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# Basic physics of X-ray imaging

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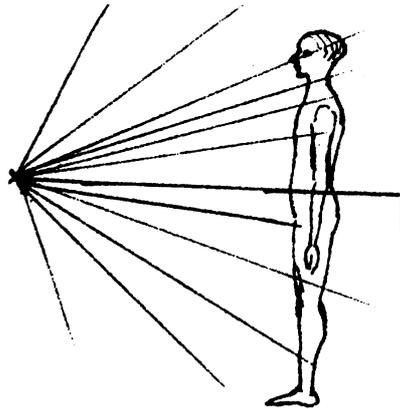

1. INTRODUCTION

In X-ray diagnostics, radiation that is partly transmitted through and partly absorbed in the irradiated object is utilised. An X-ray image shows the variations in transmission caused by structures in the object of varying thickness, density or atomic composition. In Figure 1, the necessary attributes for X-ray imaging are shown: X-ray source, object (patient) and a radiation detector (image receptor).



**Figure 1.** The necessary attributes for X-ray imaging: X ray source, object (patient) and radiation detector


After an introductory description of the nature of X-rays, the most important processes in the X-ray source, the object (patient) and radiation detector for the generation of an X-ray image will be described.


2. THE PHYSICS OF THE X-RAY SOURCE: THE X-RAY TUBE

a. The nature of X-rays

X-rays are like radio waves and visible light electromagnetic radiation. X-rays, however, have higher frequency, $n$, and shorter wavelength, $l$, than light and radio waves. The radiation can be considered as emitted in quanta, photons, each quantum having a well defined energy, h$n$, where h is a physical constant, Plancks constant, and $n$ is the frequency. The energy of X-ray photons are considerably higher than those of light.

A number of the phenomena, which are observed with X-rays are most conveniently described by the wave properties of the radiation while other phenomena can be more easily understood if the X-rays are considered as being composed of particles (photons) with well defined energies and momentum. The rest mass of a photon is zero. This means that photons can never be found at rest. All photons move at the same velocity, c, in a vacuum, given by c = 2.998 $10^8$ m/s.

b. Relationship between wave length and frequency

The wave length multiplied with the frequency (number of wave lengths per unit time) equals the velocity of light

$$l \cdot n = c \tag{1}$$

c. The propagation of X-rays

Similarly to visible light, X-rays propagate linearly. The rays from a point source form a divergent beam. The number of photons passing per unit area perpendicular to the direction of motion of the photons is called the fluence, $\Phi$. The fluence in a vacuum decreases following the inverse square law, given by

$$\Phi(r) = \Phi(1) \cdot \frac{1}{r^2} \tag{2}$$

where r is the distance from the point source and $\Phi(1)$ is the fluence at r=1 (relative units).The inverse square law is illustrated in Fig 2.
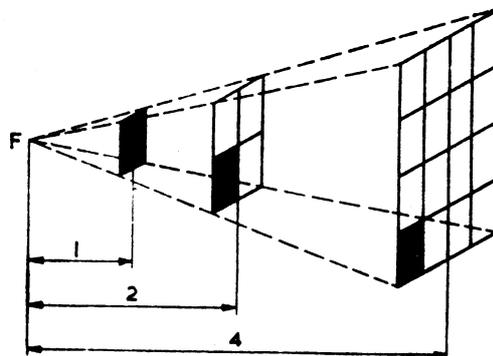


**Figure 2.** The fluence, $\Phi$, of X-rays decreases with the square of the distance from the source.

d. Refraction of X-rays

When visible light passes from one medium to another it is refracted due to the different velocities of the rays in different media and interference of waves. The velocity of propagation of X-rays varies much less in different materials and the refraction of X-rays is negligible. For this reason, X-rays cannot be focussed be means of lenses.

e. Diffraction of X-rays

Another wave phenomenon is diffraction. This means that the wave can be bent when passing an edge or a slit. The slit can then be regarded as a new source of waves propagating in all directions. If there is a periodic system of slits (lattice), interference effects will occur. That is, waves which are in phase will be amplified and those that are out of phase will be weakened. In order to demonstrate diffraction with X-rays the lattice constant (distance between the scattering slits) must be of the order of 0.1 nm. Such distances exist between the atomic planes in crystals. Crystals are frequently used for X-ray spectrometry.

f. Generation of X-rays

An X-ray tube consists of two electrodes, one negative, glow cathode, which upon being heated emits electrons, and one positive, anode. The electrodes are incapsuled in a vacuum. By applying an acceleration potential (20-200 kV), the electrons are accelerated towards the anode. The electrons gain kinetic energy which is the product of their charge and the potential difference. As a measure of the kinetic energy of the electrons and X-ray photons, the unit of 1 eV is used.

Definition: One electron volt (1 eV) is the kinetic energy, that a charged particle of one elementary charge (the charge of an electron) achieves when being accelerated in a potetial difference of one volt (1 V); $1 \text{ eV} = 1.602 \cdot 10^{-19}$ J (joule). See Fig 3.
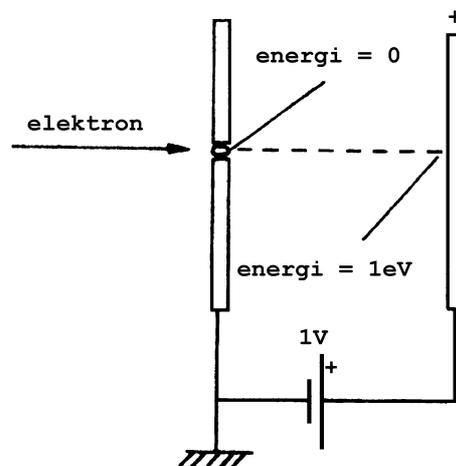


**Figure 3.** One electron volt (1 eV) is the kinetic energy of an electron which has been accelerated through a potetial difference of 1 volt.

If the potential difference is 100 kV, each electron gets a kinetic enegy of 100 keV (1000 ev = 1 keV).

When the elcetron reaches the anode it imparts the main part of its energy to the atoms of the anode by ionisations and excitations. This energy will finally appear as heat energy.
If an electron passes close to an atomic nucleus, it will change its direction of motion, i.e., exhibits an acceleration. At each such acceleration there is a small probability that

the electron looses energy in the form of a photon, Fig 4. These photons are called bremsstrahlung photons and constitute the main part of the X-rays being used in X-ray diagnostic imaging.
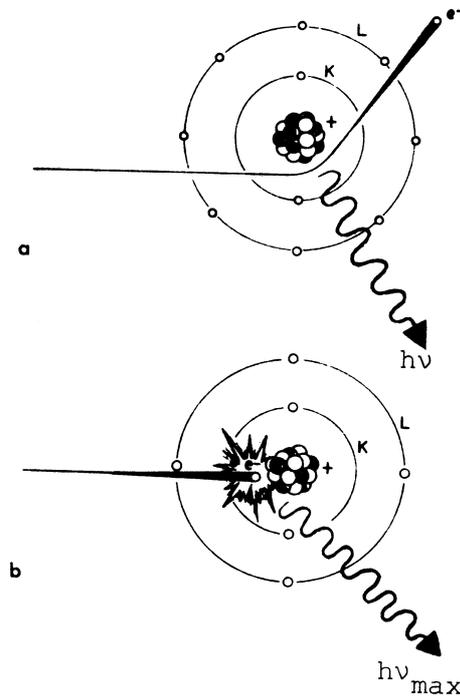


**Figure 4.** Bremsstrahlung is generated when an electron with high energy changes its direction of motion in the neighbourhood of an atomic nucleus and thereby looses energy.

The bremsstrahlung photon can obtain an arbitrary energy between zero and the whole of the kinetic energy of the electron, T.

$$h\boldsymbol{\nu}_{max} = T \qquad\qquad (3)$$

The relative amount of bremsstrahlung emitted increases with incresing electron kinetic energy and with increasing atomic number, Z, of the anode material.

Since the major part of the energy of the electrons is converted into heat in the anode (about 1% will appear as X-rays), the anode material should have a high melting point and good heat conduction ability. To get a high relative amount of X-ray energy, the anode material should be of high atomic number. Tungsten is the dominating anode material and is in modern X-ray tubes often mixed with renium ($Z_W$=74; $Z_{Re}$=75).

Modern X-ray imaging requires a small focal spot and high X-ray fluence rates (number of photons per unit area and unit time). To meet these requirements, technical solutions with a line shaped focal spot and rotating anode have been introduced.

3. THE ENERGY SPECTRUM OF X-RAYS

a. Dependence of the energy spectrum on tube potential

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Fig 5 shows energy spectra from an X-ray tube, the glass envelope of which gives a filtration corresponding to 2 mm Al (aluminium). The X-rays are then additionally filtered by an extra layer of 1 mm Al. The energy spectra show the number of photons per unit energy interval, (keV), emitted within a unit interval of the solid angle, (steradian), when the charge 1 mAs passes through the X-ray tube. The energy spectra have been measured at constant acceleration potential differences of 40, 70, 100 and 130 kV (Mika and Reiss 1969). As can be seen from Fig 5, there are only few photons close to the maximum energy.
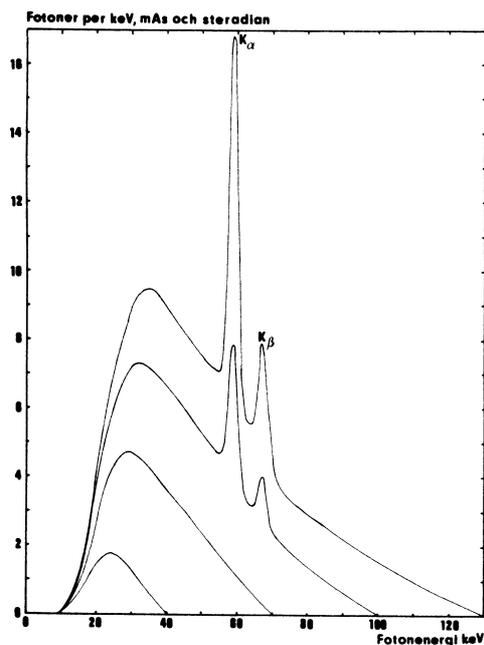


**Figure 5.** Energy spectra of X-rays at different (constant) acceleration potential differences (Mika and Reiss 1969). Total filtration. 3 mm Al

The number of X-rays emitted in the anode per unit energy interval increases with decreasing energy. The attenuation of the photons in the anode itself, the glass envelope and additional filter increases, however, still more with decreasing energy such that the number of low energy photons is heavily reduced. There are practically no photons with energies less than 10 keV, which are emitted from an X-ray tube with the above mentioned filters (Fig 5).

The sharp peaks shown in the energy spectra at 100 and 130 kV acceleration potential differences (Fig 5) are characteristic $K_\alpha$ and $K_\beta$ photons from tungsten.

Characteristic roentgen rays (fluorescence radiation) are emitted when a vacancy in an electron shell (here the K-shell) is filled with an electron from an outer shell. The emitted energy equals the difference in binding energy of the electron in the two shells. Vacancies in the K-shell can result from either ionisations caused by the accelerated electrons or from photoelectric absorption of bremsstrahlung photons (with energies higher than the binding energy of the electrons in the K shell) in the anode itself. In order to ionise the K-shell of tungsten an energy of 69.5 keV is needed. For characteristic K-photons to be emitted, the acceleration potential difference must exceed 70 kV.

From Fig 5 it can be seen that the relative proportion of characteristic K-radiation increases with incresing tube potential. This means that the imaging properties of the X-rays is only slowly varying with variations in the tube potential at tube potentials above 130 kV. Fig 5 shows how the number of photons varies at constant value of the tube charge (mAs-value = the product of tube current, mA and exposure time, s).

If instead the acceleration potential difference is kept constant and the charge through the X-ray tube (mAs) is increased, the shape of the energy spectrum remains the same, i.e., the relationship between the number of photons in the different energy intervals. The number of photons in each interval is proportional to the mAs-value.

b. Variation of the energy spectrum with filtering

Fig 6 shows the energy spectrum of photons from an X-ray tube with a beryllium window (Drexler and Perzl 1967). The beryllium (atomic number Z=4) window transmits much more low energy photons than the glass envelope. In the energy spectrum characteristic L-photons can be seen with energies around 9.5 keV. The low energy photons should normally be avoided since they do not penetrate fairly thick objects, i.e.,they do not contribute to the image formation but only to the energy deposition in the patient. As additional filters (Fig 6), sheets of 1, 2 and 4 mm of aluminium (Al) and 0.1 mm of copper (Cu) have been used.

The thin (0.1 mm) sheet of copper filters the X-rays as effectively as 3.5 mm Al at 20 keV, as 2 mm Al at 60 keV and as 1 mm Al at 95 keV.
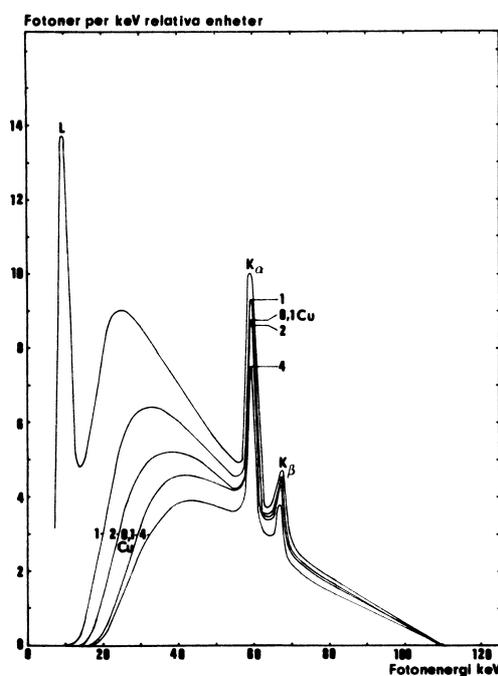


**Figure 6.** Energy spectra of X-rays with different filtrations. X-ray tube with a beryllium window. Filter: 0, 1, 2 and 4 mm aluminium and 0.1 mm copper.

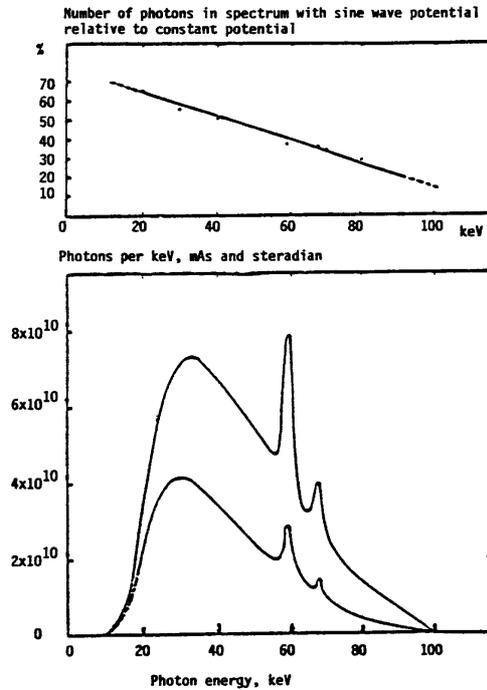## c. Variation of the energy spectrum with tube potential pulsations

The energy spectra shown in Figs 5 and 6 are valid in cases when the tube potential does not vary with time. In practice, alternating potentials derived from the electricity net (50 Hz, 220 V) are used and transformed to higher potentials.

The number of pulses per alternating current period is, with the simplest constructions, 1 or 2, with tubes of high quality 6 or 12 (Fig 7).
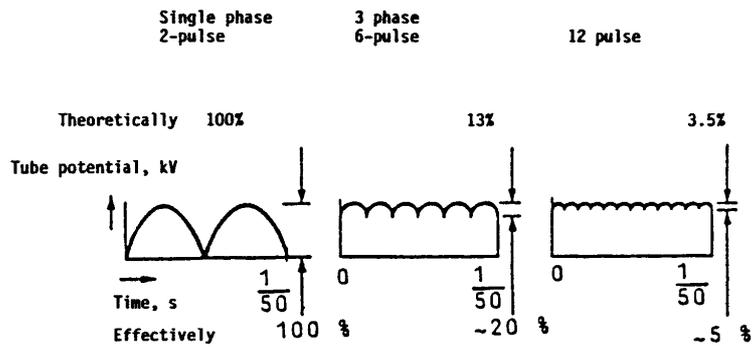


**Figure 7.** Tube potential variations (ripple) at 2, 6 and 12 pulses per alternating current period.

Wirh 12 pulses per period, the tube potential is very close to constant with time and the energy spectra in Figs 5 and 6 are valid. Figure 8 shows two energy spectra with peak potentials of 100 kV.



**Figure 8.** The upper spectrum is valid at constant tube potential, the lower at a sinusoidal variation of the potential (see 2-pulse, Fig 7). The figure above the energy spectra shows that the potential pulsations in particular reduces the number of high energy photons in the spectrum.

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Long cables or capacitances can counterbalance the fluctuations. The effect of this is most pronounced for 1- or 2 pulse generators at low tube currents (mA).

From Fig 8 it can be seen that increased potential variations (ripple) reduces the number of emitted X-ray photons per mAs and that the reduction is particularly high for the high energy photons. For instance, the number of photons obtained with a sinusoidal potential variation, relative to the number obtained at constant potential, is 20% at 90 keV and 65% at 20 keV.
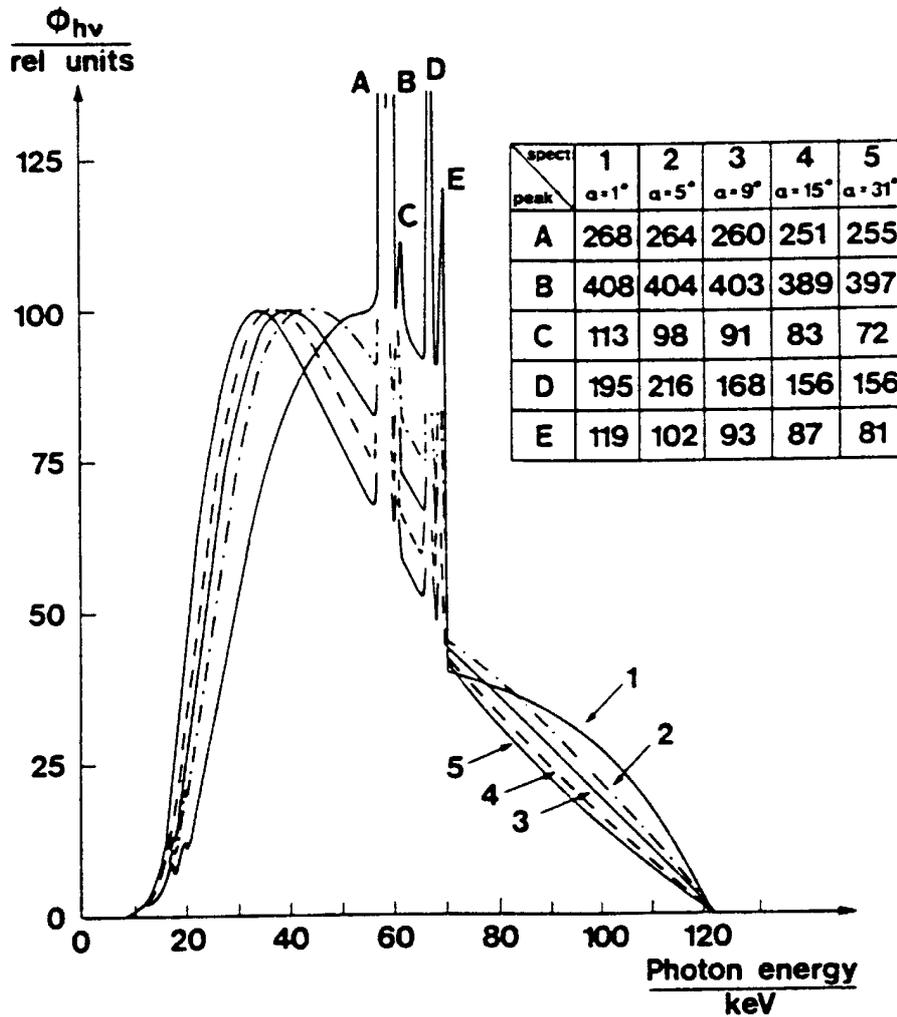
### d. Variation of the energy spectrum with the direction of emission

The anode of the X-ray tube in most cases forms an acute angle with respect to the direction of emission of the X-rays used for imaging. The X-rays emitted at acute angles to the anode plate are filtered in the anode. As a consequence, the number of photons decreases with decreasing angle between the anode plate and the direction of emission, this being more pronounced the lower the photon energies. The effect is called the heel effect and is most pronounced for small anode angles and increases in old X-ray tubes as a result of a rugged anode surface, Fig 9.
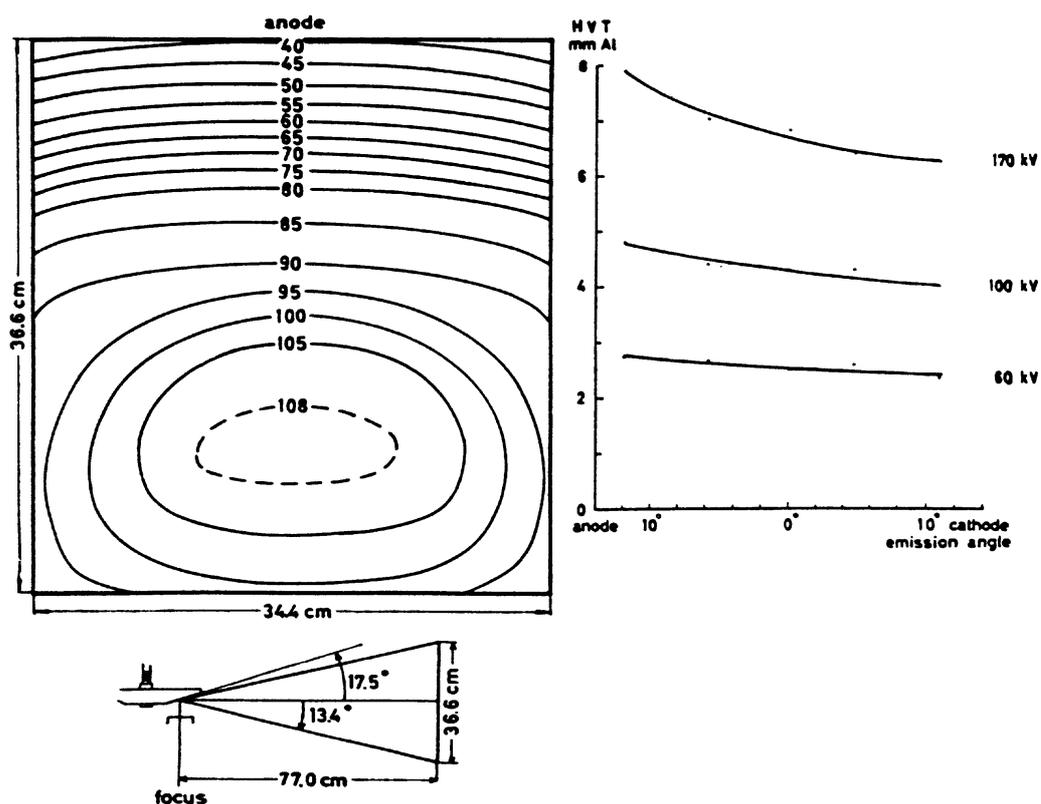
**Figure 9.** The heel effect. The left figure shows how the "amount of X-rays" is decreased on the anode side (here measured i terms of the air kerma (exposure)). The right figure shows how the "mean energy" of the photons (measured in terms of the ability to penetrate through aluminium (half value thickness HVT) increases on the anode side.

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The energy spectrum of the photons in different emission angles is shown in Fig 10 at 120 kV tube potential.



| spect peak | 1 $\alpha=1°$ | 2 $\alpha=5°$ | 3 $\alpha=9°$ | 4 $\alpha=15°$ | 5 $\alpha=31°$ |
|---|---|---|---|---|---|
| A | 268 | 264 | 260 | 251 | 255 |
| B | 408 | 404 | 403 | 389 | 397 |
| C | 113 | 98 | 91 | 83 | 72 |
| D | 195 | 216 | 168 | 156 | 156 |
| E | 119 | 102 | 93 | 87 | 81 |

**Figure 10.** Fluence energy spectra obtained at 120 kV tube potential in different directions ($\alpha$) of emission relative to the anode surface. The central ray corresponds to $\alpha=15°$. The peaks of the characteristic radiation are not fully illustrated but relative values of the photon fluence rate of the peaks, including the continuous distribution, are given in the table (Svahn 1977).

## 4. THE INTERACTIONS OF X-RAYS WITH MATTER

### a. Attenuation of photons

There are principally two interaction processes that give rise to the variation in photon transmission through the patient which is the basis of X-ray imaging. These are photoelectric absorption and scattering. The scattering is of two kinds: incoherent (Compton) scattering and coherent scattering. (At photon energies above 1 MeV, pair production is also possible. Such high photon energies are, however, not used in conventional X-ray diagnostics.)

A photon which has experienced an interaction process is lost as primary radiation: it has either been absorbed or has changed its energy and/or direction of motion. A photon that changes its direction of motion is called a scattered photon.

For monoenergetic photons, the number of photons removed from the primary beam when this is incident on a thin layer of material is proportional to the number of incident photons (N) and the thicness of the layer (dx), Fig 10.

$$dN = -\mu N dx \qquad (4)$$

where $\mu$ is a constant of proportionality called the linear attenuation coefficient.

The linear attenuation coefficient, $\mu$, is the probability, p, per unit length, x, for a photon of given energy to interact in passing through a given medium.

$$\mu = \frac{dp}{dx} \qquad (5)$$
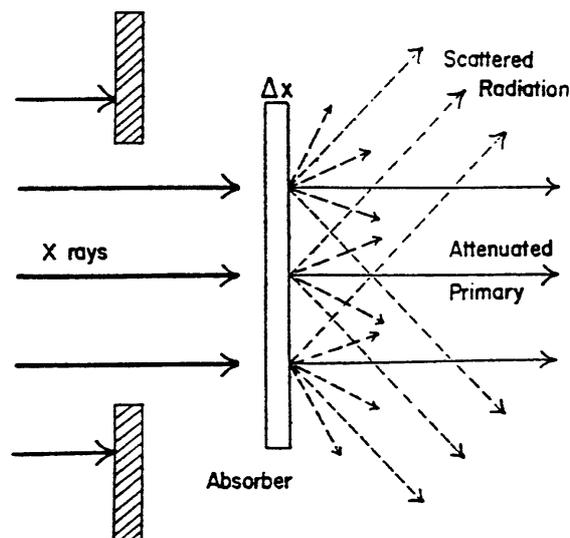


**Figure 11.** Attenuation of monoenergetic photons in a thin layer of a given material

The different interaction processes add their contributions to the total linear attenuation coefficient $m$

$$m = t + s_{coh} + s_{incoh} + k + \ldots \ldots \qquad (6)$$

where $t$, $s_{coh}$, $s_{incoh}$ and $k$ are the contributionsto the attenuation from photoelectric absorption, coherent scattering, incoherent scattering and pair production.

The solution to the differential equation, Eq(4), is

$$N(x) = N(0) \cdot e^{-mx} \qquad (7)$$

From Eq(7), it is seen that the primary photons are attenuated exponentially with the penetration depth x.

b. Photoelectric absorption

In this process, the photon is absorbed and an electron (photoelctron) is emitted from the atomic shell with kinetic energy, T

$$T = hn - B_i \qquad (8)$$

where $B_i$ is the binding energy of the electron in atomic shell $i$. In Fig 12, the process is shown schematically.
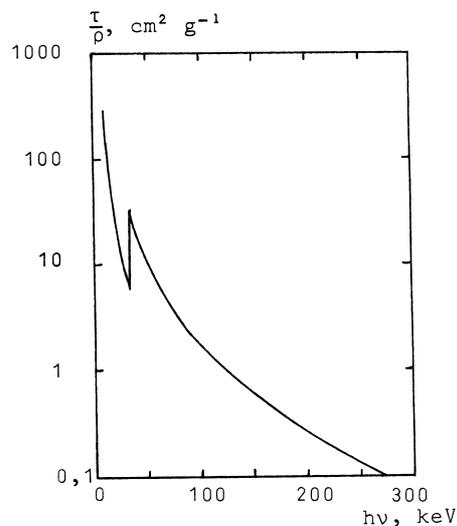
As a result of the photoelectric absorption, a vacancy is created in the atomic shell. When this vacancy is filled with an electron from an outer shell, energy is liberated corresponding to the difference in the binding energies of the inner and outer shells. This energy is emitted either in the form of a photon, *characteristic roentgen radiation (fluorescence radiation)* or is transferred to an electron in an outer shell. This electron is emitted from the shell and is called an *Auger electron*. The probability for emission of characteristic roentgen radiation subsequent to a photoelectric absorption increases with increasing atomic number and is largest for vacancies in the K-shell. The fluorecsence yield increases with increasing binding energy of the photoelectron.
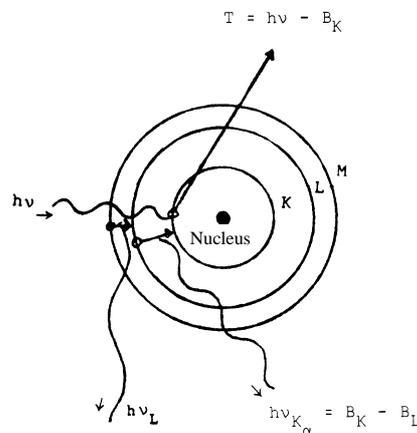


**Figure 12.** Photoelectric absorption and subsequent deexcitation of the atomic shell

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The emission of characteristic roentgen rays (and Auger electrons) is isotropic (equal probability for all directions of motion). It occurs with no memory of the direction of motion of the absorbed photon.

The probability per unit length, $t$, for photoelectric absorption decreases with increasing photon energy until an absorption edge is reached. In passing the absorption edge, more electrons are available for absorption and the probability of the process increases dramatically. In passing the K-edge (the innermost shell), the probability of photoelectric absorption increases with a factor of 4-14, higher factors for lower atomic numbers. In Fig 13, the probability for photoelectric absorption is shown for iodine (Z=53). Here, the mass attenuation coefficient, $m/r$ is given, i.e., the linear attenuation coefficient divided by the density of the material.



**Figure 13.** The contribution from photoelectric absorption, $t/r$, to the mass attenuation coefficient for iodine as a function of photon energy

In Fig 14 , values of $t/r$ are shown for a number of different materials.
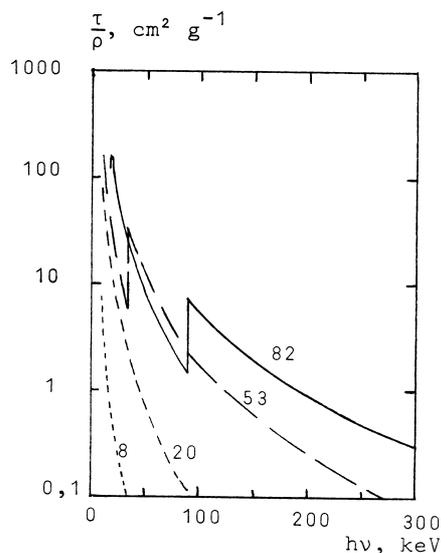


**Figure 14.** The dependence of the photoelectric absorption on photon energy, h$n$, and atomic number, Z.

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<u>c. Incoherent scattering (Compton process)</u>

At an incoherent scattering, a photon collides with an atomic electron, Fig 15. The photon is scattered with reduced energy. The electron receives the kinetic energy $T = h\boldsymbol{n} - h\boldsymbol{n}'$, where $h\boldsymbol{n}$ is the energy of the incident photon and $h\boldsymbol{n}'$ is that of the scattered photon.
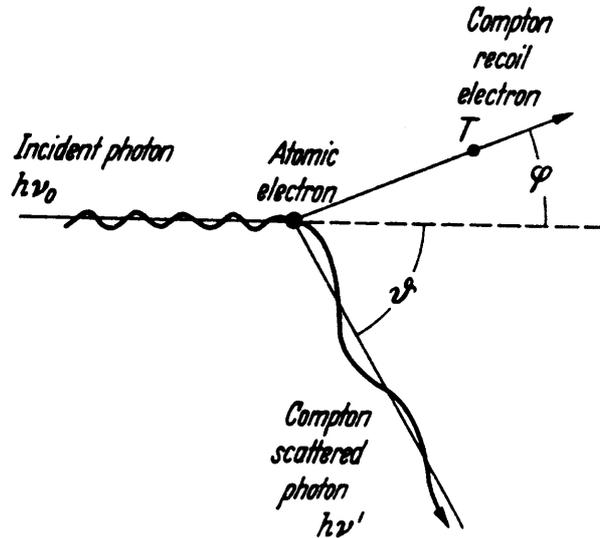


**Figure 15.** The Compton process

A simple relationship (the Comption equation) exists between the energy of the scattered photon and the scattering angle $\boldsymbol{q}$.

$$h\boldsymbol{n}' = \frac{h\boldsymbol{n}}{1 + \dfrac{h\boldsymbol{n}}{m_0 c^2}(1 - \cos \boldsymbol{q})} \tag{9}$$

The scattered photons have an angular distribution which is shown in Fig 16. At low primary photon energies, forward and backscattering are about equally probable.At higher energies, scattering in the forward direction predominates.

The contribution of the process, $\boldsymbol{s}_{incoh} / \boldsymbol{r}$, to the total mass attenuation, $\boldsymbol{m} / \boldsymbol{r}$, varies slowly with photon energy, Fig 17. Form Fig 17 it can also be seen that $\boldsymbol{s}_{incoh} / \boldsymbol{r}$ decreases slowly with increasing atomic number, Z.
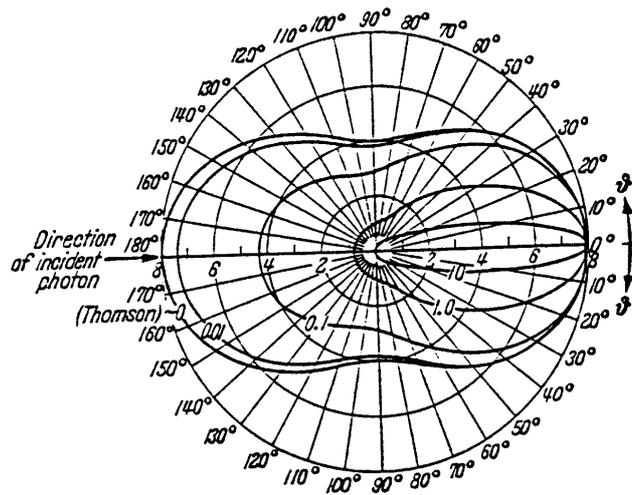
**Figure 16.** The number of scattered photons per unit solid angle, $\mathrm{d}(_e\boldsymbol{s}_{\mathrm{incoh}})\,/\,\mathrm{d}\Omega$, at scattering angle $\boldsymbol{q}$. Energies of the interacting photons are 0, 0.01, 0.1, 1.0 and 10 MeV. The radius of the polar diagram is given in units of $10^{-26}$ cm$^2$ per electron and steradian.
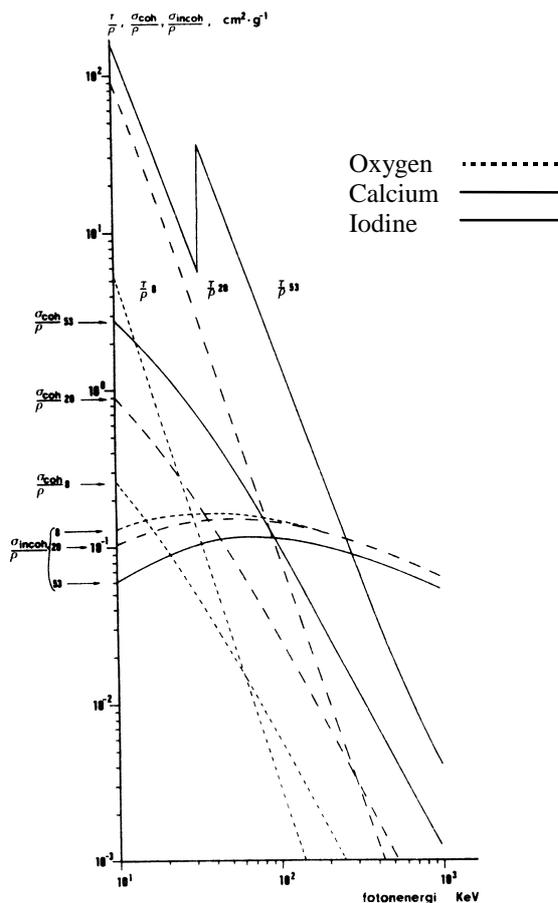


**Figure 17.** Contribution to the mass attenuation coefficient from photoelectric absorption, incoherent and coherent scattering as a function of photon energy. The influence of the atomic number is illustrated by showing values for oxygen (Z=8), calcium (Z=20) and iodine (Z=53).
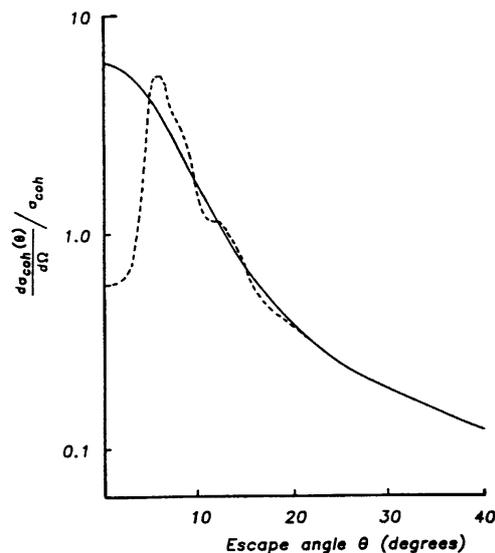
### d. Coherent scattering

*Atomic coherent scattering*

Coherent scattering is a process in which the photons are scattered by bound electrons without tansferring energy to the electrons. The recoil energy is taken up by the whole atom and the scattered photons thus loose a negligible fraction of their energy. The direction of motion is changed according to a distribution that is determined by interference among the electrons in the atom. The forward direction is preferred, the peaking in this direction increasing with increasing photon energy and with decreasing atomic number.

The probability for coherent scattering decreases with increasing photon energy and increases with increasing atomic number (increasing number of interfering electrons).As a fraction of the total mass attenuation coefficient, $s_{coh} / r$ is maximal at atomic numbers around Z=10 and photon energies in the interval 30-50 keV. At higher atomic numbers, the relative fraction decreases due to the strong increase of photoelectric absorption with increasing atomic number. For aluminium, the relative probability for coherent scattering is 13% at 50 keV.

*Molecular effects*

In a molecule, coherent scattering is determined by the interference of all the electrons in the molecule. The effect is not very well known for most molecules. However, for water it has been determined experimentally and expressed in terms of scattering probabilities (Morin 1982). In Fig 18, the angular distribution for photons scattered coherently against water molecules is compared to the scattering against a mixture of independent hydrogen and oxygen atoms.



**Figure 18.** The relative number of coherently scattered photons per unit solid angle, $\dfrac{d s_{coh} / d\Omega}{s_{coh}}$, as a function of scattering angle $q$ Dashed line: water molecules; solid line: mixture of independent hydrogen and oxygen atoms (proportion 2 : 1).

*Crystalline effects*

In crystalline material, the interference is extended to electrons from many regullay spaced atoms and the coherent scattering is more concentrated in given directions. This is called Bragg- or Laue-diffraction. The probabilty for Bragg-diffraction decreases rapidly with increasing photon energy, i.e., when the wave-length gets small compared to the spacing between the atomic planes.

## 4. NET EFFECTS OF INTERACTIONS

### a. Penetration ability of X-rays

In section 3a) it was shown that the number of monoenergetic photons that penetrate an object decreases exponentially with increasing object thickness. From Fig 6, it can be deduced that the penetration ability in aluminium and copper increases with increasing photon energy. Generally, the penetration ability increases with increasing tube potential. Exceptions from this rule exist if the object contains absorption edges within the actual photon energy spectrum.

By increasing the tube potential, a larger fraction of the incident photons will reach the detector (image receptor). If the detector absorbs all of this radiation, exposure time may be decreased resulting in lower patient absorbed doses (for imaging systems that require a certain amount of energy to be absorbed, e.g., conventional screen-film imaging). The reduction in exposure time achieved by increasing the tube potential will, however, mainly be due to the increased bremsstrahlung yield at higher tube potentials (see Fig 5).

### b. The filtering effect of the patient

Figure 19 shows how 20 cm of water (simulating a patient) changes the spectral distribution of the transmitted primary photons (beam hardening) compared to the incident ones. Almost all photons with energies below 30 keV are removed from the primary beam in passing through the water layer.

Water is fairly equivalent to most soft tissues regards attenuation, scattering and energy absorption of X-rays. After passage of 20 cm water, the transmitted photons have energies above 30 keV (Fig 19). Above 20 keV, incoherent scattering is the dominating interaction process. Since incoherent scattering does not vary much with photon energy, the X-ray image of a thick layer of body tissues mainly shows density differences between the tissues. Fat which has low density can be seen against other soft tissues. The skeleton, which contains calcium is seen with good contrast due to the photoelectric effect. In order to utilise photoelectric absorption to distinguish between soft tissues with small differences in atomic composition and density, low energy photons, below 30 keV (Fig 17), are needed as in, for instance, mammography.
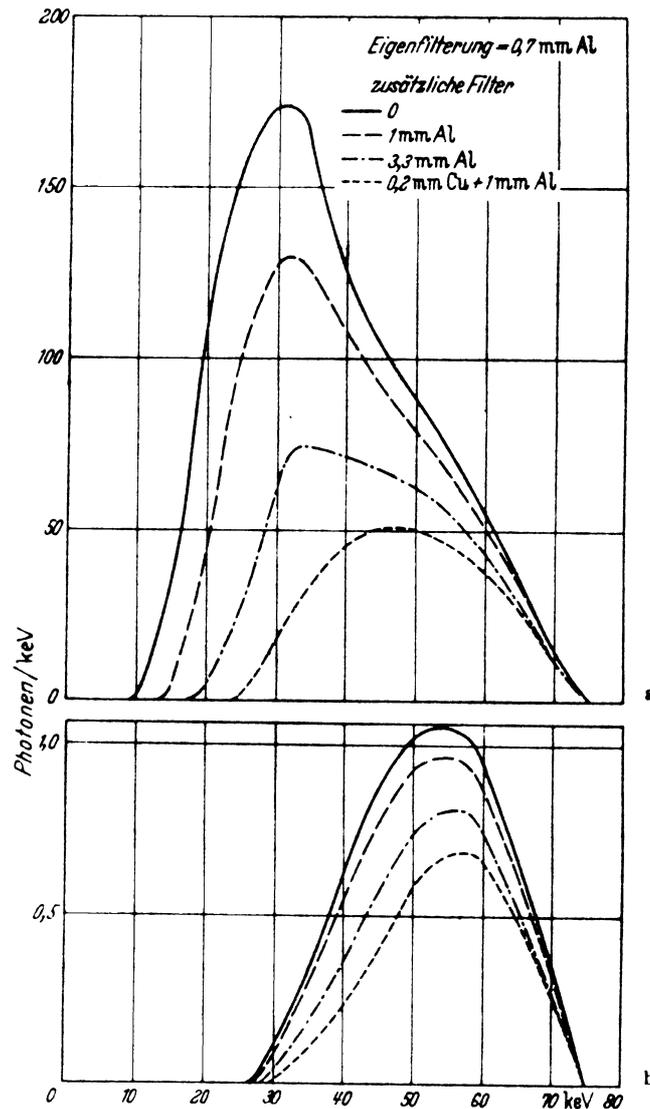
**Figure 19.** Attenuation of primary photons in passing through 20 cm of water. Tube potential: 75 kV; different added filters: zero, 1 mm Al, 3.3 mm Al, 0.2 mm Cu + 1 mm Al (Carlsson 1962).

c. Scattered radiation

The scattered radiation transmitted through the patient degrades image contrast and contributes to the irradiation of organs distant from the primary beam as well as to personel present in the examination room.

5. CONTRAST

Contrast is the deviation in some quantity of the radiation, fluence, energy fluence (radiation relief) or signal from the detector behind and beside a contrasting detail (Fig 20).

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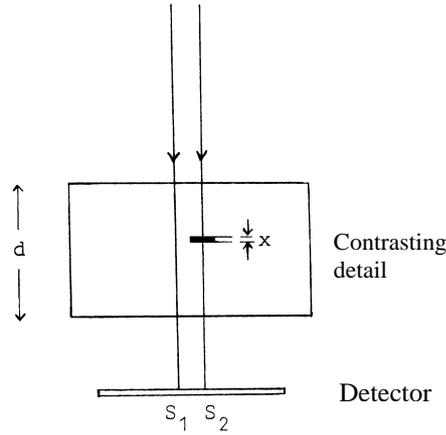
**Figure 20.** Illustration of the concept of contrast.

The contrast, C, can be written

$$C = \frac{S_1 - S_2}{S_1} \qquad (10)$$

where $S_1$ is the signal from the detector (per unit area or pixel element) beside the contrasting detail and $S_2$ that behind the detail. The signal is assumed to be proportional to the energy imparted to the detector (per unit area or pixel element).

In the case of film as image receptor, the signal is the optical density OD. The image contrast is then usually defined as the optical density difference beside and behind the detail

$$\Delta OD = OD_1 - OD_2 \approx \gamma^{10} \log \frac{S_1}{S_2} = \gamma \cdot 0.434 \cdot \ln \frac{S_1}{S_2} \qquad (11)$$

where the last equalities are valid for thin contrasting details such that C<<1; $\gamma$ is the film gradient as determined from the H&D curve, showing OD as a function of log exposure or energy imparted to the screen per unit area (proportional to the amount of light emitted from the screen).

Provided $\gamma$ is constant independent of OD, Eq (11) is valid without restrictions on C. This is approximately true for screen-film combinations over large intervals of OD. For direct film, $\gamma$ increases steadily with OD until the region of saturation is reached.

a. Calculation of contrast for monoenergetic photons

For the situation in Fig 19, with monoenergetic photons and no scattered radiation reaching the detector, the absorbed energy in the detector can be written

$$e_1 = e_0 \cdot e^{-m_1 d} \qquad (12)$$

$$e_2 = e_0 \cdot e^{-m_1(d-x)} \cdot e^{-m_2 x} = e_0 \cdot e^{-m_1 d} \cdot e^{-(m_2-m_2)x} \qquad (13)$$

where $e_0$ is the energy absorbed in the detector witth no object present, d = the thickness of the object (linear attenuation coefficient $\mu_1$) and x=the thickness of the contrasting detail (linear attenuation coefficient $\mu_2$).

Then, C is obtained as

$$C = \frac{S_1 - S_2}{S_1} = 1 - \frac{e_2}{e_1} = 1 - e^{-(m_2-m_1)x} \qquad (14)$$

which for small values of x reduces to

$$C \approx (m_2 - m_1)x \qquad (15)$$

The value for $\Delta$OD in Eq(11) can similarly be reduced to

$$\Delta OD \approx g \cdot 0.434(m_2 - m_1)x \qquad (16)$$

The contrast is proportional to the difference in the linear attenuation coefficients of the contrasting detail and the background and to the thickness of the detail. When scattered radiation can be neglected, the contrast is independent of the thickness, d, of the object and where in the object the contrasting detail is situated.

*Calculation of contrast with polyenergetic photons*

With an energy spectrum of primary photons, the contrast depends on object thickness and is determined by the energy spectrum of transmitted primary photons. The energy dependent contrast, C, Eq(15), is for polyenergetic photons weighted over the relative contributions of the photons to the energy imparted to the detector. These calculations require knowledge about the fraction of the energy of incident photons which is imparted to the detector. This fraction (imparted fraction IF) is energy dependent and may be difficult to calculate, often requiring Monte Carlo simulation of the transport of the photons and their secondary photons in the detector.

b. Contrast with scattered radiation

With scattered radiation present, contrast is degraded. Here, it is assumed that scattered radiation contributes equally, with the signal $S_s$, to the signals $S_1$ and $S_2$. This means that the contrasting detail is assumed to be small and to be placed in the interior parts of the object. Then

$$C = \frac{S_1 - S_2}{S_1} = \frac{S_{1,p} + S_s - (S_{2,p} + S_s)}{S_{1,p} + S_s} = \frac{S_{1,p} - S_{2,p}}{S_{1,p} + S_s} =$$

$$= \frac{S_{1,p} - S_{2,p}}{S_{1,p}(1 + \frac{S_s}{S_{1,p}})} = \frac{C_p}{1 + S_s / S_{1,p}} = C_p \cdot CDF$$

(17)

Here $C_p$ is the primary contrast due to the primary photons and CDF is the contrast degradation factor due to the scattered photons. The contrast degradation factor is the inverse of 1 plus the scatter-to-primary ratio S/P (the ratio of the contributions to the signal from scattered and primary photons).

The scatter-to-primary ratio may take values as high as 5-10 in X-ray diagnostic examinations of thick body parts using large field areas. In cases with large amounts of scatter, scatter rejection is indispensable to achieve acceptable contrast (image quality)

## 6. ENERGY ABSORPTION OF X-RAYS

As a consequence of photoelectric absorption and incoherent scattering, energetic secondary electrons are liberated in the irradiated medium. These electrons impart their energies to the medium through *ionisations* and *excitations* of atoms and molecules along their tracks. An ionisation means that the electron is liberated from its atom or molecule, an excitation that it is lifted into an outer shell. The energy of the secondary electrons thus deposited in the medium gives rise to absorbed doses in patients and radiation detectors.

a. Energy imparted by ionising radiation to the matter in a volume, $\varepsilon$

*Definition:* $\qquad\qquad \boldsymbol{e} = R_{in} - R_{out} + \sum Q$ (18)

$R_{in}$ is the radiant energy (the sum of the energies (minus rest energies) of all ionising particles) incident on the volume

$R_{out}$ is the radiant energy escaping from the volume

$\sum Q$ is the sum of all changes in the rest mass energies of nuclei and elementary particles which have occurred due to nuclear reactions and elementary particle transformations in the volume

Since X-rays used for imaging do not cause nuclear reactions or elementary particle transformations, $\sum Q$ is zero. The incident and escaping ionising particles are photons and secondary electrons.

b. Absorbed dose, D

*Definition:*
$$D = \frac{d\bar{\varepsilon}}{dm} \tag{19}$$

where $d\bar{\varepsilon}$ is the mean energy imparted (statistical expectation value) to a volume of mass dm. Absorbed dose takes a value at each point in an irradiated medium. The mean energy imparted to a body is the mass integral of the absorbed dose distribution in that body. It has therefore also been called the integral dose.

*Units:* Absorbed dose is energy per unit mass and is in SI units given by J/kg. It has also been given the special unit of gray (Gy)
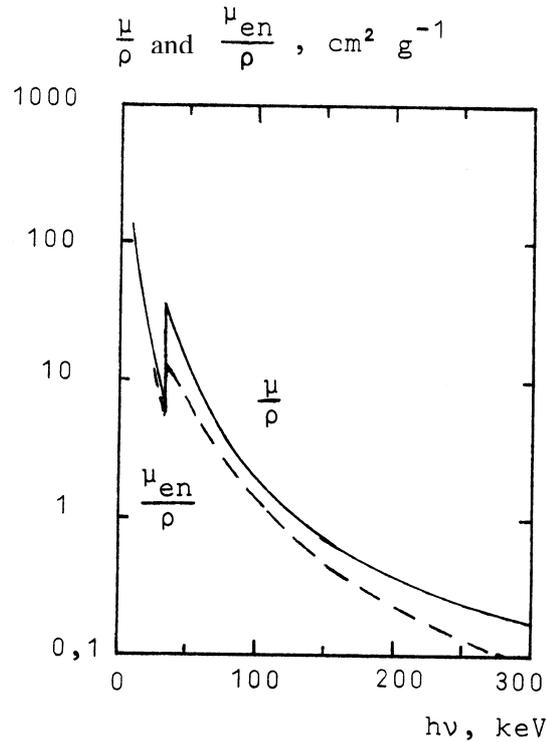
$$1 \text{ Gy} = 1 \text{ J/kg}$$

c. Mass energy absorption coefficient, $\mu_{en}/\rho$

The mass energy absorption coefficient for photons is given by

$$\frac{\mu_{en}}{\rho} = \frac{h\nu - \sum h\nu'}{h\nu} \cdot \frac{\mu}{\rho} \tag{20}$$

$\sum h\nu'$ is the mean energy of all secondary photons emitted as a result of an interaction by a photon. As secondary photons, also those generated by the secondary electrons are included, e.g., bremsstrahlung photons. The mass energy absorption coefficient thus represents the energy transferred by photons to secondary electrons, which is locally absorbed close to the interaction point. Values of $\mu_{en}/\rho$ can be obtained from tabulations (Storm and Israel 1970, Hubbell and Seltzer 1995).

The value of $\mu_{en}/\rho$ is always smaller than $\mu/\rho$ (Eq(19). This difference increases for photon energies above the K-absorption edge since the characteristic K-rays contribute to $\sum h\nu'$. The relationship between $\mu_{en}/\rho$ and $\mu/\rho$ is shown in Fig 21 for iodine.

**Figure 21.** Mass attenuation- and mass energy absorption coefficients for iodine

c. Thick objects

The secondary electrons liberated by diagnostic X-rays, have ranges (<0.1 mm in soft tissue) that are much smaller than the mean free paths ($1/\mu$) of the photons (<100 mm in soft tissue). For objects that absorb a large fraction of the incident photons, e.g., patients and intensifying screens, the absorbed energy (energy imparted) is obtained from Eq(18) by considering only the incident and escaping photons. The electrons can be considered to be locally absorbed.

d. Thin objects

For objects that are so thin that they do not appreciably attenuate the incident photons but are still thick compared to the ranges of the secondary electrons, the absorbed dose can be calculated from (monoenergetic photons)

$$D(h\nu) = \frac{m_{en}}{r} \cdot \Psi \qquad (21)$$

where $\Psi$ is the energy fluence of the photons.

e. Energy absorption in the image receptor

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## Effects of the angle of incidence

Energy spectra of primary and scattered photons are often given in terms of fluence per energy interval. Since the fluence is the number of photons per unit area, with the area perpendicular to the direction of motion of the photons, the photons are measured with a totally absorbing detector oriented perpendicularly to the different directions of motion of the scattered photons, Fig 22. The fluence is obtained by summing the fluences measured in the different directions.
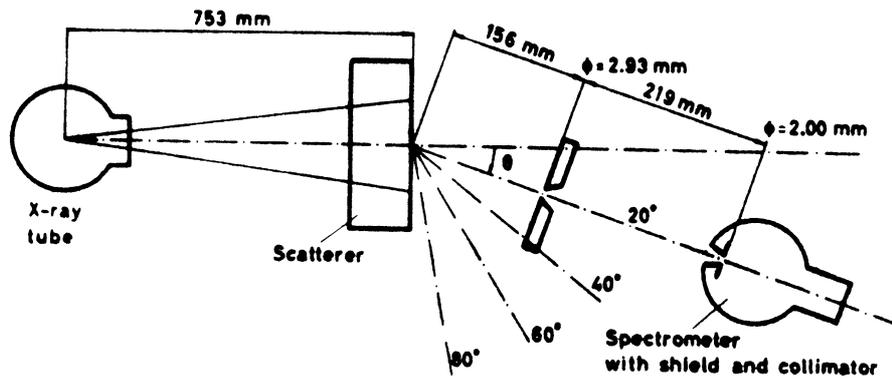


**Figure 22**. Measurement of the fluence of scattered photons

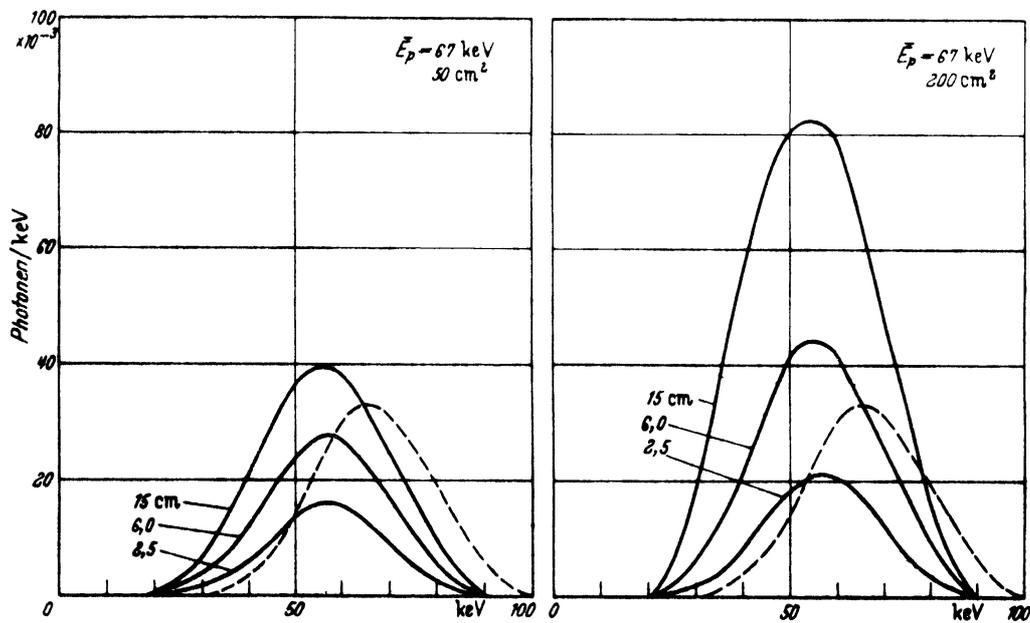The resulting fluences of primary and scattered photons are shown in Fig 23.



**Figure 23.** Energy spectra of fluences of primary and scattered photons

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*Thin receptor*

In a thin receptor, only a small fraction of the incident X-ray photons are absorbed and the conditions for Eq(20) are valid as for, e.g., direct film. This means that the absorbed dose in the receptor is proportional to the fluence, independent of the directions of motion of the photons. For the blackening of direct film (film without intensifying screens), the same is valid as for the absorbed dose.

*Thick receptor*

In a thick receptor, i.e., a receptor that absorbs the main part of the incoming photons, as, e.g., intensifying screens at low tube potentials, each photon contributes to the signal in proportion to its energy. The absorbed energy is thus proportional to the energy of the photons. At oblique incidence, the number of photons per unit area at the surface of the plane receptor, is lower than per unit area perpendicular to the direction of motion of the photons, such that the energy absorbed per unit area of the receptor decreases with increasing angle of incidence for a given value of the fluence.

While a thin (plane) receptor yields a signal per unit area that is independent of the angle of incidence, a thick receptor yields a signal that decreases with increasing angle of incidence (relative to the normal).

**Effects of photon energy energy**

In a totally absorbing receptor, the absorbed energy, $\varepsilon$, from a photon equals the photon energy, $h\nu$. If the signal S is proportional to the absorbed energy, it is also proportional to the photon energy. That is, compared to an energy spectrum that gives the fluence per unit energy interval, $d\Phi(h\nu) / d(h\nu)$, the photons influence the signal more the higher the photon energies.

In a thin receptor, the absorbed dose is for a given fluence proportional to the product of the mass energy absorption coefficient, $\mu_{en} / \rho$, and the photon energy, $h\nu$. The image receptors used in X-ray diagnostics contain materials of high atomic numbers. For these, the mass attenuation coefficient decreases so rapidly with increasing photon energy that even the product $(\mu_{en} / \rho) \cdot h\nu$ decreases with increasing photon energy (provided no absorption edges are present in the energy spectrum of the photons). Therefore, in general using thin receptors, low energy photons contribute more to the absorbed energy per unit area (the product of absorbed dose and the mass per unit area of the receptor) and to the signal than is shown by the energy spectrum of the fluence.

**Net effects of photon energy and angles of incidence**

In a thin receptor the signal per unit area is independent of the angle of incidence and in general decreases with increasing energy. On the contrary, in a thick receptor, the signal decreases with increasing angle of incidence and increases with increasing photon energy.

<u>f. Influence of the receptor on contrast</u>

Image contrast depends on the characteristic curve of the receptor, i.e., how the signal S varies with the absorbed energy per unit area, but also on how the receptor absorbs the incident X-ray energy depending on the energy spectrum and directions of motion of the photons. While a thick receptor degrades the contrast of the primary photons (compared to a thin receptor), contrast is degraded more by the scattered photons in a thin receptor compared to a thick receptor. These two effects counterbalance each other.

An interesting special case is the contrast obtainable when the contrasting detail and the recpetor contain the same material and this material has an absorption edge which falls within the energy spectrum of the X-rays used. For instance, consider an image intensifier screen of CsI as recpetor and iodine as contrasing agent. Since the contrasting agent transmits most photons just below the absorption edge of iodine and these are less efficiently absorbed by the receptor, the contrast is enhanced. However, if the object is fairly thick (above 10 cm, Fig 19), there are few photons remaining below the absorption edge (33 keV) and the effect is negligible in practice with iodine. With a higher atomic number ($Z \approx 70$) in receptor and contrasting detail, the effect can possibly be utilised.

7. STOCHASTICS IN THE X-RAY IMAGE

With increasing photon energy, the photon transfers increasingly higher energies to the secondary electrons (the common exception at absorption edges will be treated below). As a consequence, increasing energy is absorbed per interaction process in the receptor. If, as is commonly the case in conventional X-ray diagnostics, the mean energy absorbed per unit area of the receptor is kept constant (to keep the optical density constant) independent of the X-ray energy spectrum, the signal is produced with a decreasing number of interacting photons with increasing photon energies.

The number of photons, N, which interact per unit area of the receptor is a stochastic variable with a standard deviation $\sigma$. If N is assumed to be Poisson distributed, one has

$$s = \sqrt{N} \qquad (22)$$

The relative standard deviation is

$$\frac{s}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} \qquad (23)$$

which decreases with increasing N.
For small values of N (high photon energies) the fluctuations in signal per unit area in the receptor will be large. This so called quantum noise increases, for a given receptor, in general with increasing photon energies.

Photons with energies just above an absorption edge in the receptor material, give a small contribution to the absorbed energy as a result of photoelectric absorption. The photo electrons have negligible energies (Eq(8)). Most of the energy of the absorbed photon appears in the form of a characteristic K-photon. The locally absorbed energy is mostly due to low energy Auger electrons. Photoelectric absorption has high probability at

energies just above the K-edge and since the absorbed energy per interaction process is small, these photons contribute to reducing the quantum noise. The quantum noise impairs image quality more, the better the signal transfer characteristics (modulation transfer function, MTF) of the receptor. By choosing a receptor with less good MTF, the quantum noise can be lowered. This is equivalent to averaging the number of interacting photons in larger area elements. This can simply be accomplished by watching an image, reproducing the object with high MTF, through an unfocussed lens. Figure 24 shows how an image with high quantum noise impairs the possibility to interpret its information.
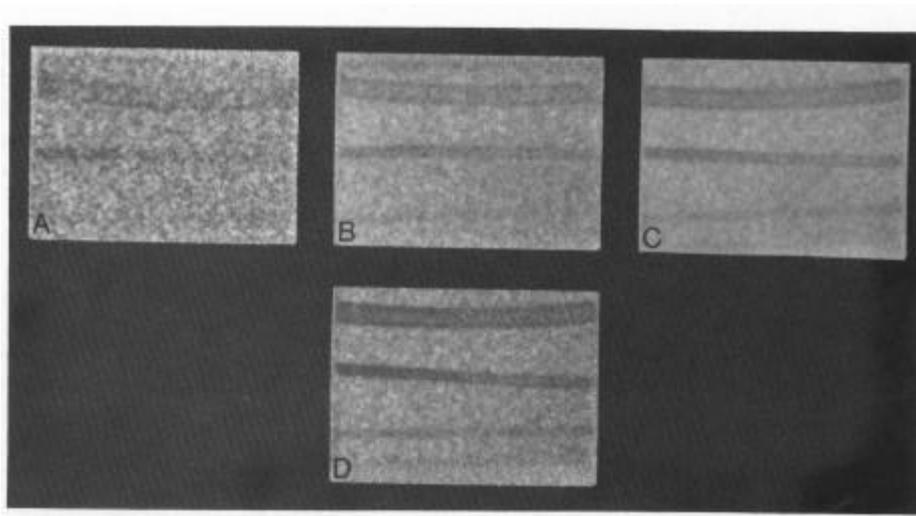


**Figure 24.** Noise in the image impairs the possibility to interpret its information.

8. APPENDIX

**Definitions of physical quantities**

Fluence $\Phi$

$$\frac{d\Phi}{da}$$

where dN is the number of particles passing through a sphere of cross sectional area da.

Flux density or fluence rate $f$ is the time derivative of the fluence

$$f = \frac{d\Phi}{dt}$$

Energy fluence $\Psi$

$$\Psi = \frac{dR}{da}$$

where dR is the radiant energy (sum of energies of ionising particles excluding rest masses) passing through a sphere of cross sectional area da

For mono-energetic particles of energy E (rest mass $m_0$) and kinetic energy T

$$\Psi = (E - m_0 c^2) \cdot \Phi = T \cdot \Phi$$

Mean free path of a photon is the distance travelled on the average before it interacts. The mean free path is given by $1/\mu$ where $\mu$ is the linear attenuation coefficient.

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