The Mathematics behind Anamorphic Art

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Abstract

In this paper, we will explore the connections between the mathematical and artistic formulations of anamorphosis. The goal of this project is to provide an artist with the tools that can assist in the creation of an anamorphic image. These tools are mathematical formulas that are used to form printable grids which can allow the artist to focus on a more precise image formation. These formulas describe bijections between the points on the surface and the points in the horizontal plane where the distorted image is placed. This paper provides a general set-up for using any parametric surface as a reflecting object. We then look at a specific example using the surface of a cylinder.

Introduction

Mirror anamorphosis is a distorted projection requiring the viewer to use special devices to reconstitute an image (see [1]). The word anamorphosis is derived from the Greek prefix *ana*, meaning back or again, and the word *morphe*, meaning shape or form (see [1]). In mirror anamorphosis, the most common surfaces used are a cone and a cylinder (see [1]). Mirror anamorphic images are created in a flat horizontal plane in which a reflective surface is then placed at a specific location on the horizontal plane so that the image in the horizontal plane can be reflected onto the surface to reveal the original image.

Several participants of the Bridges conference dedicated their research to subjects involving anamorphosis and anamorphic art (see [2], [3]). Notably, Phillip Kent has developed a software program *AnamorphMe!* to automate the creation of anamorphic images. In 2011, anamorphic art was presented by Jan Marcus at Bridges using the distortion of fractals under cylindrical distortion (see [4]). Our goal is to provide formulas that provide a bijection between the pixels of the distorted image and the pixels of the original image. These formulas allow us to create grids that can be printed out and used by the artist.

Mirror Anamorphosis for General Surfaces

In this section, we describe the mirror anamorphosis for general parametric surfaces. We then use this set-up in the particular case of the cylinder. We place the original image in the plane y=0, with $z \geq 0$, translate the image onto a parametric surface and describe the connection between the image on the surface and with the distorted image in the plane z=0. More precisely, consider a parametric surface S given by $\vec{r}(x,z) = < x(x,z), y(x,z), z(x,z) >$, for (x,z) in some domain D and in the plane y=0 with $z \geq 0$. Given a viewer's position $V = (x_o, y_o, z_o)$ our goal is to describe the relation between the points on the plane z=0 and the reflection of those points on the surface S. We say a point $P = (x, y, z) \in S$ is **<u>visible from *V*</u>** if the line segment VP does not contain points of S other than P. To each of the visible points we attach three vectors. The first vector is the vector $\overrightarrow{VP}$, called the **<u>observation vector</u>** at P. The second vector is the **<u>normal vector</u>** to S at P, denoted $\vec{n}(P)$. The third vector is the **<u>reflection vector</u>** at P, the unique vector $\vec{b}$ such that $\vec{b}$ is in the plane containing the observation vector to P and the normal vector $\vec{n}(P)$. Also, the angle created between the observation vector and the normal vector is equal to the angle between the normal vector and the reflection vector. We say a point $P \in S$ is **<u>reflecting</u>** if the z-coordinate of the reflection vector at P is negative. If P is reflecting, then the line ℓ_p through P with direction vector $\vec{b}$

intersects the plane z=0. The point of intersection of ℓ_p with z=0 is called the **reflection point** of P onto the plane z=0.

Cylinder Anamorphosis Formulas

The fixed input parameters are the viewer's position $V = (x_o, y_o, z_o)$ and the radius of the cylinder R. The cylinder is parameterized by $r(x,z) = < x, \sqrt{R^2 - x^2}, z >$, the normal vector is $< \frac{x}{\sqrt{R^2-x^2}}, 1, 0 >$ and the reflecting vector is $\vec{b} = < \frac{-R^2(x+x_0)+2x(xx_0+yy_0)}{R^2}, \frac{-R^4-2x^2(xx_0+yy_0)+R^2(x^2+2xx_0+yy_0)}{R^2 y}, z - z_0 >$, where $y = \sqrt{R^2 - x^2}$. We assume that the x-coordinate of the viewer's position is restricted to $-R \leq x_0 \leq R$. Applying the process described in the general set-up to the case of the cylindrical surface, we obtain the following formulas for the x and y coordinates of the reflection of the point $P = r(x,z)$ as viewed from $V = (x_0, y_0, z_0)$ onto the plane $z = 0$:

$$x_{\text{ref}} = F(x,z) = \frac{-2R^2xz - R^2x_0z + 2x^2x_0z + 2x\sqrt{R^2-x^2}y_0z + R^2xz_0}{R^2(z-z_0)}$$

$$y_{\text{ref}} = G(x,z) = \frac{2x(-\sqrt{R^2-x^2}x_0 + xy_0)z + R^2(2\sqrt{R^2-x^2}z - zy_0 - \sqrt{R^2-x^2}z_0}{R^2(z-z_0)}$$

Figure 1 contains the image of a rectangular grid transformed using cylinder anamorphosis described in this section.

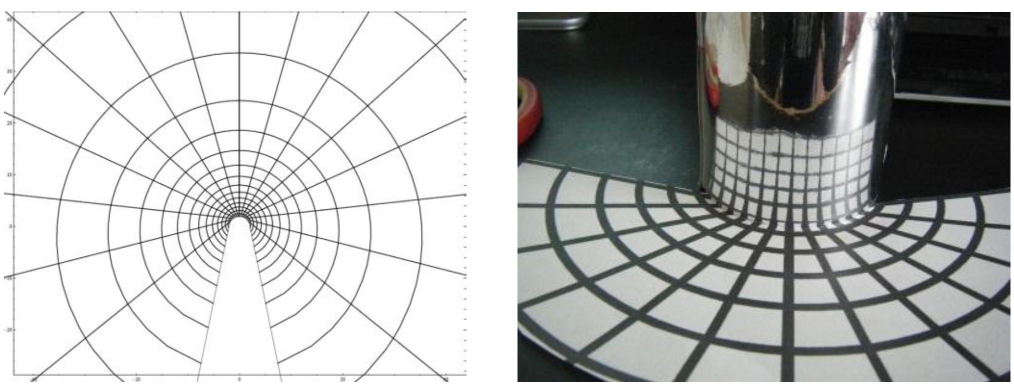

Figure 1: *Transformation of a rectangular grid using the formulas found (left) with the cylinder reconstituting the image (right).*

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References

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