

# Pressure Calculation of a Constant Density Star in the Dynamic Theory of Gravity

*Ioannis Iraklis Haranas*  
*Department of Physics and Astronomy*  
*York University*  
*314 A Petrie Science Building*  
*York University*  
*Toronto – Ontario*  
*CANADA*  
*e-mail: ioannis@yorku.ca*

## Abstract

In a new theory called Dynamic Theory of Gravity, the gravitational potential is derived from gauge relations and has a different form than the classical Newtonian potential. In the same theory an analytical expression for the pressure is derived from the equation of the hydronic equilibrium which is solved for a star of constant density and the results are compared with those of Newtonian gravity. Changes then in the central pressure and radius are also calculated and finally a redshift calculation is performed so that the dynamic gravity effects if any might be shown to be of some detectable magnitude.

**Key Words** Dynamic theory of gravity, gauge fields, Weyl's quantum principle, field equations, Christoffel symbols, energy-momentum tensor.

## 1. Introduction

There is a new theory called the Dynamic Theory of Gravity [DTG]. It is derived from classical thermodynamics and requires that Einstein's postulate of the constancy of the speed of light holds. [1]. Given the validity of the postulate, Einstein's theory of special relativity follows right away [2]. The dynamic theory of gravity (DTG) through Weyl's quantum principle also leads to a non-singular electrostatic potential of the form:

$$V(r) = -\frac{K_o}{r} e^{-\frac{\lambda}{r}}. \quad (1)$$

where  $K_o$  is a constant and  $\lambda$  is a constant defined by the theory. The DTG describes physical phenomena in terms of five dimensions: space, time and mass. [3] By conservation of the fifth dimension we obtain equations which are identical to Einstein's field equations and describe the gravitational field. These field equations are similar to those of general relativity and are given below:

$$K_o T^{\alpha\beta} = G^{\alpha\beta} = R^{\alpha\beta} - \frac{g^{\alpha\beta}}{2} R. \quad (2)$$

Now  $T^{\alpha\beta}$  is the surface energy-momentum tensor which may be found within the space tensor and is given by:

$$T^{\alpha\beta} = T_{sp}^{\alpha\beta} - \frac{1}{c^2} \left[ F_4^\alpha F^{4\beta} - \frac{h^{\alpha\beta}}{2} F^{4\nu} F_{4\nu} \right] \quad (3)$$

and  $T_{sp}^{\mu\nu}$  is the space energy-momentum tensor for matter under the influence of the gauge fields and is given by:[4]

$$T_{sp}^{ij} = \gamma u^i u^j + \frac{1}{c^2} \left[ F_k^i F^{kj} + \frac{1}{4} a^{ij} F^{kl} F_{kl} \right] \quad (4)$$

which further can be written in terms of the surface metric as follows:[4]

$$T_{sp}^{\alpha\beta} = \gamma u^\alpha u^\beta + \frac{1}{c^2} \left[ F_k^\alpha F^{k\beta} + F_4^\alpha F^{4\beta} + \frac{1}{4} (g^{\alpha\beta} - h^{\alpha\beta}) (F^{\mu\nu} F_{\mu\nu} + F^{4\nu} F_{4\nu}) \right] \quad (5)$$

since:

$$u^4 = \frac{dy^4}{dt} \Rightarrow \frac{\partial y^4}{\partial t} + \bar{\nabla} \cdot (y^4 \bar{u}) = 0 \quad (6)$$

is the statement required by the conservation of the fifth dimension, and the surface indices  $\nu, \alpha, \beta = 0,1,2,3$  and space index  $i, j, k, l = 0,1,2,3,4$ , and  $g_{\alpha\beta} = a_{ij} y_\alpha^i y_\beta^j = a_{\alpha\beta} + h_{\alpha\beta} = a_{\alpha\beta} + 2a_{\alpha 4} y_\beta^4 + a_{44} y_\alpha^4 y_\beta^4$  where the surface field tensor is given by:

$$F_{\alpha\beta} = F_{ij} y_\alpha^i y_\beta^j \text{ and } y_\alpha^i = \frac{\partial y^i}{\partial x^\alpha} = \delta_\alpha^i \text{ for } i=0,1,2,3 \text{ and } y_\alpha^4 = \frac{\partial y^4}{\partial x^\alpha}. \quad (7)$$

$$F_{ij} = \begin{bmatrix} \mathbf{0} & E_1 & E_2 & E_3 & V_o \\ -E_1 & \mathbf{0} & B_3 & -B_2 & V_1 \\ -E_2 & -B_3 & \mathbf{0} & B_1 & V_2 \\ -E_3 & B_2 & -B_1 & \mathbf{0} & V_3 \\ -V_o & -V_1 & -V_2 & -V_3 & \mathbf{0} \end{bmatrix}. \quad (8)$$

It was shown by Weyl that the gauge fields may be derived from the gauge potentials and the components of the 5-dimensional field tensor  $F_{ij}$  given by the  $5 \times 5$  matrix given in (8). [4]

Now the determination of the fifth dimension may be seen, for the only physically real property that could give Einstein's equations is the gravitating mass or it's equivalent, mass [5]. Finally the dynamic theory of gravity further argues that the gravitational field is a gauge field linked to the electromagnetic field in a 5-dimensional

manifold of space-time and mass, but, when conservation of mass is imposed, it may be described by the geometry of the 4-dimensional hypersurface of space-time, embedded into the 5-dimensional manifold by the conservation of mass. The 5 dimensional field tensor can only have one nonzero component  $V_0$  which must be related to the gravitational field and the fifth gauge potential must be related to the gravitational potential.

The theory makes its predictions for red shifts by working in the five dimensional geometry of space, time, and mass, and determines the unit of action in the atomic states in a way that can be calculated with the help of quantum Poisson brackets when covariant differentiation is used: [6]

$$[x^\mu, p^\nu] \Phi = i\hbar g^{\nu q} \left\{ \delta_{\mu q} + \Gamma_{s,q}^\mu x^s \right\} \Phi. \quad (9)$$

In (9) the vector curvature is contained in the Christoffel symbols of the second kind and the gauge function  $\Phi$  is a multiplicative factor in the metric tensor  $g^{\nu q}$ , where the indices take the values  $\nu, q = 0,1,2,3,4$ . In the commutator,  $x^\mu$  and  $p^\nu$  are the space and momentum variables respectively, and finally  $\delta_{\mu q}$  is the Kronecker delta. In DTG the momentum ascribed, as a variable canonically conjugated to the mass is the rate at which mass may be converted into energy. The canonical momentum is defined as follows:

$$p_4 = mv_4 \quad (10)$$

where the velocity in the fifth dimension is given by:

$$v_4 = \frac{\dot{\gamma}}{\alpha_o} \quad (11)$$

and gamma dot is a time derivative and gamma has units of mass density ( $\text{kg/m}^3$ ) and  $\alpha_o$  is a density gradient with units of  $\text{kg/m}^4$ . In the absence of curvature (8) becomes:

$$[x^\mu, p^\nu] \Phi = i\hbar \delta^{\nu q} \Phi \quad (12)$$

## 2. The Gravitational Potential of Dynamic Gravity

As it turns out in the DTG the gravitational potential takes the form:

$$V(r) = -\frac{GMm}{r} \exp\left[-\frac{\lambda}{r}\right] \quad (13)$$

Where  $M$  is the mass of the main body and  $m$  is the mass of the test particle body and  $r$  is the distance from the center of main body to the center of the test particle body, and finally  $\lambda$  is a distance factor, which may be different for each particle, and can be defined as: [6]

$$\lambda = \frac{GM}{c^2} \quad (14)$$

We can easily note that equation (3) is a non-singular equation, almost of the same form as a Yukawa equation. Yukawa equation goes as  $r$  rather than  $1/r$  in the exponent. For distances much greater  $\lambda$  has the familiar  $1/r$  form. When  $\lambda = r$  the potential has its maximum value. At  $r = 0$  the potential becomes zero due to the overriding effect of the exponential.

### 3. Equations of Hydrostatic Equilibrium for a Star

The structure of any stable star where non-relativistic effects are not included in hydrostatic equilibrium, obeys the following differential equations:

$$\frac{dP(r)}{dr} = -g(r)\rho(r) = -\frac{GM(r)}{r^2}\rho(r) \quad (15)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

If relativistic effects are taken into account then first of the equations in (15) becomes the so called "Oppenheimer-Volkoff" differential equation of stellar structure below:

$$\frac{dP(r)}{dr} = \frac{Gm(r)\rho(r)\left(1 - \frac{p(r)}{\rho(r)c^2}\right)\left(1 + \frac{4\pi r^3 p(r)}{m(r)c^2}\right)}{r^2\left(1 - \frac{2Gm(r)}{rc^2}\right)} \quad (16)$$

Where: in equations, (1) and (2), the L.H.S of the first and second equations indicate, the pressure and the mass gradients as a function of radius  $r$ . And in the R.H.S, of the first equation  $\rho(r)$  is the density of the star as a function of distance  $r$ , and  $M(r)$  is the mass of the star within the distance  $r$ . In particular in equation (16) the extra appearing terms are the relativistic correction terms.

### 4. Stellar Structure in the Dynamic Theory of Gravity

To study any possible effects in Dynamic Gravity of gravity lets first find using equation (3) the force due to such potential as given by the DTG:

$$F(r) = -\frac{\partial V(r)}{\partial r} = -\frac{\partial}{\partial r} \left[ -\frac{GMm}{r} \exp\left[-\frac{\lambda}{r}\right] \right] \quad (17)$$

Then equation (3) finally gives the following force function:

$$F(r) = \frac{GMm}{r^3}(\lambda - r)\exp\left[-\frac{\lambda}{r}\right] \quad (18)$$

From which we find that the acceleration of gravity per unit mass takes the form:

$$g(r) = \frac{GM(r)}{r^3}(\lambda - r)\exp\left[-\frac{\lambda}{r}\right] \quad (19)$$

If we now substitute equation (19) into the first equation of hydrostatic equilibrium we obtain:

$$\frac{dP(r)}{dr} = -g(r)\rho(r) = -\frac{GM(r)}{r^3}(\lambda - r)\rho(r)\exp\left[-\frac{\lambda}{r}\right] \quad (20)$$

As the next step we will have to express the mass  $M(r)$  as function of distance  $r$ . First we will assume constant density function  $\rho(r) = \rho_0$ . Then the second equation for hydronic equilibrium gives that:

$$M(r) = \frac{4\pi}{3}\rho_0 r^3 \quad \text{for } 0 \leq r \leq R \quad (21)$$

Which finally makes equation (20) equal to:

$$\frac{dP(r)}{dr} = -\frac{4\pi G}{3}\rho_0^2(\lambda - r)\exp\left[-\frac{\lambda}{r}\right] \quad (22)$$

Integrating now from  $(0, r')$  and changing variable back to  $r$  we obtain:

$$P(r) - P(0) = -\frac{4\pi G}{6}\rho_0^2 \exp\left[-\frac{\lambda}{r}\right] \left[ r(r - 3\lambda) + 3\lambda^2 \exp\left[\frac{\lambda}{r}\right] \Gamma\left(0, \frac{\lambda}{r}\right) \right] \quad (23)$$

Where  $\Gamma(0, \lambda/r)$  is the Incomplete Gamma function of argument  $(0, \lambda/r)$ . This expression can be further simplified to:

$$P(r) - P(0) = -\frac{2\pi G}{3}\rho_0^2 r^2 \left[ \left(1 - \frac{3\lambda}{r}\right) \exp\left[-\frac{\lambda}{r}\right] + 3\left(\frac{\lambda}{r}\right)^2 \Gamma\left(0, \frac{GM_{star}}{rc^2}\right) \right] \quad (24)$$

If we now apply the boundary condition that  $P(r=R) = 0$ , we obtain the central pressure  $P(0)$  to be equal to:

$$P(0) = \frac{2\pi G}{3}\rho_0^2 R^2 \left[ \left(1 - \frac{3\lambda}{R}\right) \exp\left[-\frac{\lambda}{R}\right] + 3\left(\frac{\lambda}{R}\right)^2 \Gamma\left(0, \frac{\lambda}{R}\right) \right], \quad (25)$$

which can be further simplified as follows:

$$P(0) \approx \frac{2\pi G}{3} \rho_0^2 R^2 \left[ 1 - \frac{4\lambda}{R} + 40.441 \frac{\lambda^2}{R^2} \right] \quad (25a)$$

where the value of the incomplete gamma function  $\Gamma(0, \lambda/R) = \Gamma(0, 2.117 \times 10^{-6}) = 12.4883$  for star of radius  $R = R_{Sun} [7]$ . The central pressure effect due to dynamic gravity is zero when  $R = 10.1112\lambda = 14.986$  Km. That would probably mean that, dynamic gravity effects on the central pressure die out within a sphere of radius  $R = 14.986$  Km from the center of the star having a mass equal to  $M_{sun}$ . Finally the total pressure at any point  $r$  can be written as follows:

$$P(r) = \frac{2\pi G}{3} \rho_0^2 \left[ \begin{array}{l} R^2 \left( 1 - \frac{3\lambda}{R} \right) \exp\left[-\frac{\lambda}{R}\right] + 3 \left( \frac{\lambda}{R} \right)^2 \Gamma\left(0, \frac{GM_{star}}{Rc^2}\right) \\ - r^2 \left( 1 - \frac{3\lambda}{r} \right) \exp\left[-\frac{\lambda}{r}\right] + 3 \left( \frac{\lambda}{r} \right)^2 \Gamma\left(0, \frac{GM_{star}}{rc^2}\right) \end{array} \right] \quad (26)$$

In the case where the radius of the star  $R$  is much greater than lamda  $\lambda$  then equation (17) can be simplified as follows, after expanding the exponentials to first order:

$$P(r) = \frac{2\pi G}{3} \rho_0^2 (R^2 - r^2) \left[ 1 - \frac{4\lambda}{(R+r)} + 3 \left( \frac{\lambda}{r} \right)^2 \left[ \Gamma\left(0, \frac{\lambda}{r}\right) + \left( \frac{r}{R} \right)^2 \Gamma\left(0, \frac{\lambda}{R}\right) \right] \right] \quad (27)$$

Taking into account that  $\lambda = G M / c^2$  equation (14) can be further written as follows:

$$P(r) = \frac{2\pi G \rho_0^2}{3} (R^2 - r^2) \left[ 1 - \frac{4GM}{c^2(R+r)} + 3 \left( \frac{GM}{c^2 r} \right)^2 \Gamma\left(0, \frac{\lambda}{r}\right) + 3 \left( \frac{GM}{c^2 R} \right)^2 \Gamma\left(0, \frac{\lambda}{R}\right) \right] \quad (28)$$

If we look at equation (18) we can see that the first term in the right hand side goes as  $(R^2 - r^2)$  and is similar to the density profile of a Newtonian gravity star of constant density [8]. There is now a difference in the pressure profile due to the DTG and its new gravitational potential. This difference can be shown as two extra correction terms in equation (28).

## 5. Change in Central Pressure

The change in central pressure  $P(0)$  between dynamic and Newtonian gravity star becomes:

$$\frac{\Delta P_c}{P_{cN}} = \frac{P_{cD} - P_{cN}}{P_{cN}} = -\frac{4\lambda}{R} + \frac{121.323\lambda^2}{R^2}. \quad (29)$$

Therefore we now further have: [9]

$$P'_{cD} = P_{cN} + \Delta P_c = \left( \frac{3GM^2}{8\pi R^4} \right) \left[ 1 - \frac{4\lambda}{R} + \frac{121.323\lambda^2}{R^2} \right]. \quad (30)$$

**Table 1**

Radius of the Star In Solar Radii ( $R_{sun}$ )	Mass of the Star In Solar Masses ( $M_S$ )	Value of $\lambda$ (km)	$1 - \frac{4\lambda}{R} + \frac{121.323\lambda^2}{R^2}$
1.00	1.000	1.4822	0.9999
300 <sub>n</sub>	0.180	0.2668	0.9999
8.80	0.720	1.0672	2.7842

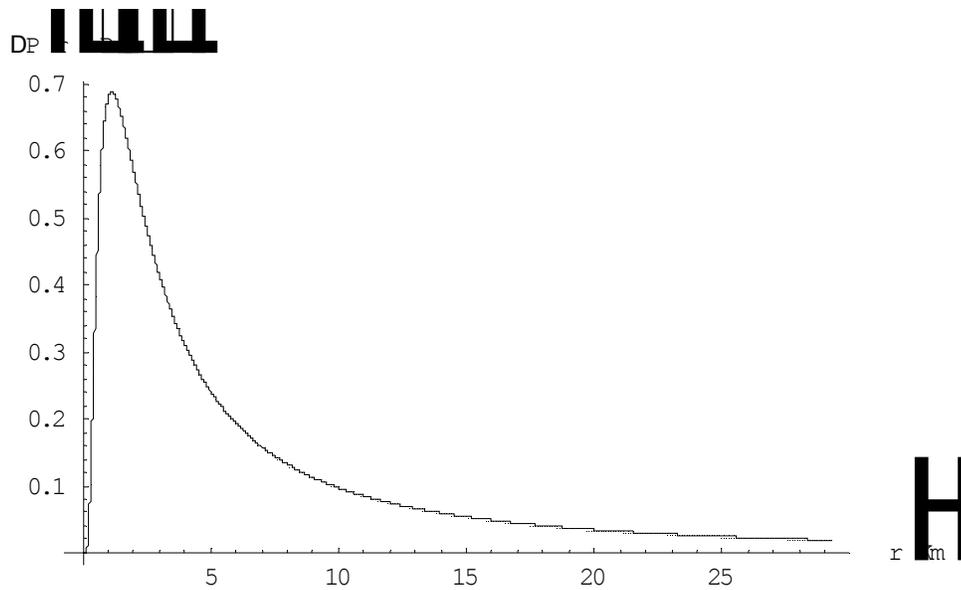
Apart from the solar type constant density star two more stars were used so that the dynamic gravity effects could be calculated on the central pressure. These were neutron stars [9] and we must say these stars can not be really thought as constant density stars to really fit a simple constant density calculation. A different relativistic treatment has to be performed in order to study these effects and that is the title of our ongoing publication. We can now see that the value for the dynamic correction of the central pressure takes a value of 0.9999 for the two first stars, and 2.7842 times for the neutron star of radius equal to  $8.8R_{Sun}$ . Next an expression for the pressure change at any r is also derived below:

$$\frac{\Delta P(r)}{P(r)} = -\frac{4\lambda}{(R+\lambda)} + 3\left(\frac{\lambda}{r}\right)^2 \Gamma\left(0, \frac{\lambda}{r}\right) + 3\left(\frac{\lambda}{R}\right)^2 \Gamma\left(0, \frac{\lambda}{R}\right) \quad (31)$$

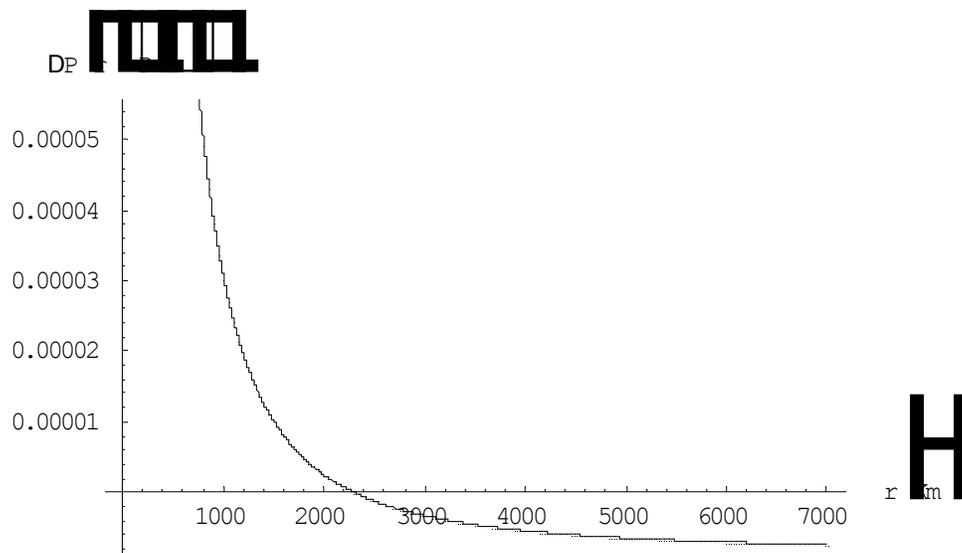
which can be further written if the numerical factors taken into account:

$$\frac{\Delta P(r)}{P(r)} = 3\left(\frac{\lambda}{r}\right)^2 \Gamma\left(0, \frac{\lambda}{r}\right) - 8.468 \times 10^{-6}. \quad (32)$$

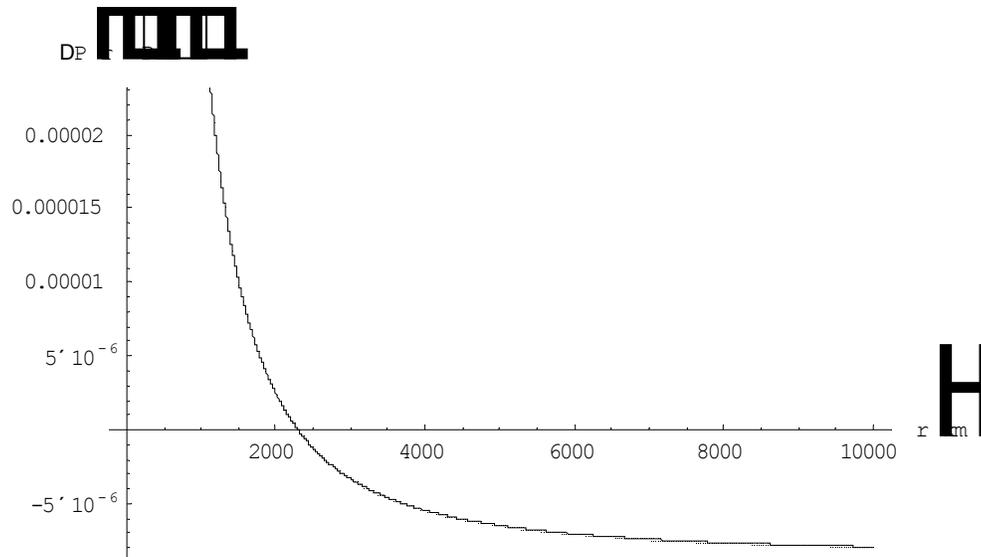
Plots of  $\Delta P(r)/P(r)$  vs r are shown for the radial ranges of zero to thirty kilometers from the center of a sun like star, and where hundred thousand points have been plotted for this and all the graphs below.



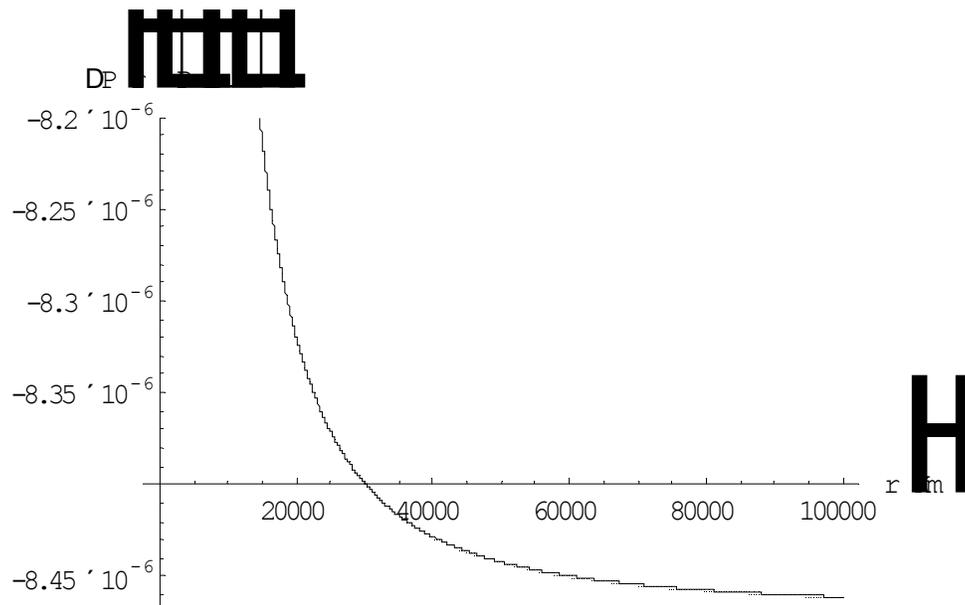
**Fig. 1**  $\Delta P(r)/P(r)$  versus radial distance  $r$  from the center of a solar mass constant density star for  $0 \text{ Km} \leq r \leq 30 \text{ Km}$ .



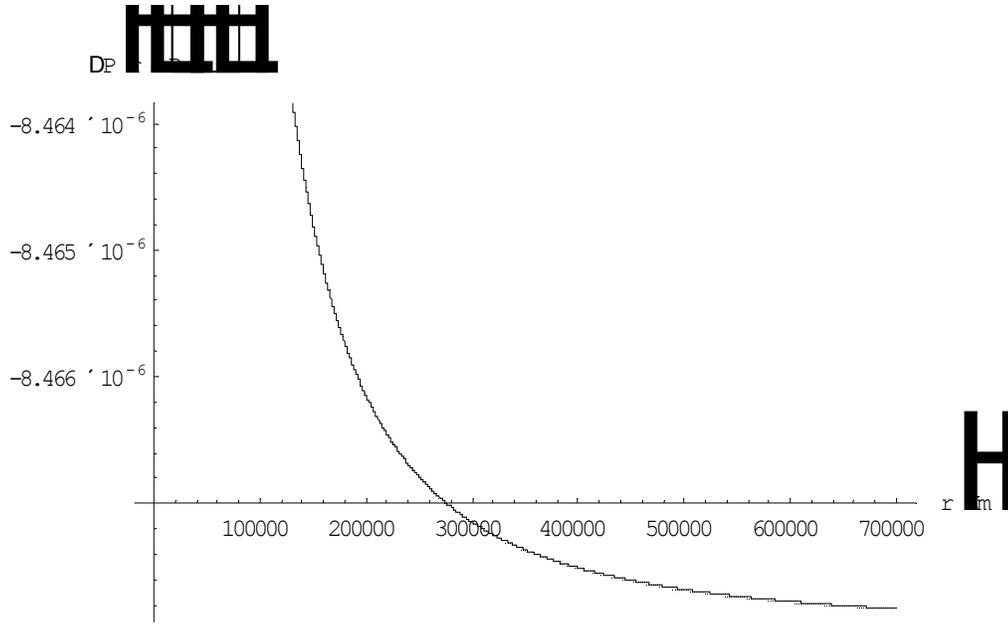
**Fig. 2**  $\Delta P(r)/P(r)$  versus radial distance  $r$  from the center of a solar mass constant density star for  $30 \text{ Km} \leq r \leq 7000 \text{ Km}$ .



**Fig. 3**  $\Delta P(r)/P(r)$  versus radial distance  $r$  from the center of a solar mass constant density star and for  $100 \text{ Km} \leq r \leq 10000 \text{ Km}$ .



**Fig 4**  $\Delta P(r)/P(r)$  versus radial distance  $r$  from the center of a solar mass constant density star and for  $70000 \text{ Km} \leq r \leq 100000 \text{ Km}$ .



**Fig 5**  $\Delta P(r)/P(r)$  versus radial distance  $r$  from the center of a solar mass constant density star and for  $100000 \text{ Km} \leq r \leq 700000 \text{ Km}$ .

### 6. Change in the Star Radius

A change in the radius of the star if any due to the change in the central pressure becomes:

$$\frac{\Delta R}{R_N} = \frac{R_D - R_N}{R_N} = 2.624 \times 10^{-6} \quad (33)$$

which makes the radius of the star to increase by:

$$R_D = R_N (1 + 2.624 \times 10^{-6}) = 1.000002624 R_N . \quad (34)$$

### 7. Redshift in the Dynamic Gravity

The redshift in the dynamic theory of gravity is given by the formula below [10]:

$$z = \frac{\Delta \lambda}{\lambda} = \exp \left\{ \left( -\frac{G}{c^2} \right) \left[ \frac{M_r e^{\frac{\lambda_r}{R_r}}}{R_r} - \frac{M_e e^{\frac{\lambda_e}{R_e}}}{R_e} \right] \right\} - 1 \quad (35)$$

where  $M_r$  is the mass of the body where the redshifted photons are received,  $M_e$  the mass of the body where the redshifted photons are emitted, and similarly  $R_r$  and  $R_e$  the radii of the corresponding bodies, and  $\lambda_r$  and  $\lambda_e$  the values of dynamic theory parameter at the receiving and emitting bodies. Using the really small increase in the radius of the star  $R_e = R_D = 1.2000002624 R_N$  we find that:

$$z = \frac{\Delta\lambda}{\lambda} = \exp(2.117 \times 10^{-6}) - 1 = 2.117 \times 10^{-6} \quad (36)$$

where the mass and radius of the receiving body is taken to be the mass and the radius of the earth. The dynamic gravity redshift contribution due to the minute change in the star's radius is calculated above and it is exactly identical to that of a star where the radius remains unchanged under Newtonian gravity and equal to  $R = 1R_{\text{solar}}$ . It is also of the same numerical value as the redshift created by Newtonian gravity. Taking or not the really small change of the radius into account in dynamic gravity we see that the corresponding redshift is very small and of the order of approximately two parts per million. For such a star it will be a really difficult task for an observer to differentiate with certainty if he is really observing dynamic or Newtonian gravity effects, not to mention that a redshift of two parts per million is a really small change to be observed for Newtonian or dynamic gravity... For such a star dynamic gravity effects do not produce a drastic change in the redshift which can be easily identified from those of which are caused by Newtonian gravity.

## 8. Conclusions

In the dynamic theory of gravity a very simple calculation has been performed using the non-relativistic equation of hydrodynamic equilibrium for a star of constant density and thus an analytical expression for the pressure at any point  $r$  has been obtained. The pressure  $P(r)$  is found to be given in terms of the incomplete gamma function of argument  $(0, r/\lambda)$  where  $\lambda$  is the dynamic gravity parameter.

Next by applying boundary conditions, an expression of the central pressure was also obtained. Both expressions resemble those of a constant density star in Newtonian gravity but, they involve extra correction terms. The central pressure of such a star appears to be slightly smaller than a constant density Newtonian star. If that's the case we anticipated some increase in the radius of the star which was estimated to be of the order of two parts per million small enough, to produce any noticeable redshift. For our simple model calculations it appears to be no difference in magnitude between Newtonian and dynamic gravity redshift, and especially when the photons are emitted from such  $1M_{\text{solar}}$  star and observed on the earth. The small increase in radius does not effect the redshift which is identically the same as the Newtonian or that of dynamic gravity with  $R_{\text{star}} = 1R_{\text{solar}}$ .

An expression for the changes in pressure  $\Delta P(r)/P(r)$  at any  $r$  has also been derived and plotted for four different regions of  $r$ , namely  $0 \text{ km} \leq r \leq 29 \text{ Km}$  from the center of the star, next for the ranges  $29 \text{ km} \leq r \leq 7000 \text{ km}$ ,  $7000 \text{ Km} \leq r \leq 100000 \text{ km}$ , and finally,  $100000 \text{ Km} \leq r \leq 700000 \text{ km}$ . All graphs have been plotted for one hundred thousand points. From the behaviour of the graphing function we can see that close to the center there is a positive change in pressure due to dynamic gravity which has a maximum value of approximately 0.700 and at approximately 2 Km from the center. That would agree with the maximum value of the dynamic gravity potential at  $\lambda = r = 1.482 \text{ km}$  for a solar type of star. Next at approximately 220 Km the change in pressure drops to positive  $10^{-5}$  and finally at becoming zero at approximately 2200 km. Finally in the range of 30000 km and 280000 km from the center it takes the values of  $-8.350 \times 10^{-6}$  and  $-8.466 \times 10^{-6}$  respectively. Our expression derived for the central pressure predicts a

correction of 0.999, for  $M = 1M_{solar}$  and  $R = 1R_{solar}$  star. It seems that throughout the star and at different radial distances the corrections to the pressure due to dynamic gravity changes sign from positive to negative. To conclude we admit inspire the fact that this is not a very realistic kind of star, it could give us an idea of how the dynamic gravity might effect stellar structure.

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