CHAPTER 2

DESCRIPTION AND CRITIQUE OF PREVIOUS STUDIES

2.1 Introduction

In this chapter a brief description of each of the studies listed in footnote 1 on page 1 is given. The theoretical model of each of the studies is described, but no attempt is made to present the empirical results, since the data used and the periods of estimation vary widely from study to study. The theoretical models of many of the studies are quite similar, and for ease of exposition the basic model which is common to these studies is presented first. Having done this, it is relatively easy to see how the individual models differ from the basic model and thus from one another.

After summarizing the studies which develop and use a model similar to the basic model, an evaluation of these studies is made. The necessity of making some kind of a cost-minimizing assumption with respect to the workers–hours mix is emphasized, and the studies are criticized for using seasonally adjusted data or seasonal dummy variables. Results are then presented of estimating the basic model using the same data and periods of estimation which are used to estimate the model developed in this study. The results strongly suggest that the basic model is incorrectly specified, even under a slightly different interpretation of some of the coefficient estimates.

The chapter concludes with a description of those studies listed in footnote 1 on page 1 which are not based on a model similar to the basic model. Included in this list are those studies which do not develop a theoretical model of the short-run demand for workers at all, but instead examine output per worker or per man hour directly. In this section the Wilson and Eckstein model is examined in somewhat more detail than the others.

In any study of short-run behavior it is important to make explicit the time periods to which the variables refer. This is especially true in a study such as this one where monthly data are used. If, for example, M_t is used to denote the number of workers employed, it is important to know whether it refers to the number employed at the beginning or end of period t, to the average number employed during period t, or to the number employed

TABLE 2.1

Notation used in ch. 2

Y_t	the amount of output produced during period t.
L_t	the amount of labor services employed during period t.
M_t	the number of workers employed during period t.
H_t	the number of hours worked per worker during period t.
K_t	the stock of capital during period t.
T_t	the level of technology during period t.
L^*_t	the amount of labor services needed during period t, given Y_t, K_t, and T_t.
HS_t	the standard (as opposed to overtime) number of hours of work per worker during period t.

at some other time during period t. When quarterly data are used this distinction is not as critical, and the question has largely been ignored in previous studies. Consequently, in this chapter the notation will be rather loose and reference will be made merely to the values of variables "during period t". The symbols used for the various variables are presented in the text as the variables are introduced, but for reference purposes the symbols for the more important variables are presented in table 2.1. Beginning in ch. 3, the notation will be made more precise.

2.2 Description of the models similar to the basic model

2.2.1. The basic model

The model presented here as the "basic model" makes no assumption about cost-minimizing behavior of firms with respect to the short-run workers–hours mix. It is thus inconsistent, as will be seen in § 2.3. Because some of the studies described below make no assumption about cost-minimizing behavior of firms, the basic model was framed in this way as well. It will be modified in § 2.3 to correct for this inconsistency.

The basic model begins by postulating a short-run production function, where the amount of output produced during period t, Y_t, is taken to be a function of the amount of labor services used during period t, L^*_t, the stock of capital on hand during period t, K_t, and the existing level of technology, T_t:

$$Y_t = F(L^*_t, K_t, T_t). \tag{2.1}$$

Specifically, it is assumed that the production function is of the Cobb–

Douglas form and that technology grows smoothly over time at rate γ. Under these assumptions the production function (2.1) can be written

$$Y_t = AL_t^{*\alpha} K_t^{\beta} e^{\gamma t}. \tag{2.2}$$

The elasticity of output with respect to labor services is α, and if there are diminishing returns to labor in the short run, α is less than one. If the assumption of constant returns to scale is made, then $\alpha + \beta = 1$.

The firm is assumed to take the amount of output produced, the capital stock, and the level of technology as given in the short run and to adjust its employment according to changes in the three exogenous variables. The production function (2.2) can be solved for L_t^* to yield

$$L_t^* = A^{-1/\alpha} Y_t^{1/\alpha} K_t^{-\beta/\alpha} e^{-(\gamma/\alpha)t}. \tag{2.3}$$

Given the stock of capital and the level of technology, L_t^* is the amount of labor services required for the production of Y_t. A change in the amount of output produced, the stock of capital, or the level of technology from one period to the next will lead to a change in L_t^*. Rapid adjustments in L_t^* may be costly for the firm, however, and only part of the change in L_t^* may be made during any one period. To take this into account an adjustment process of the following form is postulated:

$$L_t/L_{t-1} = (L_t^*/L_{t-1})^{\lambda}, \quad 0 \leqq \lambda \leqq 1. \tag{2.4}$$

L_t is the amount of labor services employed during period t, whereas L_t^* is the amount of labor services actually required in the production process during period t. The adjustment process (2.4) implies that only part of any required change in labor services will be made in any one period. A ten-percent increase in L_t^*/L_{t-1}, for example, will lead to a less than ten-percent increase in L_t/L_{t-1}, unless of course λ equals one.

Solving for L_t^* in (2.4), substituting into (2.3), and taking logarithms yields

$$\log L_t - \log L_{t-1} = -\frac{1}{\alpha}\lambda \log A + \frac{1}{\alpha}\lambda \log Y_t - \frac{\beta}{\alpha}\lambda \log K_t$$

$$-\frac{\gamma}{\alpha}\lambda t - \lambda \log L_{t-1}. \tag{2.5}$$

Given time series on the amount of labor services employed, the amount of output produced, and the stock of capital, eq. (2.5) can be estimated directly, and as is seen below, many empirical studies of the short-run demand for employment have been concerned with estimating equations very similar to (2.5).

2.2.2. The Brechling model

Brechling's model (BRECHLING, 1965) is similar to the basic model above, except that he does make an assumption regarding firms' cost-minimizing behavior with respect to the workers–hours mix. He begins by postulating a short-run production function like (2.1), where the amount of output produced, the stock of capital, and the level of technology are assumed to be exogenous. He then postulates that the amount of labor services, L_t^*, in the production function is some function of the number of workers employed, M_t, and the average number of hours worked per worker, H_t:

$$L_t^* = f(M_t, H_t). \tag{2.6}$$

Brechling assumes that there are two hourly wage rates per period t, w_{1t} and w_{2t}. w_{1t} is the rate which is payable up to the standard number of hours of work per worker during period t, denoted as HS_t, and w_{2t} is the overtime rate. The total wage bill (short-run cost function) during period t is then

$$W_t = (H_{1t}\, w_{1t} + H_{2t}\, w_{2t})M_t. \tag{2.7}$$

W_t is the total wage bill, M_t is again the number of workers employed during period t, and H_{1t} and H_{2t} are the average number of hours worked per worker during period t for standard and overtime pay respectively.

Given the amount of labor services needed during period t, L_t^*, the wage bill (2.7) can be minimized with respect to M_t and with respect to the average number of hours worked per worker, H_t. The cost-minimizing number of workers, denoted as M_t^d, turns out to be a function of L_t^*, HS_t, and w_{1t}/w_{2t}:[1]

$$M_t^d = g(L_t^*, HS_t, w_{1t}/w_{2t}). \tag{2.8}$$

[1] BRECHLING (1965, p. 190, footnote 1) points out that for a unique cost-minimizing solution to exist, L^*_t cannot equal M_tH_t in eq. (2.6), i.e., labor services cannot be approximated by man hours. It should also be pointed out that since the iso-cost curve has a kink in it at the point where H_t equals HS_t in the iso-quant–iso-cost diagram for M_t and H_t, it is likely, given reasonably smooth iso-quant curves, that the cost-minimizing solution will be at the point where $H^d{}_t$ equals HS_t. In other words, it is likely that the cost-minimizing number of hours worked per worker, $H^d{}_t$, will be equal to the standard number of hours of work per worker.

Solving for L_t^* in the production function (2.1) yields

$$L_t^* = G(Y_t, K_t, T_t), \tag{2.9}$$

and substituting (2.9) into (2.8) yields

$$M_t^d = g(Y_t, K_t, T_t, HS_t, w_{1t}/w_{2t}). \tag{2.10}$$

Brechling assumes that g is a linear function and that the ratio of the standard wage rate to the overtime rate, w_{1t}/w_{2t}, is constant over time and thus can be ignored. He assumes an adjustment process like (2.4) of the basic model for M_t^d:[1]

$$M_t - M_{t-1} = \lambda(M_t^d - M_{t-1}), \quad 0 \leqq \lambda \leqq 1. \tag{2.11}$$

The final equation which he estimates is like eq. (2.5) of the basic model with M_t replacing L_t as the labor variable, except that the variables are not in log form and a term in HS_t has been added. (HS_t has fallen slowly over time in the United Kingdom.) In addition, Brechling adds the variable t^2 to his equation to allow for the possibility that technical progress has been accelerating over time, and he adds the change in output, $Y_t - Y_{t-1}$, to the equation, arguing that firms may build up their labor requirements in anticipation of high levels of activity.[2]

BRECHLING and O'BRIEN (1967) have gone on to estimate an equation like (2.5) of the basic model (this time in log form and without the capital stock variable) for a number of different countries and have analyzed the differences in results across countries.

2.2.3. The Ball and St Cyr model

Ball and St Cyr's model (BALL and ST CYR, 1966) is very similar to Brechling's model with a few modifications. They approximate capital stock by an exponential trend and assume that labor services, L_t^*, can be adequately

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[1] Brechling gives empirical results for both the linear and log forms of his equations. In this discussion attention is concentrated on the linear version of his model, since this is the version which Brechling concentrates on. The adjustment process for the linear version is thus in linear rather than ratio form.

[2] Brechling makes the assumption that $Y^e_{t+1} = Y_t + \delta(Y_t - Y_{t-1})$, where Y^e_{t+1} is the amount of output which is expected to be produced during period $t+1$. Adding Y^e_{t+1} to an equation like (2.5) introduces the additional variable $Y_t - Y_{t-1}$ in the equation. Brechling also tries in his equation a four-quarter moving average of the first differences in output.

approximated by man hours, $M_t H_t$, instead of some more complicated expression which Brechling is required to assume in eq. (2.6). M_t is again the number of workers employed, and H_t is the average number of hours worked per worker. The production function (2.2) for Ball and St Cyr is therefore of the form

$$Y_t = A(M_t H_t)^{\alpha} e^{\rho t}, \tag{2.12}$$

where ρ equals the rate of growth of technology (γ in eq. (2.2)) plus the rate of growth of the capital stock times the elasticity of output with respect to capital.

They postulate a short-run cost function of the form

$$W_t = w_{H_t} M_t H_t + F_t, \tag{2.13}$$

where w_{H_t} is the "effective wage per man hour"[1] during period t and is a function of H_t. F_t is the fixed cost during period t. Up to HS_t, the standard number of hours of work per worker during period t, the cost to the firm of one worker working one period is $w_{1t} HS_t$ (workers are assumed to be paid for HS_t hours during period t regardless of how many hours they actually work), and after that the cost is $w_{1t} HS_t + w_{2t}(H_t - HS_t)$, where again w_{1t} is the standard wage rate and w_{2t} is the overtime rate during period t.

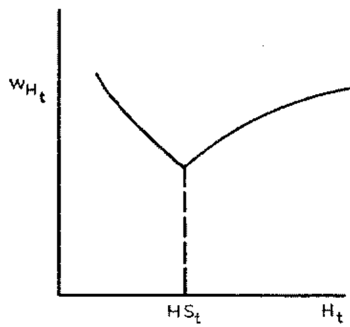

Fig. 2.1. The relationship between the effective wage per man hour, w_{H_t}, and the number of hours worked per worker, H_t, in Ball and St Cyr's model.

In figure 2.1 the relationship between w_{H_t} and H_t is depicted. Ball and St Cyr argue that a reasonable approximation for w_{H_t} is the quadratic

$$w_{H_t} = v_0 - v_1 H_t + v_2 H_t^2. \tag{2.14}$$

[1] BALL and ST CYR (1966, p. 180).

Substituting this expression for w_{H_t} into the cost function (2.13), solving for H_t in the production function (2.12) and substituting the resulting expression for H_t into the cost function, and then minimizing the resulting expression of the cost function with respect to M_t yields

$$M_t^d = 2v_2/(A^{1/\alpha}v_1)\, e^{-(\rho/\alpha)t}\, Y_t^{1/\alpha}. \tag{2.15}$$

M_t^d is the cost-minimizing number of workers. Eq. (2.15) is of the same form as eq. (2.3) of the basic model without the capital stock variable.

Ball and St Cyr then assume an adjustment process like (2.4) of the basic model for M_t^d:

$$M_t/M_{t-1} = (M_t^d/M_{t-1}),\ \ 0 \leqq \lambda \leqq 1, \tag{2.16}$$

and arrive at an estimating equation like (2.5) without the log K_t variable and with M_t being used as the labor variable in the equation.

Ball and St Cyr's results show strongly increasing returns to labor services, even when direct (as opposed to overhead) labor is considered alone, and they believe that this may be due to the fact that measured man hours, denoted as $(M_tH_t)_m$, may not at all times be a good approximation of "productive" man hours. They postulate that

$$M_tH_t = (M_tH_t)_m\,(1 - U_t)^u, \tag{2.17}$$

where U_t is the "difference between the percentage unemployment . . . and the percentage chosen to represent full employment".[1] In other words, "as unemployment rises the degree of underutilization of employed labor is likely to increase".[2] Using relation (2.17), they estimate the parameters of the production function (2.12) directly (ignoring the adjustment process and using the variable M_tH_t instead of M_t in the estimated equation) to get an alternative estimate of returns to labor. The results in general give lower estimates of returns to labor services, but of the eleven industries for which estimates are made, two of them give non-sensible results and five of the remaining nine give labor input elasticities (i.e., elasticities of output with respect to labor services) greater than one. Ball and St Cyr remain agnostic as to "the extent to which the estimated labour input elasticities are determined by the time structure of the production functions [i.e., by equations like (2.5) of the basic model, which incorporate lagged

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[1] Ball and St Cyr (1966, p. 189).

[2] Ball and St Cyr (1966, p. 189).

adjustment mechanisms like (2.4)] or a widespread propensity to hoard labour permanently [as exemplified by eq. (2.17)]".[1]

2.2.4. The Ireland and Smyth model

The Ireland and Smyth model (IRELAND and SMYTH, 1967) is a slight modification of the Ball and St Cyr model, with a different interpretation being given to the estimate of returns to labor services. Instead of a Cobb–Douglas production function, they postulate a CES production function:

$$Y_t = e^{\gamma t} A[\delta(M_t H_t)^{\alpha} + (1 - \delta)(KU)_t^{\alpha}]^{\eta/\alpha}. \tag{2.18}$$

The capital input variable, $(KU)_t$, is the capital utilized during period t instead of the actual stock of capital in existence during the period. The CES production function is homogenous of degree η, η being the measure of short-run returns to scale. Ireland and Smyth assume that the percentage change in $(KU)_t$ is proportional to the percentage change in $M_t H_t$ through time, arguing that as long as there is excess capacity this assumption seems more plausible than assuming that capital services grow at a constant rate through time, as, for example, Ball and St Cyr do.

They postulate the same short-run cost function, eqs. (2.13), (2.14), as Ball and St Cyr:

$$W_t = w_{H_t} M_t H_t = v_0(M_t H_t) - v_1(M_t H_t) H_t + v_2(M_t H_t) H_t^2. \tag{2.19}$$

Utilizing the above assumptions and minimizing W_t in eq. (2.19) with respect to M_t, Ireland and Smyth arrive at the following equation:

$$M_t^d = \text{constant} \times e^{(\gamma/\eta)t}\, Y_t^{1/\eta}. \tag{2.20}$$

They next assume the familiar lagged adjustment process (2.4) for M_t^d:

$$M_t/M_{t-1} = (M_t^d/M_{t-1})^{\lambda},\ 0 \leqq \lambda \leqq 1, \tag{2.21}$$

and arrive at an estimating equation similar to (2.5) of the basic model with M_t being used as the labor variable:

$$\log M_t - \log M_{t-1} = \text{constant} + \frac{1}{\eta}\lambda \log Y_t - \frac{\gamma}{\eta}\lambda t - \lambda \log M_{t-1}. \tag{2.22}$$

The only significant difference between eqs. (2.22) and (2.5) is that in eq. (2.22) η has replaced α in the coefficients of log Y_t and t. In the Ireland

[1] BALL and ST CYR (1966, p. 192).

and Smyth model η is the measure of short-run returns to scale, whereas α in the basic model is the elasticity of output with respect to labor alone. Most estimates of α in eq. (2.5) (or η in eq. (2.22)) turn out to be greater than one, and Ireland and Smyth argue that a more realistic interpretation of the coefficient estimates is that they are measures of short-run returns to scale rather than returns to labor alone. If, for example, there are constant returns to scale and if, as Ireland and Smyth assume, the percentage change in capital services is always proportional to the percentage change in labor services, then η equals one. To the extent that η is greater than one, there are, under these assumptions, increasing short-run returns to scale.

SMYTH and IRELAND (1967) have estimated eq. (2.22) using Australian data. The results show, on their interpretation, evidence of increasing short-run returns to scale (i.e., values of η greater than one).

2.2.5. The Solow model

Solow's model (SOLOW, 1964) is very similar to the basic model. He estimates an equation like (2.5) in both linear and log forms, trying as the labor services variable both the number of workers employed and total man hours paid-for. To the log form of his equation he adds the variable $\log Y_t - \log Y_{t-1}$, which he argues can be interpreted either as a carrier of expectations or as a variable which "simply converts a geometric distributed lag between employment and output to a slightly more general lag pattern, geometric only after the first term".[1]

It is clear from his discussion that Solow is not very satisfied with this model and the results he obtains, and in the latter part of his paper he discusses, as a possible alternative to the Cobb–Douglas production function model, a vintage capital model with fixed coefficients both *ex ante* and *ex post*.

2.2.6. The Soligo model

Soligo's model (SOLIGO, 1966) is in the spirit of the basic model. He begins by postulating a Cobb–Douglas production function like (2.2):

$$Y_t = AM_t^{*\alpha} K_t^{\beta} e^{\gamma t}, \tag{2.23}$$

where the labor input variable is taken to be the number of workers, M_t^*. He is concerned with the problem that in the short run capital may not be

[1] SOLOW (1964, p. 18).

perfectly adaptable; and if capital is not perfectly adaptable, employment will not be adjusted as much in the short run as it would if capital were perfectly adaptable.[1]

In the production function (2.23), M_t^* is the desired work force if capital were perfectly adaptable. Call M_t^d the desired work force for the capital stock in existence during period t. Soligo postulates that

$$M_t^*/M_t^d = (C_t)^v, \quad v > 0, \tag{2.24}$$

where C_t is the rate of capacity utilization during period t. What eq. (2.24) says is that the further the firm deviates from the maximum rate of capacity utilization, the greater will be the gap between the desired work force if capital were perfectly adaptable and the desired work force for the capital stock in existence. Solving for M_t^* in (2.24), substituting this expression into eq. (2.23), and then solving for M_t^d yields:

$$M_t^d = A^{-1/\alpha}\, Y_t^{1/\alpha}\, K_t^{-\beta/\alpha}\, C_t^{-v}. \tag{2.25}$$

Eq. (2.25) is similar to eq. (2.3) of the basic model with the addition of the C_t variable.

With respect to future output expectations Soligo assumes that

$$Y_{t+1}^e = Y_t(Y_t/Y_{t-1}), \tag{2.26}$$

where Y_{t+1}^e is the output expected to be produced in the following period. If output increases by one percent during period t, for example, then according to eq. (2.26) it is expected to increase by one percent again during period $t+1$. Soligo assumes that the desired work force depends on future output expectations and adds the term $(Y_{t+1}^e/Y_t)^\delta$ [which by eq. (2.26) becomes $(Y_t/Y_{t-1})^\delta$] to eq. (2.25), where δ is the "elasticity of the desired work force with respect to the predicted change in output".[2]

Soligo assumes an adjustment process like (2.4) of the basic model for M_t^d:

$$M_t/M_{t-1} = (M_t^d/M_{t-1})^\lambda, \quad 0 \leqq \lambda \leqq 1, \tag{2.27}$$

and arrives at an estimating equation like (2.5) of the basic model with M_t used as the labor input variable and with the additional terms $-(\lambda/\alpha)v \log C_t$ and $\lambda\delta(\log Y_t - \log Y_{t-1})$ on the right-hand side.

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[1] Perfectly adaptable capital stock is like putty – the "marginal product curve of labor is congruent to the long-run or *ex ante* curve". SOLIGO (1966, p. 166).

[2] SOLIGO (1966, p. 172).

2.2.7. *The Dhrymes model*

Dhrymes' model (DHRYMES, 1967) deviates somewhat more from the basic model than do the models previously discussed. Dhrymes first postulates a CES production function:

$$Y_t = A(\delta_1 M_t^{*\alpha} + \delta_2 K_t^{\alpha})^{1/\alpha}. \tag{2.28}$$

The labor input variable is taken to be the number of workers, M_t^*. Dhrymes assumes that optimal employment is given by

$$\partial Y_t/\partial M_t^* = s w_t, \tag{2.29}$$

where "s is a well defined function of the elasticity of the demand for output and supply of labor",[1] and w_t is the product wage. s is assumed to be a constant function. Solving (2.29) yields

$$M_t^d = A^{\alpha/(1-\alpha)} s^{\alpha/(\alpha-1)} w_t^{1/(\alpha-1)} Y_t \delta_1^{1/(1-\alpha)}. \tag{2.30}$$

M_t^d is the desired number of workers for period t.

Dhrymes argues that Y_t and w_t in eq. (2.30) should be replaced by Y_t^e and w_t^e, since M_t^d is based on expected output and the expected wage rate for period t. He assumes that $w_t^e = A_1 w_t$ and $Y_t^e = A_2 Y_t^{\mu} Y_{t-1}^{\nu}$, i.e., that "expected wages are proportional to actual wages and expected output is proportional to some root of the actual output in the current period and the actual output of the period for which planning takes place".[2] He assumes an adjustment process like (1.4) of the basic model for M_t^d:

$$M_t/M_{t-1} = (M_t^d/M_{t-1})^{\lambda}, \quad 0 \leqq \lambda \leqq 1. \tag{2.31}$$

Dhrymes is also concerned with the possible dependence of employment on investment, for "one might expect the (marginal) productivity of labor in general to depend on the type of capital equipment the unit employs".[3] Since "capital goods of different vintages embody in them different levels of technical advance",[4] he assumes that the parameter δ_1 in the production function (2.28) depends with infinite lag on investment, I. Specifically, he assumes that:

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[1] DHRYMES (1967, p. 3).

[2] DHRYMES (1967, p. 4).

[3] DHRYMES (1967, p. 4).

[4] DHRYMES (1967, pp. 4–5).

$$\frac{1}{1-\alpha}\log\delta_1 = \frac{\gamma_1 \log I_{t-1} + \gamma_2 \log I_{t-2} + \gamma_3 \log I_{t-3} + \gamma_4 \log I_{t-4}}{\log I_t + \gamma_5 \log I_{t-1}}. \quad (2.32)$$

Combining the above information Dhrymes arrives at the following non-linear equation to estimate:

$$\begin{aligned}\log M_t = {} & \text{constant} + \frac{\lambda}{\alpha - 1}(\log w_t + \gamma_5 \log w_{t-1}) \\ & + \lambda\mu(\log Y_t + \gamma_5 \log Y_{t-1}) + \lambda\nu(\log Y_{t-1} + \gamma_5 \log Y_{t-2}) \\ & + (1-\lambda)(\log M_{t-1} + \gamma_5 \log M_{t-2}) - \gamma_5 \log M_{t-1} \\ & + \lambda \sum_{i=1}^{4} \gamma_i \log I_{t-i}. \end{aligned} \quad (2.33)$$

In other words, $\log M_t$ is a function of $\log Y_t$, $\log Y_{t-1}$, $\log Y_{t-2}$; $\log M_{t-1}$, $\log M_{t-2}$; $\log w_t$, $\log w_{t-1}$; and $\log I_{t-1}$, $\log I_{t-2}$, $\log I_{t-3}$, $\log I_{t-4}$. Dhrymes estimates the model for all employees and then for production workers and non-production workers separately.

2.2.8. *The Kuh model*

KUH (1965b) makes a distinction between production workers and non-production workers, the latter being more like "overhead" labor and thus more like a fixed factor in the short run than the former. For production workers Kuh regresses $\log M_t$ on a constant, $\log Y_t$, $\log Y_{t-1}$, $\log K_{t-1}$, $\log M_{t-1}$, and $\log H_{t-1} - \log H_{t-2}$ or $\log H_t - \log H_{t-1}$. It is clear from his discussion that his model is similar to the basic model discussed above. The lagged variables are added to the equation because they "depict the nature of the adjustment process".[1]

Kuh discusses the possibility that there may be some substitution in the short run between the number of hours worked per worker and the number of production workers employed, in the sense that the number of hours worked per worker may be used as the principle short-run adjustment tool with respect to changes in man-hour requirements.[2] With respect to the addition of $\log H_{t-1} - \log H_{t-2}$ to the equation, he argues that one would expect that "the larger the rate of change in hours in the previous period,

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[1] KUH (1965b, p. 242).

[2] KUH (1965b, p. 239).

the greater will be employment in this period as a substitute, in order to reduce hours toward normal and thus minimize overtime production".[1]

For non-production workers Kuh finds the coefficient of log Y_{t-1} to be insignificant, and for his final equation he regresses log N_t on a constant, log Y_t, log K_{t-1}, and log N_{t-1}, where N_t is the number of non-production workers employed during period t.

Kuh also estimates an equation determining the number of hours worked per week per production worker. He regresses log H_t on a constant, log Y_t − log Y_{t-1}, and log H_{t-1}. According to Kuh, the main determinant of the number of hours worked per week per worker "is a convention established through bargaining and a variety of social and institutional forces".[2] But, "there is a lagged adjustment to the desired constant level of hours (more accurately, a gently declining trend) and a strong transient response to the rate of change of output".[3] This leads to an equation of the form

$$\log H_t - \log H_{t-1} = \alpha(\beta - \log H_{t-1}) + \gamma(\log Y_t - \log Y_{t-1}), \quad (2.34)$$

or

$$\log H_t = \alpha\beta + (1 - \alpha) \log H_{t-1} + \gamma(\log Y_t - \log Y_{t-1}), \quad (2.35)$$

which is the equation he estimates.

Kuh also argues that the relative scarcity of labor may be important in determining the demand for hours worked per worker, and he adds log U_t and log U_t − log U_{t-1} to eq. (2.35), where U_t is the unemployment rate during period t, on the grounds that "tight labor markets generate a demand for additional hours".[4] When labor markets are tight, firms have more incentive to increase H_t rather than M_t, due among other things to the "deterioration in the quality of the marginal work force".[5] log U_t − log U_{t-1} enters as an "expectational variable".[6]

2.3 Critique of the models similar to the basic model

2.3.1. Introduction

While the details of the various models described in § 2.2 differ considerably

1 KUH (1965b, p. 239).
2 KUH (1965b, p. 239).
3 KUH (1965b, p. 239).
4 KUH (1965b, p. 240).
5 KUH (1965b, p. 240).
6 KUH (1965b, p. 240).

from one another, the models themselves are all based on the postulation of a short-run production function and a simple lagged adjustment process. Equations similar to (2.5) of the basic model have been the ones most often estimated in the above studies.

In this section the above studies are evaluated, and some empirical results of estimating the basic model are presented. It was mentioned at the beginning of § 2.2 that the basic model as presented there is inconsistent because no assumption about cost-minimizing behavior of firms with respect to the workers–hours mix was made. This inconsistency will be discussed and eliminated first before a further evaluation of the above studies is made.

2.3.2. The necessity of cost-minimizing assumptions regarding the workers–hours mix

There are two different, though not mutually exclusive, cost-minimizing assumptions which can be made regarding the short-run employment decisions of firms. The first assumption which can be made is that firms are concerned with the optimal short-run allocation of total factor inputs between labor services and capital services; and the second assumption which can be made is that firms are concerned with the optimal short-run allocation of labor services between the number of workers employed and the number of hours worked per worker. Brechling, Ball and St Cyr, and Ireland and Smyth make the second assumption but not the first, i.e., they assume that in the short run firms are concerned with adjusting their workers–hours worked per worker mix so as to achieve a minimum wage bill, but that firms are not concerned with achieving an optimal capital–labor mix by adjusting the amounts of capital services and labor services used to changing factor prices. Dhrymes, on the other hand, makes the second assumption that firms are concerned with achieving an optimal capital–labor mix, but he does not discuss the optimal short-run allocation of labor services between workers and hours worked per worker. Kuh, Solow, and Soligo do not make any assumptions about short-run cost-minimizing behavior.

Without the assumption of cost-minimizing behavior with respect to the workers–hours worked per worker mix, there is a contradiction between the production function (2.2), or (2.1), of the basic model and the lagged adjustment process (2.4). Eq. (2.3) is derived from the production function (2.2) and gives L_t^* (the amount of labor services needed in the production process) as a function of the exogenous variables, Y_t, K_t, and t. Assume that for period t eq. (2.3), given Y_t, K_t, and t, calls for an L_t^* greater than L_{t-1}. The lagged adjustment process (2.4) implies that L_t (the amount of labor services used) will be less than L_t^*. The production function (2.2), however, reveals that, given Y_t, K_t, and t, this cannot be the case and still have Y_t produced, i.e., it is not possible to have the amount of labor services used, L_t, be less than the amount of labor services needed, L_t^*. For L_t^* less than L_{t-1} no problem arises, but for L_t^* greater than L_{t-1} eqs. (2.2) and (2.4) are incompatible. In other words, for (2.2) and (2.4) to be compatible, the labor services input variable in the production function cannot be the same variable that is subjected to the lagged adjustment process (2.4).

The cost-minimizing assumptions made by Brechling, Ball and St Cyr, and Ireland and Smyth discussed above are sufficient for the compatibility of the production function and the lagged adjustment process. Actually, their assumptions are more complicated than is necessary. Assume, as Ball and St Cyr do, that labor services can be approximated by man hours, so that in the notation of the basic model, $L_t^* = (M_tH_t)^*$, where, as usual, M_t denotes the number of workers employed and H_t denotes the number of hours worked per worker. A simpler assumption to make than either Brechling's or Ball and St Cyr's[1] is that the cost-minimizing number of workers during period t, denoted as M_t^d, equals $(M_tH_t)^*/HS_t$. HS_t is again the standard (as opposed to overtime) number of hours of work per worker for period t.[2] In other words, it is assumed that the cost-minimizing number of workers occurs at the point where no undertime or overtime is being worked, i.e., where each worker is working the standard number of hours per period. The adjustment process (2.4) can then be in terms of M_t^d:

$$M_t/M_{t-1} = (M_t^d/M_{t-1})^\lambda, \quad 0 \leqq \lambda \leqq 1, \tag{2.36}$$

and whenever M_t^d is greater than M_{t-1} (so that M_t is less than M_t^d), the number of hours worked per worker, H_t, can be assumed to make up the difference in the short run.

Ball and St Cyr approximate figure 2.1 by the quadratic (2.14) above, and their cost-minimizing level of hours is a function of the parameters of the quadratic function. The simpler assumption made here takes the least cost level of hours at HS_t in figure 2.1, which is the least cost point before any quadratic approximation is made.

When the basic model is referred to from now on, the reference will be to the model as modified above. The lagged adjustment process will thus

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[1] Ireland and Smyth's assumption is the same as that of Ball and St Cyr.

[2] The standard number of hours of work per worker may be subject to long-run trend influences, and this is the reason for the time subscript on HS.

be taken to be (2.36) instead of (2.4) as before. The final equation of the basic model is an equation similar to (2.5) above, except that M_t has replaced L_t as the labor variable and the log HS_t variable has been added:

$$\log M_t - \log M_{t-1} = -\frac{1}{\alpha}\lambda \log A + \frac{1}{\alpha}\lambda \log Y_t -$$

$$\frac{\beta}{\alpha}\lambda \log K_t - \frac{\gamma}{\alpha}\lambda t - \lambda \log M_{t-1} - \lambda \log HS_t. \tag{2.37}$$

It should be pointed out that Dhrymes' cost-minimizing assumption regarding the optimal capital services–labor services mix is not sufficient to remove the incompatibility between the production function (2.28) and the lagged adjustment process (2.31) in his model. If the desired number of workers for period t, M_t^d, is less than M_{t-1}, then by the adjustment process the actual number of workers employed during period t, M_t, will be less than the desired number. If the amount of output produced, the stock of capital, and the wage rate are assumed to be exogenous in the short run, then his adjustment process (2.31) may yield an M_t which, from the production function (2.28), is not sufficient to produce the output. It is possible to remove this incompatibility by assuming that the capital stock varies in such a way in the short run as to allow the output to be produced, given the M_t resulting from the adjustment process. This, of course, is a very unrealistic assumption to make, and Dhrymes' model the way it stands has not accounted for the possible incompatibility between the production function and the lagged adjustment process.

In an appendix, Brechling (1965) presents estimates of his equations for man hours as well as for workers, and since the man-hours variable does not enter his model either as an input of the production function nor as the variable in the lagged adjustment process, it is not at all clear how these estimates relate to his theoretical model.

2.3.3. The seasonal adjustment problem

A more serious criticism relating to all of the above studies relates to the use of seasonally adjusted data. In all of the studies discussed above the authors either use seasonally adjusted data or seasonally unadjusted data with seasonal dummy variables to estimate their equations.

Many, if not most, industries have large seasonal fluctuations in output and, to a lesser extent, in employment. In table 2.2 the percentage changes from the trough month to the peak month of the year in output, Y, in the

TABLE 2.2

The percentage changes from the trough month to the peak month of the year in Y, M, and HP for the years 1950, 1955, 1960, and 1964

	1950			1955			1960			1964		
Industry	*Y*	*M*	*HP*	*Y*	*M*	*HP*	*Y*	*M*	*HP*	*Y*	*M*	*HP*
201	42.5	14.6	13.7	34.6	8.3	11.4	20.1	6.6	6.7	24.9	7.9	8.6
207	93.1	32.1	9.1	79.0	25.5	7.2	76.0	23.4	6.8	79.7	17.2	3.1
211	35.5	7.9	23.2	18.8	6.6	10.2	14.3	5.0	21.7	44.9	3.9	28.9
212	32.1	10.4	18.1	19.5	11.2	10.3	15.1	5.0	13.3	79.3	19.2	15.3
231	22.8	7.1	7.2	25.7	9.3	9.1	34.3	2.8	7.6	30.4	4.4	4.2
232	41.3	7.3	8.0	24.3	6.4	7.7	28.9	6.0	7.4	24.8	6.6	7.1
233	53.7	25.4	12.5	31.6	16.4	5.1	27.7	12.2	7.1	19.2	5.8	9.7
242	66.2	23.3	9.4	24.4	11.5	4.8	42.1	19.2	8.8	28.7	10.8	8.1
271	24.9	4.6	2.9	27.7	5.1	4.7	23.9	3.1	2.8	23.3	2.8	2.8
301	27.1	11.9	9.8	28.9	5.8	8.2	30.4	10.5	9.7	19.9	5.0	12.0
311	17.8	7.3	5.7	10.2	2.0	2.8	12.7	5.5	4.9	19.8	7.3	3.5
314	21.4	7.1	13.5	23.5	8.8	6.9	22.5	5.8	10.4	17.0	4.8	6.7
324	58.0	7.0	2.9	43.2	4.9	1.7	93.7	17.0	4.3	99.0	15.9	3.7
331	19.8	9.2	9.6	19.6	14.5	4.0	108.3	38.0	16.0	25.3	13.3	3.7
332	51.3	36.6	14.0	21.5	19.0	5.8	53.8	14.3	8.0	24.6	7.7	5 0
336	60.3	35.7	10.4	17.1	12.1	4.0	34.4	13.1	4.3	13.2	4.6	2.9
341	151.7	42.4	10.7	114.0	21.3	8.7	90.4	18.0	9.1	71.8	14.4	6.4

number of production workers employed, *M*, and in the average number of hours paid-for per week per worker, *HP*, are presented for the years 1950, 1955, 1960, and 1964 for the seventeen three-digit United States manufacturing industries considered in this study.[1] The output fluctuations in most cases are quite large, with output during the peak month being between 10.2 and 151.7 percent larger than during the trough month. The fluctuations in the number of workers employed and the number of hours paid-for per worker are in general much less, but still are reasonably large.

A major criticism of the above studies of short-run employment demand which are based on the concept of a short-run production function is that the use of seasonally adjusted data or seasonal dummy variables is in-

[1] The data are discussed in ch. 4. As mentioned at the beginning of this chapter, for monthly data it is important to make explicit the time periods to which the variables refer. This will be done in ch. 4.

compatible with the production function concept. A production function is a technical relationship between certain physical inputs and a physical output and is not a relationship between seasonally adjusted inputs and a seasonally adjusted output. Unless one has reason to believe that the technical relationship itself fluctuates seasonally, and at least for manufacturing industries it is difficult to imagine very many instances where this is likely to be true, the use of seasonally adjusted data or seasonal dummy variables is unwarranted.

Likewise, when seasonally adjusted data or seasonal dummy variables are used, the lagged adjustment process (2.36) of the basic model must be interpreted as implying the lagged adjustment of the seasonally adjusted number of workers rather than the actual number of workers. Interpreted in this way, it implies that the adjustment coefficient λ fluctuates seasonally. Here again there seems little reason to believe that λ should fluctuate seasonally. It is possible to argue that the adjustment costs might be less in the spring and fall when a large number of students can be hired and then laid off, but in general the interpretation of (2.36) in seasonally adjusted terms seems theoretically less warranted than in seasonally unadjusted terms.

2.3.4. Equation estimates of the basic model

The proof of any model is how well it stands up under empirical tests. If the basic model above is to lead to any empirically meaningful results, seasonally unadjusted data must be used. In tables 2.3 and 2.4 the results of estimating two equations similar to eq. (2.37) of the basic model using seasonally unadjusted monthly data for the seventeen three-digit manufacturing industries considered in this study are presented. In both equations the log K_t variable in eq. (2.37) has been assumed to be absorbed in the time trend, as Ball and St Cyr have assumed, and in the second equation the lagged output variable, log Y_{t-1}, has been added, as Bechling, Solow, Soligo, and Kuh have done under various expectational hypotheses. Also, the effects of the log HS_t variable have been assumed to be absorbed in the constant term and the time trend.

The data used to estimate the two equations are the basic data used to estimate the model developed in this study. The exact period of estimation used for each industry and the adjustments which have been made in the data are discussed in ch. 4 and the data appendix. In what follows, M_{2wt} denotes the number of production workers employed during the second week of month t and Y_{dt} denotes the average daily rate of output during month t. The following two equations were estimated:

TABLE 2.3

Parameter estimates for eq. (2.37)′

Industry	No. of obs.	$\hat{a}_0$	$\hat{a}_1$	1000 $\hat{a}_2$	$\hat{a}_3$	R^2	SE	DW	[a]Value of $-\hat{a}_3/\hat{a}_1$
201	192	.813	.032	−.062	−.131	.076	.0194	1.03	4.09
		(3.40)	(1.94)	(1.45)	(3.83)				
207	136	.701	.226	−.847	−.333	.579	.0299	1.36	1.47
		(3.05)	(13.34)	(8.92)	(7.64)				
211	136	−.109	.047	−.089	−.036	.084	.0119	2.20	0.76
		(0.55)	(2.96)	(1.62)	(1.27)				
212	136	−.283	.097	−.420	−.058	.227	.0188	2.57	0.60
		(1.65)	(6.17)	(3.52)	(2.20)				
231	136	.573	.118	−.221	−.196	.273	.0245	2.00	1.66
		(1.81)	(6.15)	(2.97)	(4.13)				
232	136	.709	.057	−.105	−.137	.199	.0132	1.43	2.40
		(3.52)	(4.72)	(2.18)	(4.87)				
233	136	.681	.163	−.271	−.220	.301	.0348	1.32	1.35
		(1.60)	(6.24)	(2.89)	(4.15)				
242	154	.601	.210	−.797	−.245	.589	.0171	0.98	1.17
		(3.88)	(14.16)	(9.35)	(10.96)				
271	166	.782	.043	.068	−.147	.312	.0059	2.02	3.42
		(3.95)	(7.43)	(2.21)	(5.29)				
301	134	.187	.057	−.307	−.073	.173	.0152	1.86	1.28
		(1.12)	(4.62)	(4.71)	(3.11)				
311	170	.196	.094	−.349	−.138	.146	.0136	1.62	1.47
		(1.33)	(4.80)	(4.07)	(4.73)				
314	136	3.129	.178	−.407	−.560	.383	.0190	1.30	3.15
		(7.13)	(7.28)	(6.67)	(8.30)				
324	187	.773	.096	−.379	−.234	.383	.0228	1.27	2.44
		(5.03)	(9.82)	(8.43)	(8.07)				
331	128	1.493	.173	−.484	−.307	.772	.0103	1.53	1.77
		(12.87)	(20.08)	(15.14)	(17.39)				
332	170	.424	.131	−.203	−.174	.382	.0175	1.99	1.33
		(4.05)	(9.84)	(5.55)	(8.52)				
336	170	.006	.081	−.173	−.085	.126	.0240	1.19	1.05
		(0.05)	(4.74)	(3.33)	(3.90)				
341	191	1.698	.121	−.088	−.402	.425	.0282	0.77	3.32
		(8.71)	(10.65)	(2.14)	(10.59)				

t-statistics are in parentheses.

[a] Implied value of the production function parameter α.

TABLE 2.4

Parameter estimates for eq. (2.37)″

Industry	No. of obs.	$\hat{a}_0$	$\hat{a}_1$	1000 $\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	R^2	SE	DW	[a]Value of $-\hat{a}_3/(\hat{a}_1+\hat{a}_4)$
201	192	.717	.125	.033	−.083	−.135	.252	.0175	1.47	−8.30
		(3.31)	(6.11)	(0.79)	(2.63)	(6.65)				
207	136	.562	.226	−.747	−.290	−.022	.582	.0299	1.47	1.42
		(2.01)	(13.32)	(5.05)	(4.42)	(0.88)				
211	136	−.135	.032	−.101	−.041	.023	.095	.0119	2.04	0.75
		(0.69)	(1.16)	(1.82)	(1.44)	(1.27)				
212	136	−.302	.153	−.264	−.031	−.079	.296	.0180	2.81	0.42
		(1.84)	(7.05)	(2.16)	(1.17)	(3.58)				
231	136	.895	.053	−.366	−.305	.131	.446	.0215	1.95	1.66
		(3.17)	(2.66)	(5.30)	(6.78)	(6.39)				
232	136	.770	.055	−.196	−.170	.032	.230	.0130	1.36	1.95
		(3.85)	(4.65)	(3.19)	(5.45)	(2.31)				
233	136	.158	.211	−.077	−.082	−.138	.371	.0331	1.62	1.12
		(0.37)	(7.58)	(0.75)	(1.33)	(3.84)				
242	154	.573	.215	−.770	−.237	−.011	.590	.0171	1.02	1.16
		(3.43)	(11.42)	(7.42)	(8.42)	(0.46)				
271	166	.532	.068	.049	−.093	−.046	.475	.0051	2.19	4.23
		(3.00)	(10.98)	(1.79)	(3.64)	(7.06)				
301	134	.208	.025	−.360	−.085	.043	.202	.0150	1.80	1.25
		(1.26)	(1.29)	(5.23)	(3.56)	(2.14)				
311	170	.198	.101	−.318	−.124	−.019	.149	.0136	1.68	1.51
		(1.34)	(4.75)	(3.40)	(3.74)	(0.84)				
314	136	2.013	.221	−.135	−.292	−.185	.513	.0169	1.73	8.11
		(4.64)	(9.62)	(1.89)	(3.88)	(5.91)				
324	187	.250	.181	−.187	−.094	−.133	.579	.0189	1.91	1.96
		(1.80)	(14.74)	(4.38)	(3.31)	(9.22)				
331	128	1.257	.208	−.421	−.265	−.054	.782	.0101	1.76	1.72
		(8.38)	(12.52)	(10.33)	(10.83)	(2.42)				
332	170	.363	.158	−.182	−.155	−.039	.391	.0174	2.02	1.30
		(3.26)	(7.33)	(4.67)	(6.56)	(1.58)				
336	170	.000	.190	−.107	−.053	−.145	.237	.0225	1.60	1.04
		(0.00)	(6.89)	(2.12)	(2.49)	(4.89)				
341	191	.659	.165	−.015	−.137	−.136	.657	.0218	1.84	4.72
		(3.72)	(17.15)	(0.46)	(3.62)	(11.23)				

t-statistics are in parentheses.

[a] Implied value of the production function parameter α.

$$\log M_{2wt} - \log M_{2wt-1} = a_0 + a_1 \log Y_{dt} + a_2 t + a_3 \log M_{2wt-1}, \quad (2.37)'$$

$$\log M_{2wt} - \log M_{2wt-1} = a_0 + a_1 \log Y_{dt} + a_2 t + a_3 \log M_{2wt-1} + a_4 \log Y_{dt-1}. \quad (2.37)''$$

For eq. (2.37)′, which does not include the log Y_{dt-1} variable, the implied value of the production function parameter α is $-a_3/a_1$, as can be seen from eq. (2.37). (The effects of omitting the log K_t variable in eq. (2.37) are merely reflected in the coefficient of the time trend if K_t is growing smoothly through time, as Ball and St Cyr assume.) For eq. (2.37)″, which includes the log Y_{dt-1} variable, the steady state solution can be derived (by setting $M_{2wt} = M_{2wt-1} = \overline{M}$ and $Y_{dt} = Y_{dt-1} = \overline{Y}$), giving log $\overline{M}$ as a function of a constant, log $\overline{Y}$, and t, and the resulting coefficient of log $\overline{Y}$ can then be taken to be $1/\alpha$. This coefficient of log $\overline{Y}$ is $-(a_1 + a_4)/a_3$, so the implied value of α in eq. (2.37)″ is $-a_3/(a_1 + a_4)$. In table 2.3 the results of estimating eq. (2.37)′ are given, along with the implied estimate of α, $-\hat{a}_3/\hat{a}_1$, and in table 2.4 the results of estimating eq. (2.37)″ are given, along with the implied estimate of α, $\hat{a}_3/(\hat{a}_1 + \hat{a}_4)$.

In all but five of the thirty-four cases the implied value of α turns out to be greater than one, and in one of the remaining five cases it is negative. In nine of the thirty-four cases α is greater than two, and in seven of these cases it is greater than three. The results clearly do not appear to be consistent with the interpretation of α as the elasticity of output with respect to labor services.

Under the Ireland and Smyth interpretation, the implied value of α should be interpreted not as measuring returns to labor services alone but as measuring short-run returns to scale (capital services being expanded and contracted along with labor services in the short run). Even under this interpretation, however, one would expect that α (or η in the Ireland and Smyth notation) should be equal to or slightly less than one, since during high rates of output, less (or at least not more) efficient capital is likely to be utilized and the additional workers hired are likely to be less (or at least not more) efficient. One would certainly not expect η to be considerably greater than one, as is the case for most of the estimates presented in tables 2.3 and 2.4. The model, even under this alternative interpretation of α, appears to be incorrectly specified.

In addition to the unrealistically large values of α, the estimate of the constant term turns out to be negative as expected in only four of the thirty-four cases.

The Durbin–Watson statistics given in the tables are biased towards two because of the existence of a lagged dependent variable among the set of regressors in each equation.[1] Even without considering this bias, however, the DW statistics presented in the tables reveal the existence of first-order serial correlation in about half of the thirty-four equations estimated. The existence of serial correlation appears to be less pronounced in the equations which include the log Y_{t-1} variable, but the problem still remains for at least five of the industries. In general, the DW statistics cast some doubt on the specification of the model.

Although seasonally unadjusted (monthly) data have been used to estimate the above equations, as this seemed to be the theoretically preferred procedure, in the previous studies, where seasonally adjusted (quarterly) data or seasonally unadjusted (quarterly) data and seasonal dummy variables have been used, the results in most cases also show strongly increasing returns to labor services (or, on the Ireland and Smyth interpretation, strongly increasing short-run returns to scale). The results presented in tables 2.3 and 2.4 are not unique to the type of data used.

2.4 Description of other studies of employment fluctuations

2.4.1. The Neild model

Neild's approach (NEILD, 1963) is highly empirical in nature, his main concern being with forecasting. His basic postulate is that employment depends on a productivity trend and on "past and present levels of output".[2] He estimates two basic equations:[3]

$$\log M_t - \log M_{t-1} = \alpha_0 + \alpha_1 (\log Y_t - \log Y_{t-1}) + \alpha_2(\log Y_{t-1} - \log Y_{t-2}) + \alpha_3(\log Y_{t-2} - \log Y_{t-3}), \tag{2.38}$$

and

$$\log M_t - \log M_{t-1} = \alpha_0 + \alpha_1(\log Y_t - \log Y_{t-1}) + \alpha_2(\log Y_{t-1} - \log Y_{t-2}) + \beta_1(\log M_{t-1} - \log M_{t-2}). \tag{2.39}$$

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[1] See NERLOVE and WALLIS (1966).

[2] NEILD (1963, p. 56).

[3] Neild estimates the same equations for both workers, M_t, and total man hours, $M_t H_t$. The equations presented in this summary are for M_t only.

Eq. (2.39), which includes the lagged dependent variable on the right-hand side, implies that the number of workers employed is a geometrically declining function of all past levels of output after the second period, while eq. (2.38) implies that the number employed is a function of only the present and the past two levels of output.

2.4.2. The Wilson and Eckstein model

Description of the model. The Wilson and Eckstein model (WILSON and ECKSTEIN, 1964) is considerably different from the basic model presented above. Wilson and Eckstein begin by postulating a long-run production function

$$C_t = \frac{1}{\alpha}(M_t H_t)_p, \tag{2.40}$$

which, when solved for $(M_t H_t)_p$, they call the "long-run labor requirements function":

$$(M_t H_t)_p = \alpha C_t. \tag{2.41}$$

C_t is capacity output, and $(M_t H_t)_p$ is the number of man hours required to produce the capacity output.

In the short run the plant is fixed, and Wilson and Eckstein assume that the "plant man-hour requirements function" can be approximated by a straight line which intersects the long-run function from above at capacity output:

$$(M_t H_t)_e = \alpha C_t + \beta(Y_t^e - C_t). \tag{2.42}$$

Y_t^e is the output which is planned at the beginning of period t to be produced during period t, and $(M_t H_t)_e$ is the number of man hours required to produce the planned output. β is assumed to be less than α.

Wilson and Eckstein then define a "short-run maladjustment man-hour requirements function", which intersects the plant function from above at planned output:

$$M_t H_t = \alpha C_t + \beta(Y_t^e - C_t) + \gamma(Y_t - Y_t^e). \tag{2.43}$$

Y_t is the actual output produced during period t, and $M_t H_t$ is the actual number of man hours required to produce Y_t. γ is assumed to be less than β. The relationships among the three man-hour requirements functions can be seen graphically in figure 2.2.

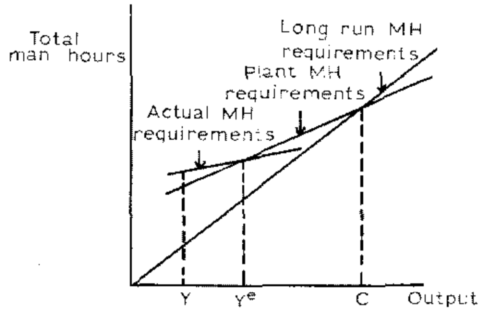

Fig. 2.2. Wilson and Eckstein's man-hour requirements functions.

Wilson and Eckstein include technical change in their model by assuming that

$$\alpha = \alpha_0 + \alpha_1 t, \tag{2.44a}$$
$$\beta = \beta_0 + \beta_1 t, \tag{2.44b}$$
$$\gamma = \gamma_0 + \gamma_1 t. \tag{2.44c}$$

They also assume that

$$Y_t^e = \frac{S_t}{6} (3Y_{t-1}^* + 2Y_{t-2}^* + Y_{t-3}^*), \tag{2.45}$$

where Y_{t-i}^* is seasonally adjusted output for period $t-i$ and S_t is the seasonal factor for period t. They use seasonally unadjusted data and seasonal dummy variables in the estimation of eq. (2.43) and estimate the equation separately for production worker straight time hours and production worker overtime hours. They also estimate a modified version of eq. (2.43) for non-production workers.

Critique of the model. Wilson and Eckstein have three concepts of output – capacity output, C_t, planned output, Y_t^e, and actual output, Y_t. Man-hour requirements differ to the extent that planned output differs from capacity output and to the extent that actual output differs from planned output. As can be seen from figure 2.2, the model has the rather odd implication that if actual output is greater than planned output, actual man-hour requirements per unit of output are less than plant man-hour requirements per unit of output. It also has the implication that if actual output is greater than capacity output (which they state can happen[1]), actual man-hour

[1] Wilson and Eckstein (1964, p. 42).

requirements per unit of output are less than long-run man-hour requirements per unit of output. Wilson and Eckstein argue that by sacrificing maintenance work and using machinery more intensively actual man-hour requirements per unit of output may be less at high levels of output than plant or long-run man-hour requirements per unit of output.[1] Even if this is true, however, it does not seem likely that the effects on man-hour requirements should be symmetrical for positive and negative deviations of planned output from capacity output or of actual output from planned output, as is implied in figure 2.2. It is also open to question whether actual man-hour requirements per unit of output really are less than long-run man-hour requirements per unit of output at output greater than capacity, especially if less efficient machines are brought into use at high levels of output.

Wilson and Eckstein estimate eq. (2.43) first for production worker standard hours, which are defined to be $37.5M_t$, and then for production worker overtime hours, which are defined to be $M_t(H_t - 37.5)$. This procedure appears to be inconsistent with their overall model. Eq. (2.43) is interpreted as a man-hour *requirements* function, and if $M_t H_t$ number of man hours are required to produce the output, Y_t, then the relevant dependent variable is $M_t H_t$ and not some fraction of it.

Actually, eq. (2.41) of their model might be better interpreted as expressing *desired* man hours as a function of capacity output, with eqs. (2.42) and (2.43) showing how, due to adjustment lags in the short run, desired man hours deviate from actual man hours used. Eq. (2.43) could perhaps then be interpreted as a reduced form equation of some more complicated employment demand equation, the reduced form equation being a combination of a man-hour requirements function and a lagged adjustment process. The theoretical underpinnings of the Wilson and Eckstein model do not appear to be well developed.

2.4.3. *The Hultgren, Raines, and Masters studies*

As mentioned in ch. 1, an alternative approach to the study of short-run fluctuations in output and employment is to examine output per worker (or per man hour) directly in an attempt to discover how it fluctuates with respect to short-run fluctuations in output. HULTGREN (1960, 1965), RAINES (1963), and MASTERS (1967) have used this approach, and although this is not the basic approach used in this study, these studies will be briefly summarized.

[1] WILSON and ECKSTEIN (1964, p. 42).

After seasonally adjusting the data, HULTGREN (1960) examines how output per man hour fluctuates during contractions (falling output) and during expansions (rising output). He finds that output per man hour increases during expansions, although there is some evidence that near the end of the expansions this phenomenon is less widespread, and that output per man hour decreases during contractions, although again there is some evidence that this phenomenon is less widespread near the end of the contractions. In another study, using different data, Hultgren arrives at a similar conclusion.[1]

In the Raines model (RAINES, 1963) output per man hour is taken to be a function of capacity utilization (both the level and the change), the amount and quality of the capital stock, and time. Raines estimates the following equation:

$$\begin{aligned}\log(Y_t/M_tH_t) = {} & \alpha_1 t + \alpha_2(Y_t/C_t) - \alpha_3(Y_t/C_t)^2 \\ & + \alpha_4\Delta(Y_t/C_t)_+ + \alpha_5\Delta(Y_t/C_t)_- \\ & + \alpha_6\Delta(Y_t/C_t)_{t-1} - \alpha_7 A_t.\end{aligned} \tag{2.46}$$

Y_t/C_t is the capacity utilization in period t, and A_t is the average age of the capital stock. The notation $\Delta(Y_t/C_t)_+$ means that when $\Delta(Y_t/C_t)$ is positive, $\Delta(Y_t/C_t)_+$ is set equal to this value, and when it is negative, $\Delta(Y_t/C_t)_+$ is set equal to zero; and conversely for $\Delta(Y_t/C_t)_-$.

Raines finds that output per man hour is positively related to the level of capacity utilization and also to the change in capacity utilization. The coefficient estimate of α_4 is larger than the estimate of α_5,[2] which implies that output per man hour is more positively related to positive changes in capacity utilization than it is negatively related to negative changes in capacity utilization.

MASTERS (1967), using seasonally adjusted data, examines how output per worker behaves during contractions. For the years 1947–1961 he finds 64 contractions occurring in 24 three- and four-digit industries. For each of these 64 cases he computes the change in output and the change in output per worker, using as end points the peak and the trough of the output series. Using these 64 observations, he regresses the change in output per worker on the change in output and a constant, and finds that the change in output per worker is positively related to the change in output, i.e., that output per worker decreases during contractions.

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[1] HULTGREN (1965, pp. 39–42).

[2] RAINES (1963, Table I, p. 187).

2.5 Summary

This completes the survey of previous studies of employment demand and output per man-hour fluctuations. The approach of many of the studies has been to postulate a short-run production function and a lagged adjustment process and from these two equations to derive an equation in which the production function parameter and adjustment coefficient can be estimated. Previous results using seasonally adjusted quarterly data and the results achieved in this chapter using seasonally unadjusted monthly data have indicated that there are strongly increasing returns to labor alone or, on the Ireland and Smyth interpretation, strongly increasing short-run returns to scale. These results are inconsistent with the assumptions of classical economic theory and in general cast doubt on the specification of the model. Previous studies which have examined output per man-hour fluctuations directly have found that output per man hour varies directly with output in the short run, which also seems to be inconsistent with what would be expected from the assumptions of classical economic theory.

In the next chapter an alternative model of the short-run demand for workers is developed. The model provides an explanation of the observed phenomenon of increasing returns to labor services and will be seen in ch. 4 to yield substantially better results than those presented in tables 2.3 and 2.4 for the basic model of previous studies.