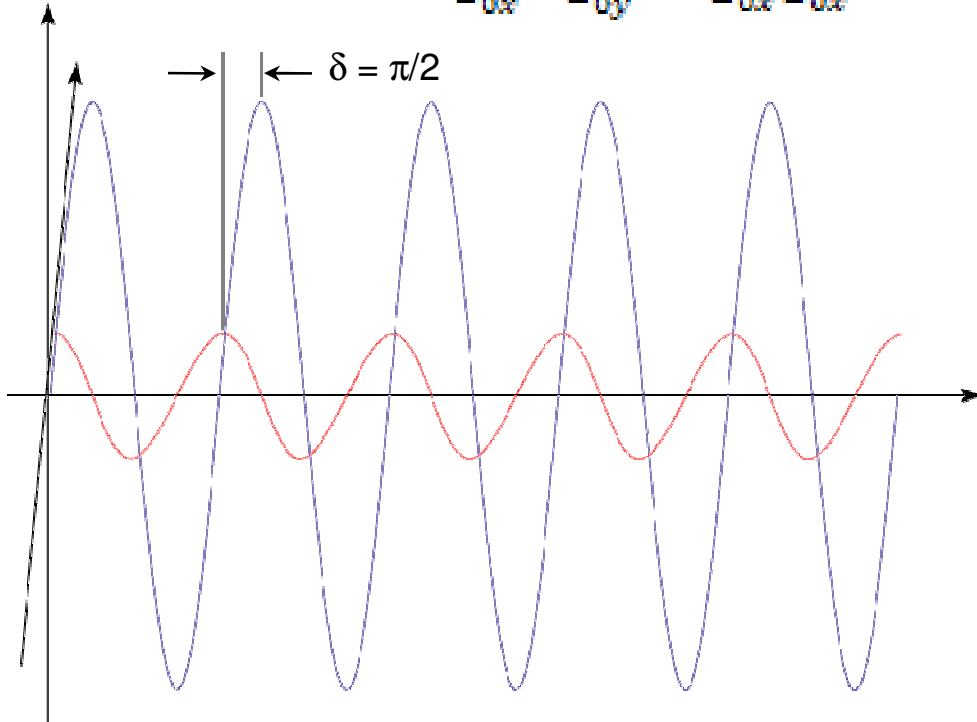
$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = \sin^2(\delta) \rightarrow \frac{E_x^2}{E_0^2} + \frac{E_y^2}{E_0^2} = 1$$

$\delta = \pi/2$

- Right circular occurs when E_y is advanced in phase by $\lambda/4$.

- Left circular occurs when E_x is advanced in phase by $\lambda/4$.

➢ Case IV: Elliptical Polarization (Linear + Circular = Elliptical)

D. Elliptically Polarized Light

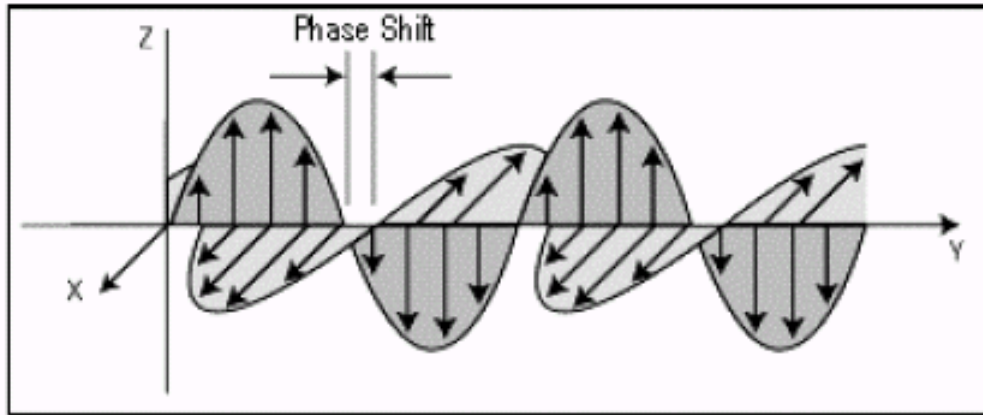➢ When δ = π/2 or 3π/2 the polarization ellipse reduces to the standard equation of an ellipse:

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = sin^2(\delta) \;\rightarrow\; \frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} = 1$$
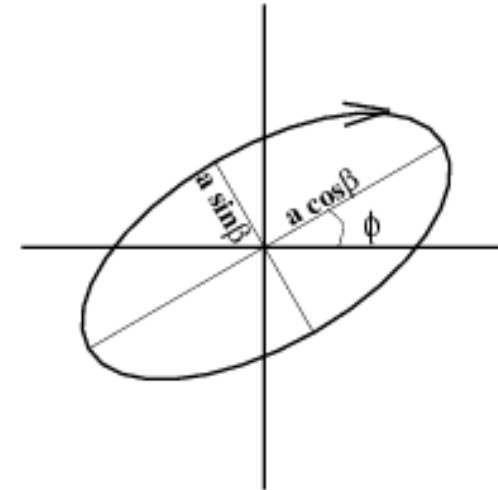
➢ Illustration Aids:

- http://webphysics.davidson.edu/physlet_resources/dav_optics/examples/polarization.html
- http://www.ub.es/javaoptics/index-en.html
- http://www.colorado.edu/physics/2000/applets/polarized.html

➢ The next few slides will discuss the relationship between the parameters of the polarization ellipse.

➢ Recall the form of the polarization ellipse (again, $\delta = \delta_y - \delta_x$):

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} - 2\frac{E_x}{E_{0x}}\frac{E_x}{E_{0x}}\cos(\delta) = \sin^2(\delta)$$

➢ Due to the cross term, the ellipse is rotated relative to the x and y directions.

➢ The purpose of the next couple slides is to show the mathematical relations between polarization ellipse, E_{0x}, E_{0y}, δ and the angle of rotation χ, and β the ellipticity angle.

➢ From the figure below, the transformation between the primed and unprimed coordinates:

$$E_x' = E_x \cos(\psi) + E_y \sin(\psi)$$
$$E_y' = -E_x \sin(\psi) + E_y \cos(\psi)$$

➢ Using the rotated components:

$$E'_x = E_x \cos(\psi) + E_y \sin(\psi)$$
$$E'_y = -E_x \sin(\psi) + E_y \cos(\psi)$$

➢ The equation of the ellipse in terms of primed coordinates:

$$E'_x = a \cos(\tau + \delta')$$
$$E'_y = \pm b \sin(\tau + \delta')$$

➢ The original equations for the optical field:

$$\frac{E_x}{E_{0x}} = \cos(\tau + \delta_x)$$
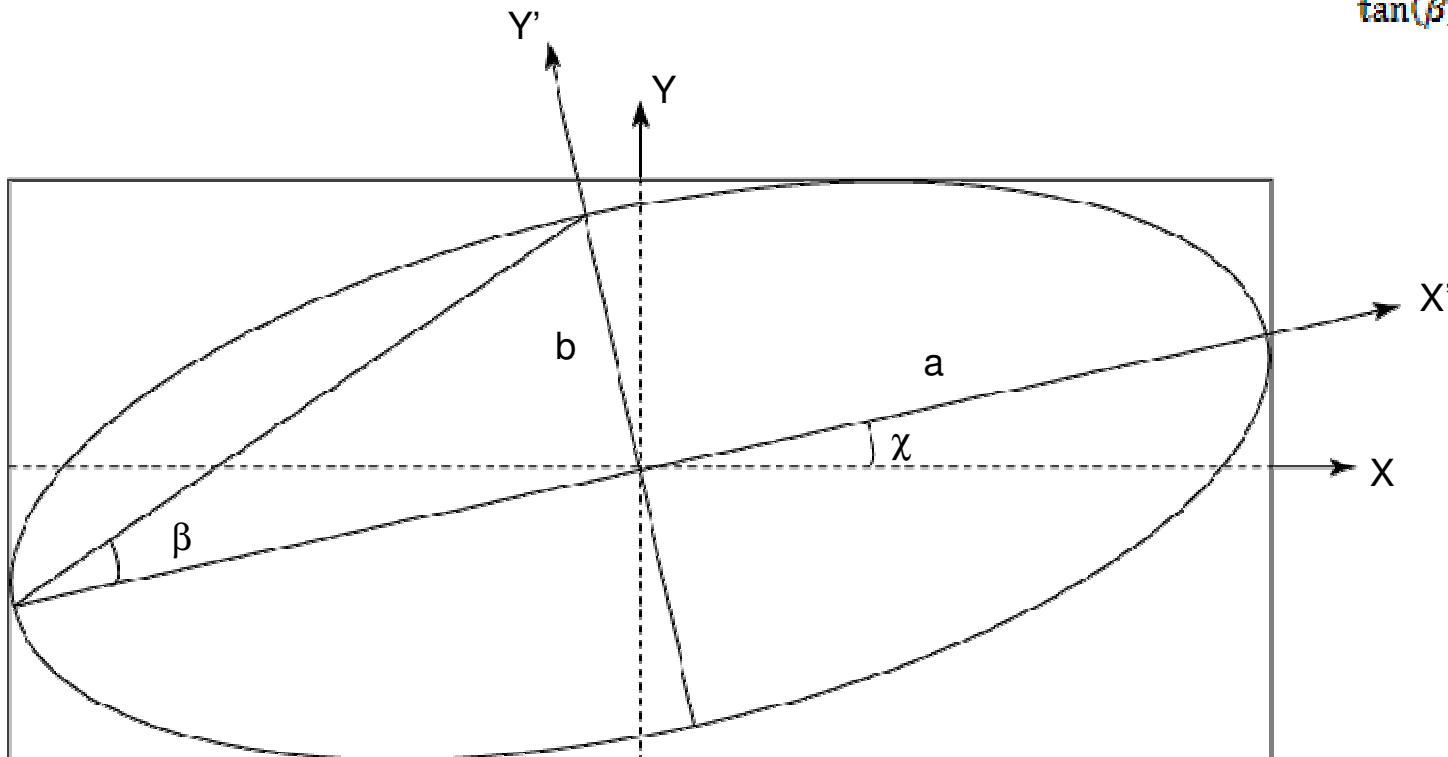$$\frac{E_y}{E_{0y}} = \cos(\tau + \delta_y)$$

➢ And a whole lot of algebra, the following can be derived:

$$\tan(2\chi) = \frac{2 E_{0x} E_{0y} \cos(\delta)}{E_{0x}^2 + E_{0y}^2}$$

$$\tan(\beta) = \pm \frac{b}{a}$$

Part II: The Jones Matrix Calculus

- The Jones Vector
- Linear, Circular, and Elliptical Polarization
  Representations
- Polarizer, Retarder, and Rotator Matrices

➢ We can also write the plane wave components of the optical field in terms of complex quantities:

$$\tilde{E}_x(z,t) = E_{0x}e^{i(kz-\omega t+\delta_x)}$$

$$\tilde{E}_y(z,t) = E_{0y}e^{i(kz-\omega t+\delta_y)}$$

➢ The total complex field for an electromagnetic wave propagating in the z-direction can then be written as:

$$\tilde{E} = E_{0x}e^{i(kz-\omega t+\delta_x)}\hat{x} + E_{0y}e^{i(kz-\omega t+\delta_y)}\hat{y} = \left[E_{0x}e^{i(\delta_x)}\hat{x} + E_{0y}e^{i(\delta_x)}\hat{y}\right]e^{i(kz-\omega t)} = \tilde{E}_0 e^{i(kz-\omega t)}$$

➢ If you recall from the polarization ellipse, the polarization state of the wave is completely determined by the amplitudes (E_{0x}/E_{0y}), and phases (δ_x/δ_y).

The bracketed term in the above equation ($\tilde{E}_0$) referred to as the complex amplitude, contains all the information needed to determine the state of polarization of the optical field.

This complex amplitude can be written as a two element matrix, known as a Jones Vector:

$$\tilde{E} = \begin{bmatrix} \tilde{E}_{0x} \\ \tilde{E}_{0y} \end{bmatrix} = \begin{bmatrix} E_{0x}e^{i\delta_x} \\ E_{0y}e^{i\delta_x} \end{bmatrix}$$

➢ It is important to realize that E_{0x} and E_{0y} are REAL quantities, the presence of the exponent with imaginary arguments causes E_x and E_y to be imaginary.

25

➢ Before determining the Jones vectors for the different polarization states, it is customary to normalize the vectors to gain simpler expressions.

➢ This is done by dividing the vector elements by a scalar (real or complex) quantity such that the sum of the squares of the components is 1.

➢ The Jones vector for linear horizontal polarized (LHP) light (E_y = 0, suppress δ_x since wave is unimodular):

$$\tilde{E} = \begin{bmatrix} \tilde{E}_{0x} \\ \tilde{E}_{0y} \end{bmatrix} = \begin{bmatrix} E_{0x}\,e^{i\delta_x} \\ E_{0y}\,e^{i\delta_x} \end{bmatrix} = \begin{bmatrix} E_{0x} \\ 0 \end{bmatrix}$$

➢ So to find the normalization factor (I understand this is a completely trivial example, but it's important to see how the method evolves, and we'll get more complicated later):

$$\left| E_{0x}^2 \right| + \left| 0^2 \right| = E_{0x}^2 = 1$$

➢ So that the normalized Jones vector for LHP is simply:

$$\tilde{E} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

➢ That wasn't a very interesting example, but we'll get there, so lets do the case for linear +45 polarized light.

➢ The Jones vector ($E_x = E_y = E_0$, $\delta_x = \delta_y = 0$):

$$\tilde{E} = \begin{bmatrix} E_{0x}e^{i\delta_x} \\ E_{0y}e^{i\delta_y} \end{bmatrix} = \begin{bmatrix} E_0 \\ E_0 \end{bmatrix}$$

➢ To determine the normalization factor:

$$\left| E_0^2 \right| + \left| E_0^2 \right| = 2E_0^2 = 1 \; the \; normalization \; factor \; is \; therefore \; \frac{1}{\sqrt{2}}$$

➢ The normalized Jones vector for linear +45 is then:

$$\tilde{E} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

➢ For linear -45, the Jones vector ($Ex = Ey = E_0$, $\delta_y - \delta_x = \pi$):

$$\tilde{E} = \begin{bmatrix} E_{0x}e^{i\delta_x} \\ E_{0y}e^{i\delta_x} \end{bmatrix} = \begin{bmatrix} E_0 \\ E_0 e^{i\pi} \end{bmatrix} = \begin{bmatrix} E_0 \\ -E_0 \end{bmatrix} \; since \; e^{i\pi} = \cos(\pi) + i\sin(\pi)$$

➢ To determine the normalization factor:

$$\left|E_0^2\right| + \left|-E_0^2\right| = 2E_0^2 = 1 \; the \; normalization \; factor \; is \; therefore \; \frac{1}{\sqrt{2}}$$

➢ The normalized Jones vector for linear -45 is then:

$$\tilde{E} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

➢ now for left-hand circular polarized (LCP) light. In this case, $E_{0x} = E_{0y} = E_0$ and $\delta y - \delta x = +90°$:

$$\tilde{E} = \begin{bmatrix} E_{0x}e^{i\delta_x} \\ E_{0y}e^{i\delta_x} \end{bmatrix} = \begin{bmatrix} E_0 \\ E_0e^{i\frac{\pi}{2}} \end{bmatrix} = \begin{bmatrix} E_0 \\ iE_0 \end{bmatrix} \; since \; e^{i\frac{\pi}{2}} = cos(\pi/2) + isin(\pi/2)$$

➢ To determine the normalization factor:

$$\left|E_0^2\right| + \left|iE_0^2\right| = 2E_0^2 = 1 \; the \; normalization \; factor \; is \; therefore \; \frac{1}{\sqrt{2}}$$

➢ The normalized Jones vector for LCP is then written:

$$\tilde{E} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ +i \end{bmatrix}$$

➢ **QUIZ 1**: What is the form for right-hand circular polarized (RCP) light?

$$\tilde{E} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

➢ A useful property of the Jones vectors is that we can superpose them to create different polarization states.

➢ For example, what happens when we superpose the Jones vectors for LHP, and LVP?

$$\tilde{E} = \tilde{E}_{LHP} + \tilde{E}_{LVP} = \begin{bmatrix} E_{0x} e^{i\delta_x} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ E_{0y} e^{i\delta_y} \end{bmatrix} = \begin{bmatrix} E_{0x} e^{i\delta_x} \\ E_{0y} e^{i\delta_y} \end{bmatrix}$$

➢ This is the Jones vector for elliptically polarized light. Therefore, superposing two orthogonal linear polarizations gives rise to elliptically polarized light.

➢ If $E_{0x} = E_{0y}$, and $\delta_y = \delta_x$:

$$\tilde{E} = E_0 e^{i\delta_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

➢ **QUIZ 2**: Which polarization state does this Jones vector describe (absent the normalization factor)?

➢ Linear +45

30

➤ For another example, superpose LCP and RCP of equal amplitudes (we found the normalized forms back on slide 19).

$$\tilde{E} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix} = \frac{2}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

➤ **QUIZ 3**: Which polarization state does this Jones vector describe?

➤ Linear horizontal.

➤ For the final example it will be shown that elliptical polarization can be created from the superposition of LCP and RCP with unequal amplitudes.

➤ The Jones vectors for LCP and RCP with unequal amplitudes can be written as:

$$\tilde{E}_{LCP} = a \begin{bmatrix} 1 \\ i \end{bmatrix} \quad and \quad \tilde{E}_{RCP} = b \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

➤ Adding them together:

$$\tilde{E} = \tilde{E}_{LCP} + \tilde{E}_{RCP} = \begin{bmatrix} a + b \\ i(a - b) \end{bmatrix} = \begin{bmatrix} \tilde{E}_{0x} \\ \tilde{E}_{0y} \end{bmatrix}$$

➤ Writing this out in component form:

$$\tilde{E}_{0x} = a + b$$
$$\tilde{E}_{0y} = (a - b)e^{i\frac{\pi}{2}}; \quad remember, e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

➤ And, restoring the propagator term (kz-ωt):

$$\tilde{E}_{0x} = (a + b)e^{i(kz - \omega t)}$$

$$\tilde{E}_{0y} = (a - b)e^{i(kz - \omega t + \frac{\pi}{2})}$$

32

➢ Taking the real part of the complex representations (setting τ=kz-ωt):

$$E_x(z,t) = E_{0x}\cos(\tau)$$

$$E_y(z,t) = E_{0y}\cos\left(\tau + \frac{\pi}{2}\right) = E_{0y}\sin(\tau)$$

➢ Now, keeping in mind the derivation of the polarization ellipse (slides 4-6), we square E_x and E_y, and add them together:

$$\frac{E_x^2(z,t)}{(a+b)^2} + \frac{E_y^2(z,t)}{(a-b)^2} = 1$$

➢ **QUIZ 4**: Is a four part question:

      1.   What is this an equation for?
      2.   what do (a+b) and (a-b) represent physically?
      3.   Which direction, if any, is the resultant vector of E_x and E_y rotating?
      4.   Given your answer to (1), is it rotated?

➢ **QUIZ 4**: Question 1, what is this an equation for?

   ➢ An ellipse.

➢ **QUIZ 4**: Question 2, what do (a+b) and (a-b) represent physically?

   ➢ The major and minor axes respectively of the ellipse.

➢ **QUIZ 4**: Question 3, Which direction is the resultant vector of E_x and E_y rotating?

   ➢ Counterclockwise.

➢ **QUIZ 4**: Question 4, Given your answer to (1),
         is it rotated?

   ➢ No.

➢ Go back to slides 6 through 13 .

➢ There are 3 types of devices that will transmit, but modify the state of polarization of light:

1. Polarizer: Removes all or most of one vibration while transmitting the other.
2. Phase Retarder: Introduces a phase difference between the two components.
3. Rotator: Rotates the direction of linearly polarized light.

➢ The action of these 3 devices can be described by 2 X 2 matrices, called Jones Matrices.

➢ In order to do this it is assumed that the components of a beam emerging from a polarizing element are linearly related to the components of the incident beam:

$$E'_x = j_{xx} E_x + j_{xy} E_x$$
$$E'_y = j_{yx} E_y + j_{yy} E_y$$

➢ Where E_x', E_y' are the components of the emerging beam, Ex/Ey are those of the incident, and the j_{ik}, l,k = x,y are the transformation factors.

➢ This can be re-written in matrix form:

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{bmatrix} j_{xx} & j_{xy} \\ j_{yx} & j_{yy} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} \quad or \quad E = JE \ where \ J = \begin{bmatrix} j_{xx} & j_{xy} \\ j_{yx} & j_{yy} \end{bmatrix} \ is \ the \ Jones \ matrix.$$

➢ We now determine the Jones matrix for a polarizer, which has the characteristics:

$$E'_x = p_x E_x$$
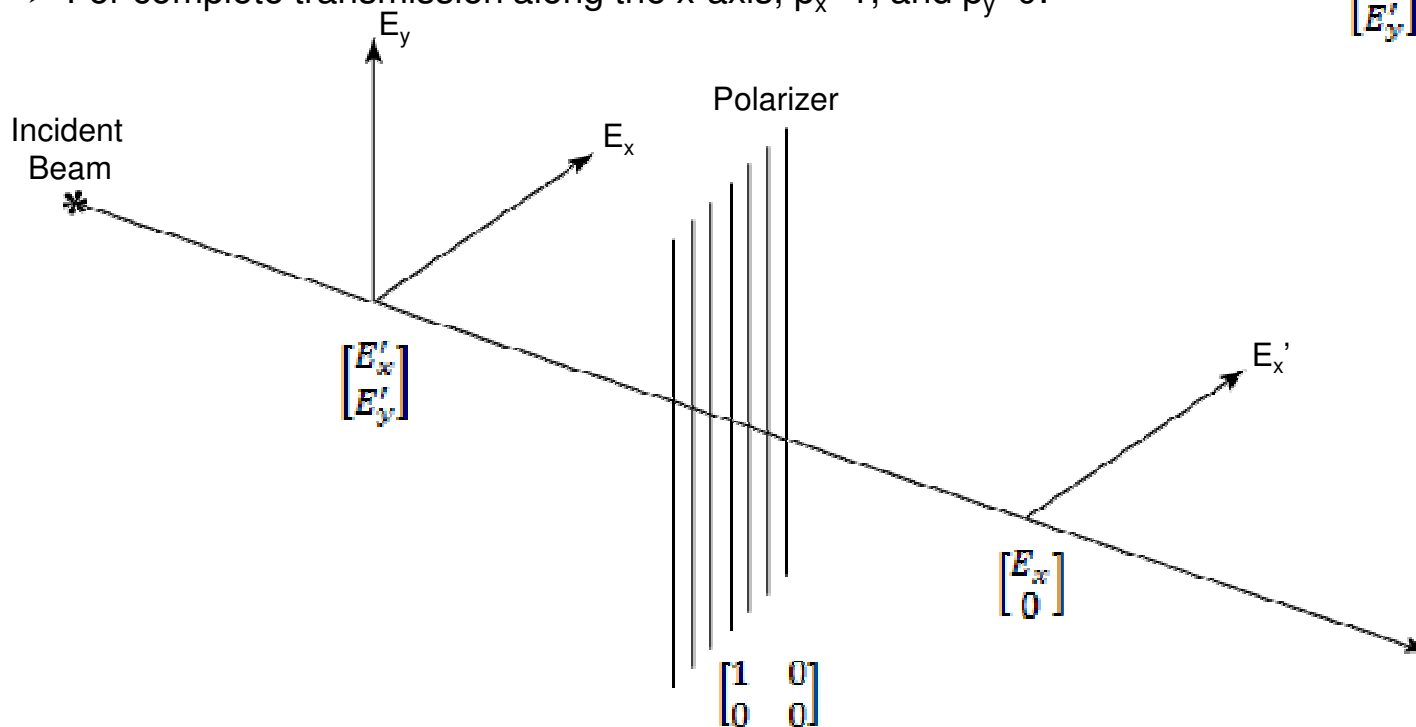$$E'_y = p_y E_y \quad where \ 0 \leq p_{x,y} \leq 1$$

➢ For complete transmission, $p_{x,y} = 1$, and for complete attenuation $p_{x,y} = 0$.

➢ In general then, the Jones vector for a polarizer can be expressed as:

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{bmatrix} p_x & 0 \\ 0 & p_y \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

➢ For complete transmission along the x-axis, $p_x=1$, and $p_y=0$:

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_x \\ 0 \end{bmatrix}$$

E_y

Incident
Beam

E_x

Polarizer

E_x'

$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix}$

$\begin{bmatrix} E_x \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

36

➢ We now determine the Jones matrix for a Phase Retarder.

➢ As mentioned in your text, the phase retarder behaves as if it were to advance one component relative to the other (fast axis and slow axis), thus introducing some total phase shift ϕ between the two orthogonal components.

➢ This behavior can be described as advancing one of the axis by $\phi/2$, and retarding the other by $-\phi/2$.

$$E'_x = e^{i\frac{\phi}{2}}E_x$$

$$E'_y = e^{-i\frac{\phi}{2}}E_y$$

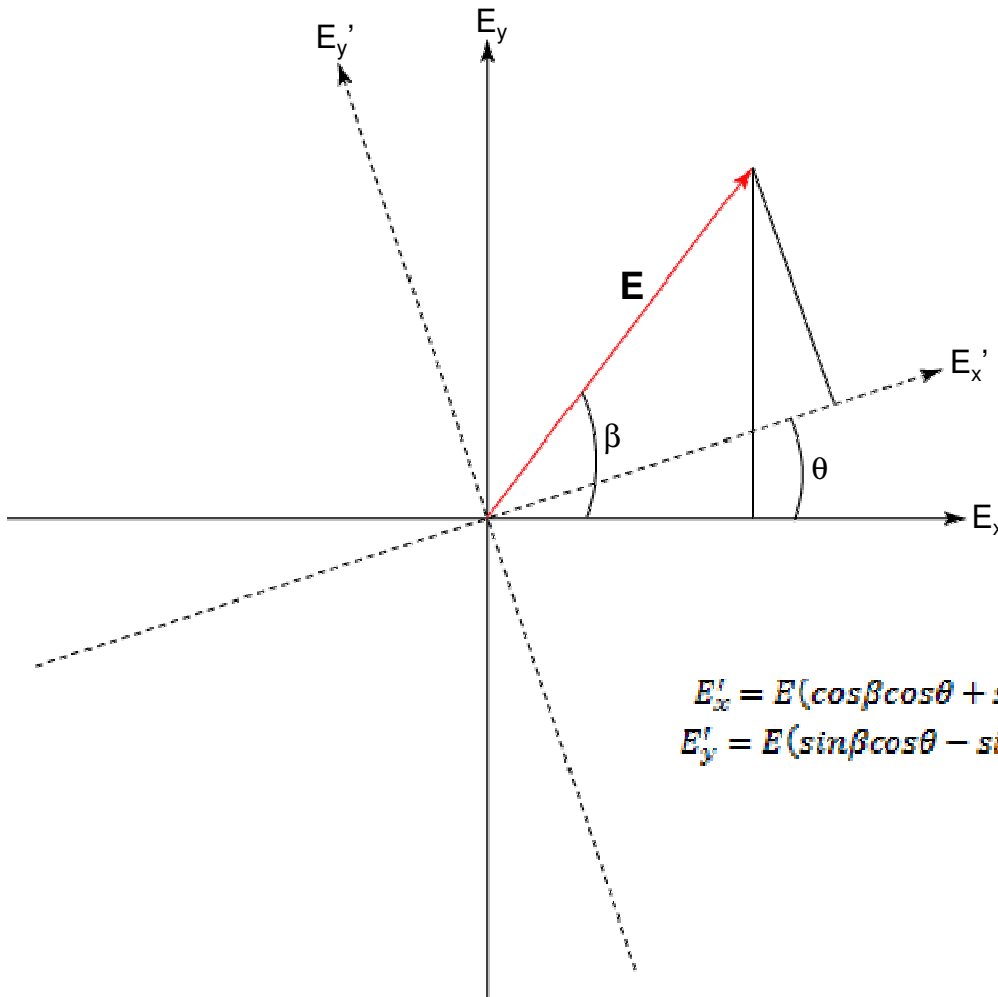➢ Where I have chosen the y-axis as the fast axis. The Jones formalism is then:

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{bmatrix} e^{i\frac{\phi}{2}} & 0 \\ 0 & e^{-i\frac{\phi}{2}} \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

➢ Two of the most common types of phase retarders are the quarter-wave (ϕ=90), and the half-wave (ϕ=180). These Jones matrices can be expressed as:

$$J\left(\frac{\lambda}{4}\right) = \begin{bmatrix} e^{i\frac{\pi}{4}} & 0 \\ 0 & e^{-i\frac{\pi}{4}} \end{bmatrix} = e^{i\frac{\pi}{4}}\begin{bmatrix} 1 & 0 \\ 0 & e^{-i\frac{\pi}{2}} \end{bmatrix} = e^{i\frac{\pi}{4}}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

$$J\left(\frac{\lambda}{2}\right) = \begin{bmatrix} e^{i\frac{\pi}{2}} & 0 \\ 0 & e^{-i\frac{\pi}{2}} \end{bmatrix} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = i\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

➤ We now determine the Jones matrix for a rotator.

➤ In this case, the optical device rotates the orthogonal components Ex/Ey through an angle θ.

➤ from the figure, θ describes rotation of E_x to E_x', and E_y to E_y'. β is the angle between **E** and E_x.

➤ In the E_x'/E_y' coordinate system:

$$E'_x = E\cos(\beta - \theta)$$
$$E'_y = E\sin(\beta - \theta)$$

➤ In the E_x/E_y coordinate system:

$$E_x = E\cos\beta$$
$$E_y = E\sin\beta$$

➤ Expanding the trig functions for E_x'/E_y':

$$E'_x = E\cos(\beta - \theta) = E(\cos\beta\cos\theta + \sin\beta\sin\theta)$$
$$E'_y = E\sin(\beta - \theta) = E(\sin\beta\cos\theta - \sin\theta\cos\beta)$$

➤ Using the equations for Ex/Ey and collecting like terms:

$$E'_x = E(\cos\beta\cos\theta + \sin\beta\sin\theta) = (E\cos\beta)\cos\theta + (E\sin\beta)\sin\theta = E_x\cos\theta + E_y\sin\theta$$
$$E'_y = E(\sin\beta\cos\theta - \sin\theta\cos\beta) = (E\sin\beta)\cos\theta - (E\cos\beta)\sin\theta = -E_x\sin\theta + E_y\cos\theta$$

38

➢ From the previous slide:
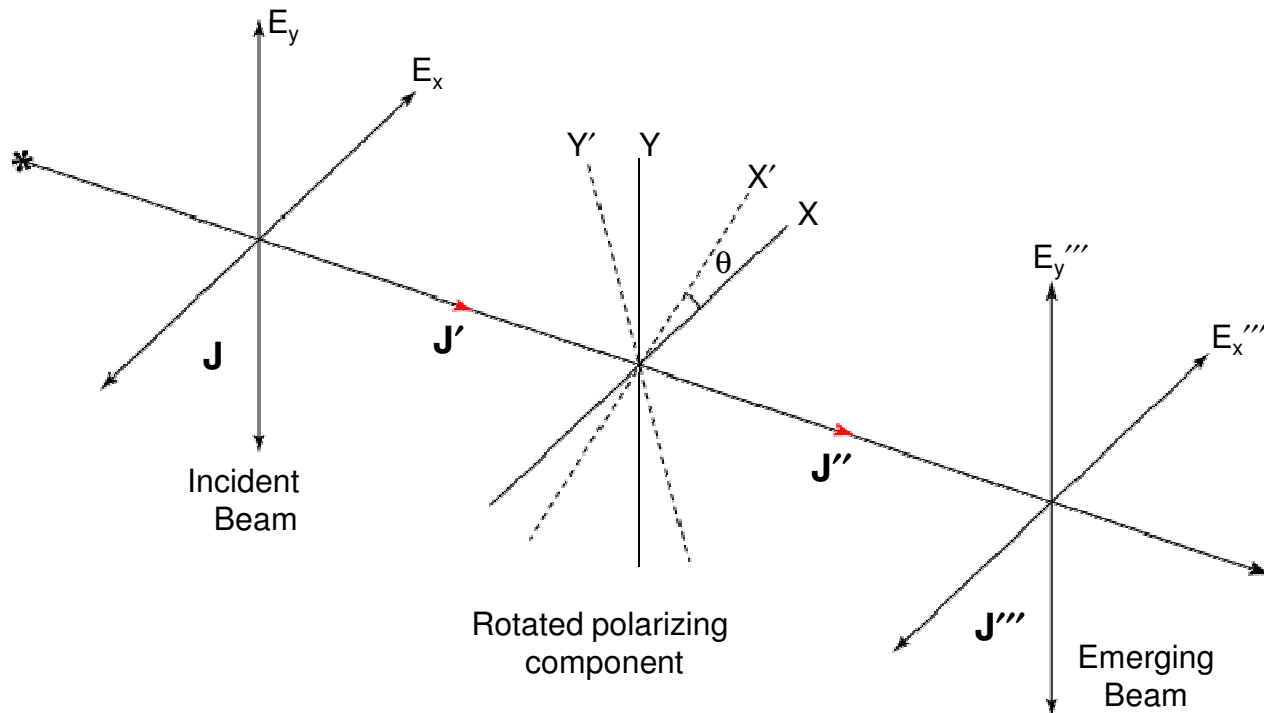
$$E'_x = E_x \cos\theta + E_y \sin\theta$$
$$E'_y = -E_x \sin\theta + E_y \cos\theta$$

➢ We can then write for the Jones formulism:

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} \text{ so that } J(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

➢ Problem 14-1 from your textbook asks you to derive Equation 14-15. I'll do that now.

➢ The problem is to derive a general matrix representing a linear polarizer with its transmission axis at some arbitrary angle θ. The problem can be set up by the following:
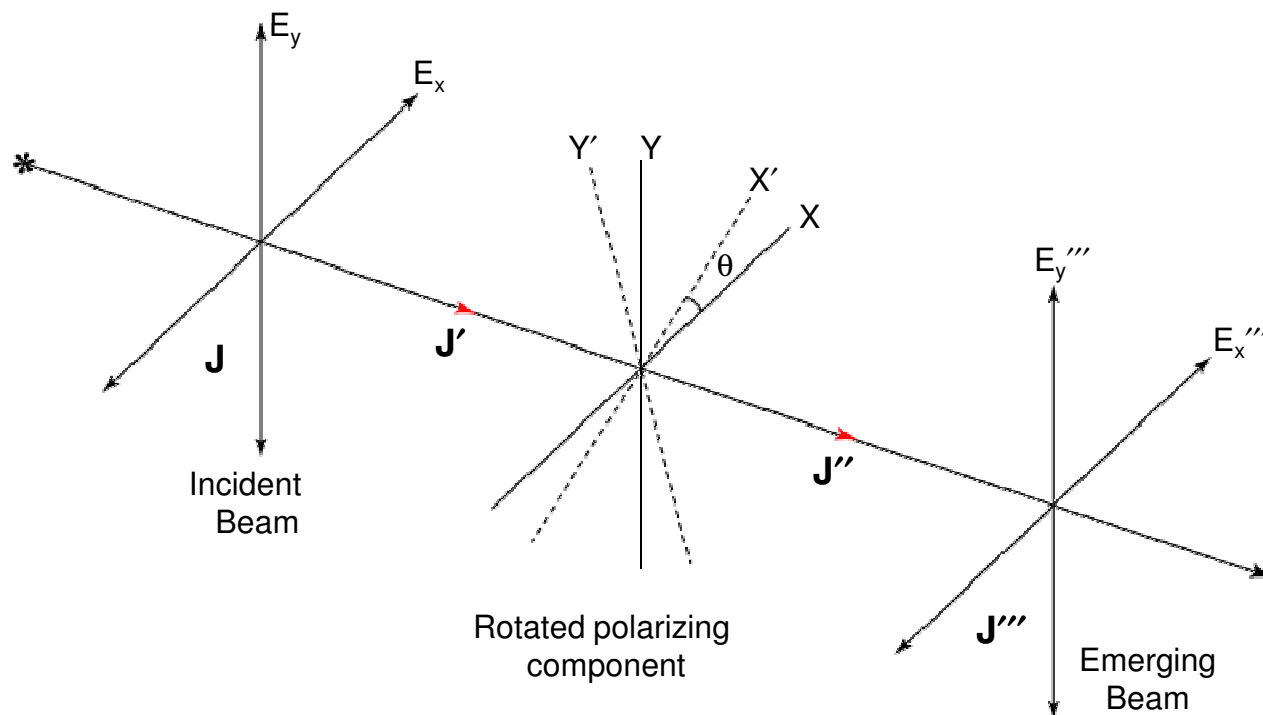

➢ Ex/Ey described by incident Jones vector **J**.

➢ **J′** is incident components along X′/Y′.

➢ **J″** is Jones vector for emergent beam from polarizer.

➢ **J‴** is Jones vector for emergent.

➤ The components of the incident beam, **J**, along the x′/y′ axes can be written in terms of the Jones vector as:

$$J' = M_\theta J \text{ where } M_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

➤ Where M_θ is the Jones matrix for rotation, and **J′** is the Jones vector for the beam whose axes are along x′/y′.

➤ The **J′** beam now interacts with the polarizing element whose matrix is M_{pol}. The Jones vector, J″, or the beam emerging from the rotated polarized component is:

$$J'' = M_{pol}J' = M_{pol}M_\theta J \text{ where } M_{pol} = \begin{bmatrix} p_x & 0 \\ 0 & p_y \end{bmatrix}$$

E_y

E_x

Y′ Y

X′

X

θ

E_y'''

E_x'''

J

J′

J″

J‴

Incident
Beam

Rotated polarizing
component

Emerging
Beam

40

➢ Finally, the components of the beam emerging from the rotated polarizer (J''), must be must be taken along the original x/y axes. This is described by a rotation of J'' through $(-\theta)$, back to the original x/y axes.
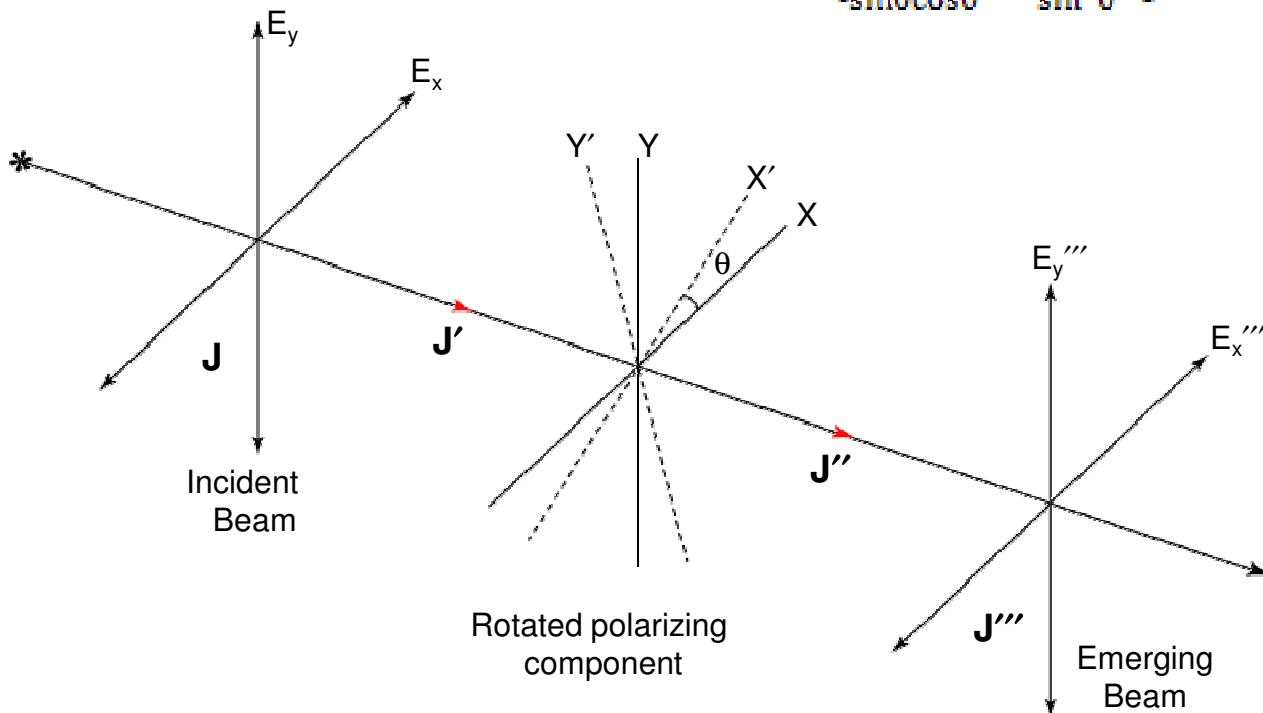
$$J'''' = M_{\theta-}J'' = M_{\theta-}M_{pol}M_{\theta}J \text{ where } M_{\theta-} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

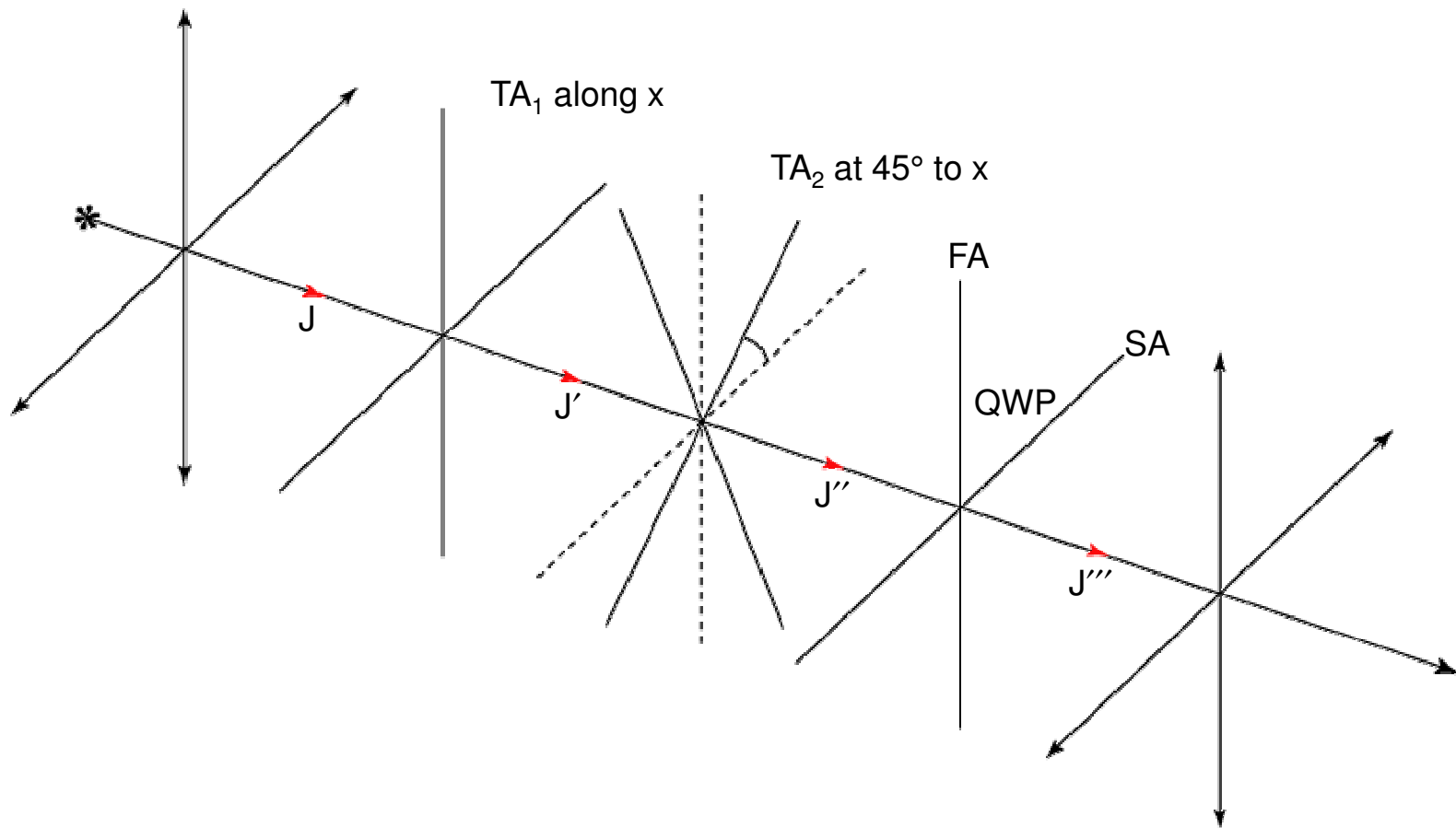➢ The Jones matrix for an arbitrarily rotated polarizer is then:

$$M = M_{\theta-}M_{pol}M_{\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} p_x & 0 \\ 0 & p_y \end{bmatrix} * \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} p_x\cos^2\theta + p_y\sin^2\theta & (p_x - p_y)\sin\theta\cos\theta \\ (p_x - p_y)\sin\theta\cos\theta & p_x\sin^2\theta + p_y\cos^2\theta \end{bmatrix}$$

➢ Assuming the ideal case for transmission along the x-axis, px=1/py=0:

$$M = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Incident Beam

Rotated polarizing component

Emerging Beam

➢ Problem 14-13: Light is first sent through an ideal linear polarizer with transmission axis along the x-axis. It is then sent through a linear polarizer rotated at 45°, and finally through a quarter wave-plate with slow axis horizontal. Use the Jones matrices to determine and describe the product.

TA$_1$ along x

TA$_2$ at 45° to x

FA

J

J′

SA

QWP

J″

J‴

➢ Problem 14-13: The problem is worded somewhat strangely, but I take it to mean that J′ is linear horizontal polarized light because of the first polarizer. Therefore we don't care what J is.

➢ Need to determine J″, J‴. From the diagram on the previous slide:

$$J' = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$J'' = M_{pol45} * J'$$
$$J''' = M_{QWP} * J''$$

➢ For the Jones matrices:

$$M_{pol45} = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

$$M_{QWP} = e^{i\frac{\pi}{4}} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

➢ Solving for the Jones vectors:

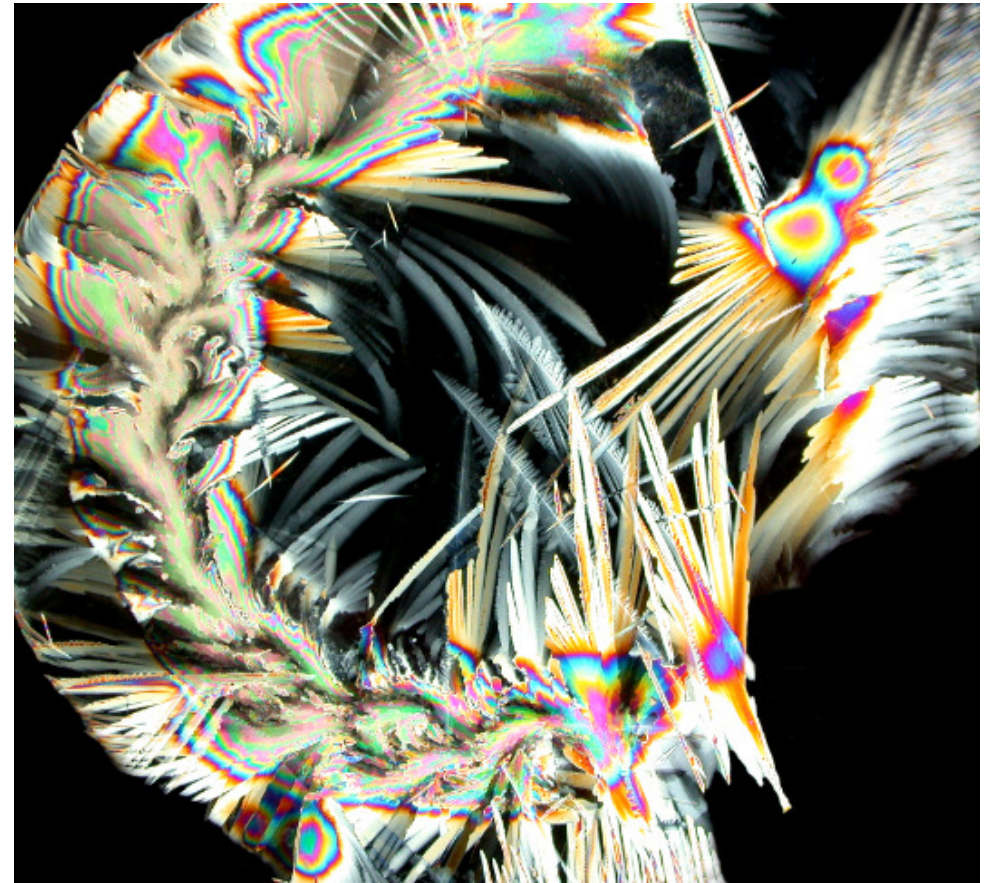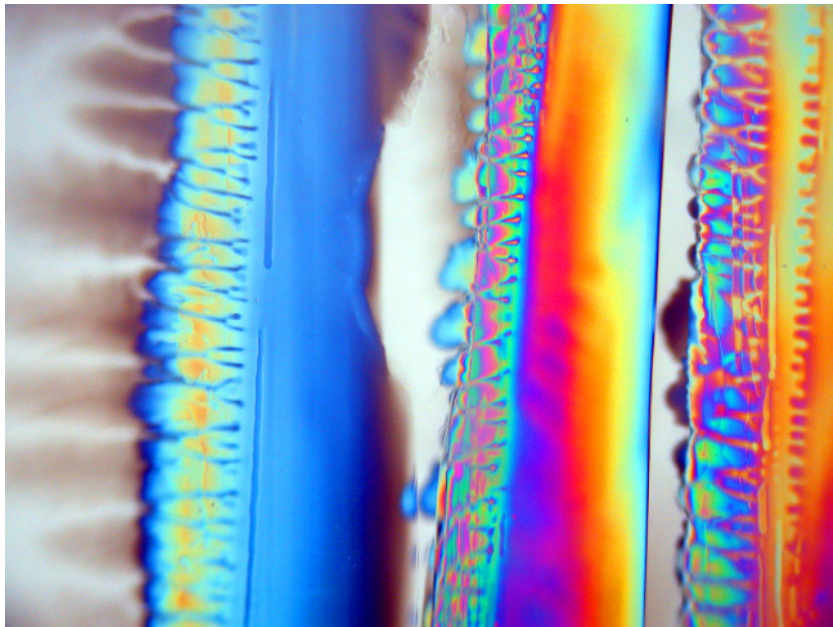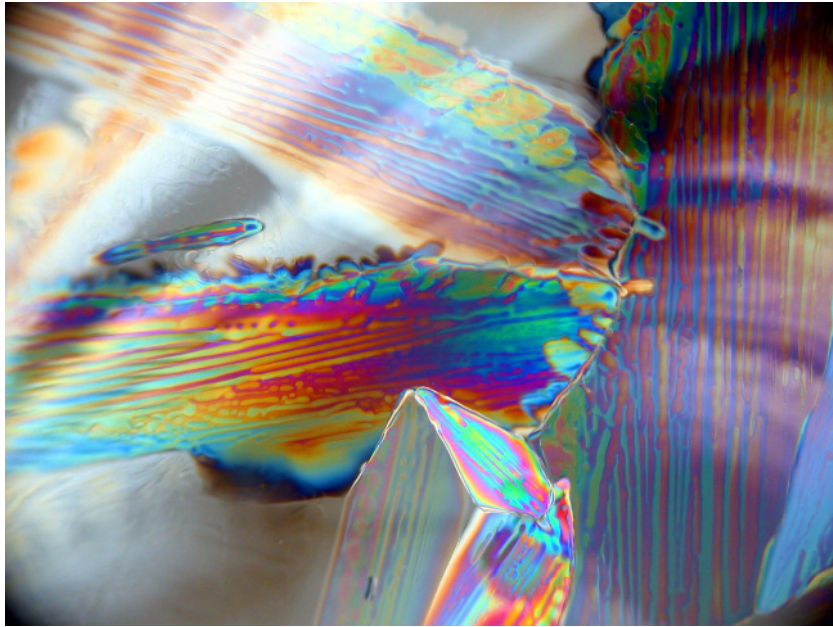$$J''' = M_{QWP} * J'' = M_{QWP} * M_{pol45} * J' = \frac{e^{i\frac{\pi}{4}}}{2} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

➢ Which we know from table 14.1 is RCP. I'll let you go into more detail if needed.

Part III: Something Cool, Something Silly, and
         Something to learn how to do.

- Polarized Art
- Polarized Clock
- Can we train our eyes to see polarization?

➤ NASA scientist Peter Wasilewski painting with ice and light:**Peter.J.Wasilewski.1@gsfc.nasa.gov**

➢ The Aurora Clock by Chrono Art (http://homepage.mac.com/chronart1/index.html):

➢ Can we train our eyes to see polarized light? Apparently…

➢ "Haidingers Brush: How to see polarization with the naked eye."

http://www.polarization.com/haidinger/haidinger.html

- References:

  - Polarized Light by Dennis Goldstein
  - Optics by Eugene Hecht
  - Principles of Optics by Born & Wolf
  - Polarization of Light: From Basics to Instruments by N. Manset CFHT
    (Do a google search to get the Powerpoint, link is to long to paste)