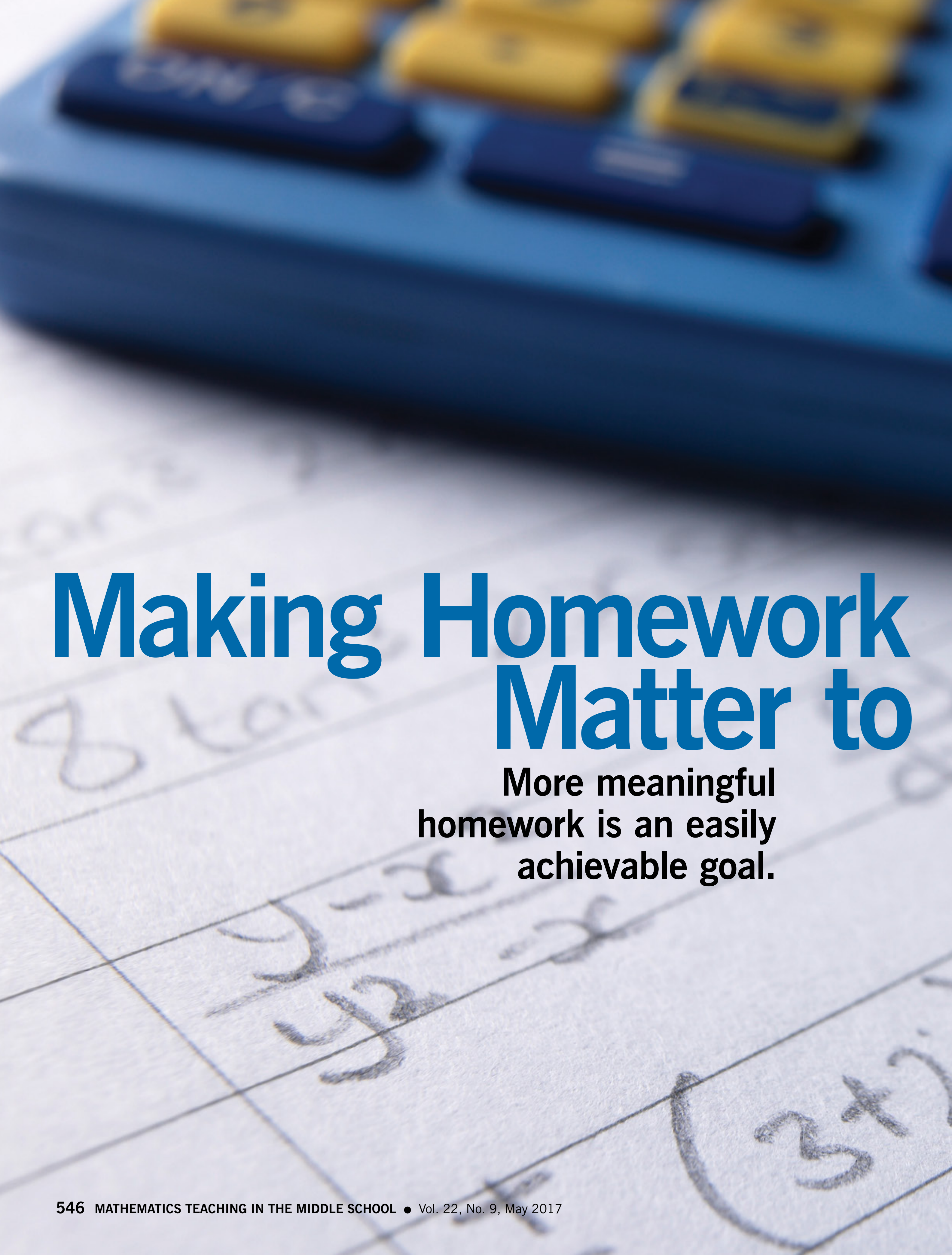


Making Homework Matter to

More meaningful homework is an easily achievable goal.

Students

Lee Walk and Marshall Lassak

During my teaching career (author Walk), I have been frustrated with student assignments being handed in incomplete, rushed through, or not at all. It made me wonder what students saw as the purpose of homework. I tried to impart the idea that the purpose of homework was to help students improve their understanding of mathematical concepts, practice skills, and act as a formative assessment that could help me see what they currently comprehended. Unfortunately, many students seemed to think that homework was just another unpleasant task to finish as quickly as possible without thinking deeply about what they were doing.

Two studies (Trautwein 2007 and Dettmers et al. 2010) show a positive correlation between high-quality homework and mathematics achievement. Students who completed their homework assignments scored better on assessments. However, these studies also showed no relationship between time spent on homework and resulting student achievement. This helped verify that although homework can be a valuable tool for learning, more time spent on daily homework is not necessarily a good idea.

In continuing to research effective homework styles and implementation, I found in many studies that student collaboration was important in developing mathematical reasoning. Wieman and Arbaugh (2014) discuss how students could use homework as an opportunity to work together. However, they also emphasize that teachers must be clear about their goals and expectations for student homework:

> Students may think that they are supposed to be able to complete all the homework problems quickly and easily, and that referring to their notes or asking a friend for help is somehow "cheating." (p. 162)

Students must understand the difference between asking another student for help in thinking through a problem and simply asking for the answer. Homework does not need to be completed independently if the primary goal is to learn about mathematics.

LEVELS OF DEMAND

Smith and Stein (1998) argue that the highest learning gains for students result from engagement in high levels of cognitive thinking and reasoning. They break down tasks in terms of four categories of cognitive demand:

1. Memorization
2. Procedures without connections to concepts or meaning
3. Procedures with connections to concepts
4. Doing mathematics

Keeping in mind the appropriate level of homework challenge, it appears that most homework tasks should be in the third category (procedures with connections to concepts). The first two categories are considered to have a lower-level demand for students because they can be solved with limited or no cognitive demand. The third and fourth categories of tasks require deeper thinking and understanding. These tasks might be more complex and often have multiple solution paths.

Fig. 1 Students were given this format and asked to find and fix the mistake.

1) $5x + 4x - x$ $9x$ **Explain mistake:** **Correct answer:**	2) $5 - (x - 2)$ $5 - x - 2$ $3 - x$ **Explain mistake:** **Correct answer:**	3) $2(x + 5)$ $2x + 5$ **Explain mistake:** **Correct answer:**

Fig. 2 Jennifer produced this explanation.

3) $2(x + 5)$ [2x+10] $2x + 5$ **Explain mistake:** [The person did not distribute the 2 to the 5] **Correct answer:** [2x+10]

Fig. 3 Connor found these errors; he was unable to obtain the correct answer for problem 5.

4) $5x + 4x - x$ [5x+4x= 9x−x=8x] $9x$ **Explain mistake:** [They forgot to subtract the one x.] **Correct answer:** [8x]	5) $5 - (x - 2)$ [5−(x−2)] $5 - x - 2$ [5−1(−x+2)] $3 - x$ [6−x] **Explain mistake:** [they didn't change the double negitive to a positive.] **Correct answer:** [6−x]	6) $2(x + 5)$ [2(x+5)] $2x + 5$ [2x+10] **Explain mistake:** [they didn't multiply 5·2.] **Correct answer:** [2x+10]

Fig. 4 Kim produced these explanations.

4) $5x + 4x - x$ $9x$ **Explain mistake:** [They didn't subtract the last x in the problem.] **Correct answer:** [8x]	5) $5 - (x - 2)$ $5 - x - 2$ $3 - x$ **Explain mistake:** [They didn't chang the signs.] **Correct answer:** [7−x]	6) $2(x + 5)$ $2x + 5$ **Explain mistake:** [They didn't multiply the 5 by 2.] **Correct answer:** [2x + 10]

Although the fourth level may be appropriate for classroom learning, it is likely to be too difficult for homework on a regular basis and could negatively impact student effort.

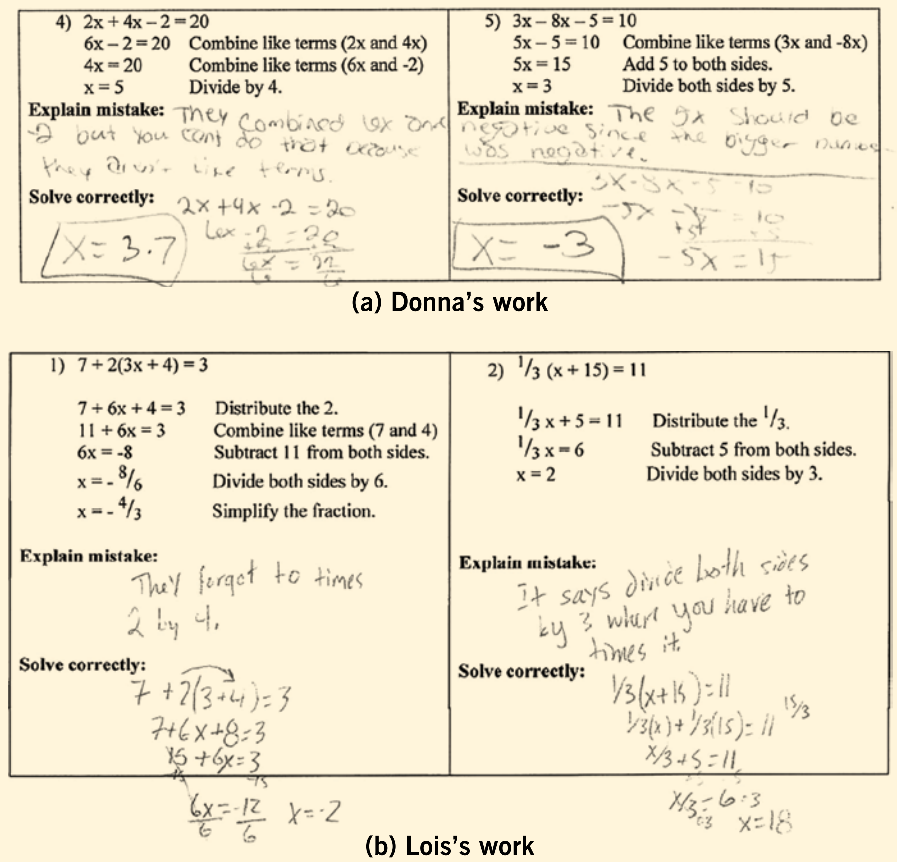
CHANGING MY APPROACH

All this research led me to wonder if my students would be more successful in completing and learning from their homework if they were given fewer problems with a higher level of cognitive demand. I already knew that some students have difficulties with procedural questions and that even those students who are able to answer such questions often have a difficult time explaining the reasoning for their methods.

When teaching prealgebra to my eighth-grade students, I typically assign skill-and-drill questions accompanied by one short-response problem. Skill-and-drill problems give students repeated practice of a particular procedure and are intended to help them gain fluency. The majority of my students completed the homework problems, but those who often did not explained that they did not have "enough time" or did not understand what to do. These students were typically habitual offenders in failing to complete their assignments, and the homework never seemed important to them. I tried to motivate my students and access self-motivation by putting more control of the homework into their hands through additional time and choices.

Focusing on the algebraic concept of solving linear equations, I implemented a change in my homework style for several weeks. Manipulating and solving equations is a central concept for my students, and it is a skill that many have difficulty mastering. I thought this would be a good time to provide a better homework experience. My class received "cognitive" homework with fewer problems that

Fig. 5 Revisiting ideas often allowed for better class discussion and participation.



had a higher level of demand. Weekly homework assignments contained suggestions about the number of questions to be completed each day. The presentation of the material for the unit and the structure of the classroom still allowed for time spent reviewing homework questions.

In changing the type of homework, I tried not to change my method of assessment or how I integrated homework into my lessons. I continued to collect and check homework for completion and accuracy. Students were allowed and encouraged to ask questions about homework ideas as part of daily lessons. Weekly quizzes were used to help determine student progress throughout the unit.

I emphasized that the new homework had fewer problems for them to complete and that daily assignments were only suggestions; nothing was due until the end of the week. This meant that if they were busy on a given night, they could get it done later in the week. I tried to motivate them by putting more control of the homework into their hands.

To foster student cooperation, I told students that they could ask me for help but reminded them that they could also rely on their classmates. On several occasions, I told students that they could work with a partner or in small groups to work out problems at the end of class. The class also engaged in think-pair-share activities to jump-start thinking and discussion.

The new homework problems I created and assigned were influenced by suggestions obtained from reading work done by Wieman and Arbaugh (2014); Lange, Booth, and Newton (2014); and Friedlander and Arcavi (2012), all of whom offered advice on particular homework style questions as well as questions to elicit the type of thinking and activity advocated by Smith and Stein (1998).

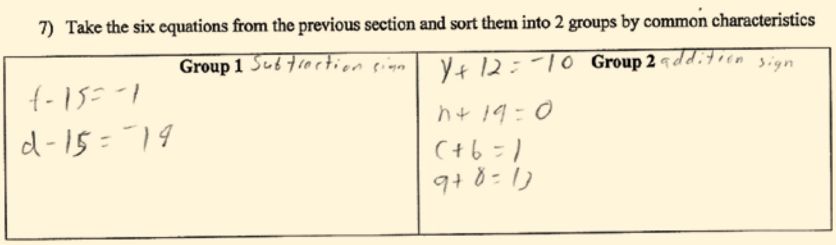
Find and Fix the Mistake

Find and Fix the Mistake problems (see **fig. 1**) were the most commonly used tasks throughout the new homework implementation. Students were instructed to identify the mistake, explain it, and simplify the expressions or solve the equations correctly. One advantage of these problems was that they required students to use correct mathematical vocabulary. Jennifer (see **fig. 2**) includes the term *distribute* in her response, which was discussed several times in class.

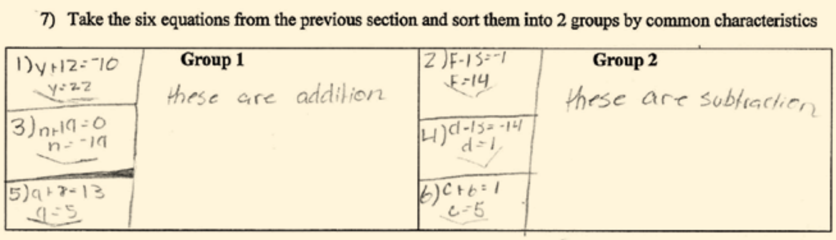
Connor (see **fig. 3**) and Kim (see **fig. 4**) both demonstrated their procedural knowledge; writing about the problem helped them participate in class discussion. During this time, I found that students were more confident in class discussions when they had had a chance to think before class about these ideas. Rather than just solving the equations with varying levels of procedural fluency, students were thinking more about the problems and how to explain errors.

Students like Donna and Lois (see **fig. 5**) were improving on their answers and explanations from previous assignments. These two students—as well as others—were also more likely to volunteer their opinions in class when they had their explanations already on paper. This led to better class discussions in that more students engaged with the ideas. From my perspective, Find and Fix the Mistake problems made it clear what students did and did not understand in terms of mathematical procedures and concepts.

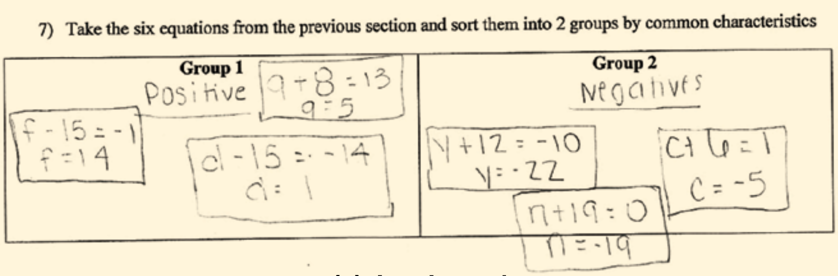
Fig. 6 These students sorted equations by using common characteristics.



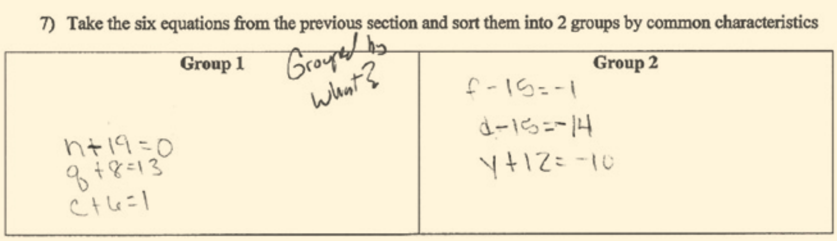
(a) Connor's work



(b) Mary's work



(c) Jane's work



(d) Debbie's work

Problem Sorts

Problem Sort questions asked students to sort equations they had already solved into two groups by using common characteristics, such as operations (shown by Connor and Mary's work in **figs. 6a–6b**), properties, nature of the solution (shown by Jane in **fig. 6c**), and so on. The open-ended nature of choosing the sorting criteria made this type of question challenging for students. Quite often I received answers like Debbie's (see **fig. 6d**), which appears to show a separation by operation but no explanation. This

AXSTOKES/THINKSTOCK

type of problem does not seem to exceed the cognitive level of the class, yet its usefulness in helping students better understand solving linear equations remains unclear to me.

Create Your Own Word Problem

Students were asked to write a story that matched a given equation and expressions; basically, a word problem in reverse (an example is shown in **fig. 7**).

The level of demand for this task is much greater than those of previous assignments because students must—

- understand how the expressions relate to the equation;
- determine what the equation is asking; and
- make a meaningful story to go with the problem.

Unfortunately, the level of demand was evidently too high for students in the class.

One determinant of how important or useful a problem was to students was whether they actually completed the problem. The Create Your Own Word Problem task was one of the most skipped problems in any of the homework assignments. **Figure 8** shows that Kim and Connor gave answers that were nearly correct. They both solved the equation correctly but had difficulty in determining the number of cups needed for each ingredient. Kim's answers for each ingredient did not match the equation. Connor found a solution to the equation but not for each ingredient in the problem.

I did notice improvement for some students as they gained more experience with this type of problem. **Figure 9** shows how Jane struggled with the first version of this problem and then a few weeks later how she improved with a different version. Improvement was evident for several students, and

Fig. 7 This grid illustrated a word problem in reverse.

2) The art teacher is making salt dough for an upcoming project. The ratio of flour to salt to water used to make salt dough is shown below.

Making Sour Dough	Story
Cups of flour: 2c	
Cups of salt: c	
Cups of water: ¾ c	
2c + c + ¾ c = 60 cups	

a. Write a story that matches the expressions and equation shown above.
b. Solve the equation. How many cups of each ingredient is the art teacher planning to use?

Fig. 8 These students had difficulty determining the number of cups needed for each ingredient.

Making Sour Dough	Story
Cups of flour: 2c	Mrs. Need is making salt dough.
Cups of salt: c	The ratio of the flour is 2c the
Cups of water: ¾ c	ratio of the salt is a amount of cups
2c + c + ¾ c = 60 cups	and the ratio of the water is 3/4 c
2c + c + 3/4c = 60 cups	and the total ratio is 60cups.

a. Write a story that matches the expressions and equation shown above.
b. Solve the equation. How many ~~cups of each~~ ingredient is ~~the art~~ teacher planning to use?

$\frac{3.75c}{3.75} = \frac{60}{3.75}$ $\boxed{C = 16 \text{cups}}$ Flour = 2c water = 3/4 c Salt = 16c

(a) Kim's work

Making Sour Dough	Story
Cups of flour: 2c	An art teacher is making special
Cups of salt: c	dough for a project. The ingredents are
Cups of water: ¾ c	two cups of flour, one cup of salt,
2c + c + ¾ c = 60 cups	and three quarters of water. Make an
2c + c + 3/4 c = 60cups 3c + 3/4 c = 60cups 3.75c/3.75 = 60cups/3.75 c = 16cups	equation to find how many cups of ingredients the teacher is going to use.

(b) Connor's work

although the level of demand may have been high, the problem was useful in determining students' understanding and misconceptions of certain types of linear equations.

Justify Your Reasoning

This type of problem is designed to help students analyze why linear equations can have one, none, or infinitely many solutions. The concept of a math equation not having a solution was a new idea for many of the students at this grade level. This question was asked after students were already solving linear equations that might not have had a solution.

Many students, including Sharon (see **fig. 10a**), repeated phrases that were discussed in class. Although these statements may be true, students did not offer any justification or evidence that they really understood how the inequality and the no-solution equation were related. Amy (see **fig. 10b**) also tried to relate the problem directly to class conversation. Like Sharon, Amy may simply be repeating what others have said in class, making it more difficult to perceive individual reasoning. Other students were able to provide some insight into how they viewed the problem. Kim attempted to describe actions she took to solve these types of problems (see **fig. 10c**). This does not necessarily demonstrate understanding; rather, it is an observed connection between the process and end result.

Connor's response (see **fig. 10d**) dug a little bit deeper. Connor was able to explain his answer by talking about how inequalities went with no solutions because the equations were "never true in the first place." He continued to describe how these equations may initially look like the others but do not hold up under mathematical investigation.

Students completed the Justify Your Reasoning problems relatively quickly, and most students attempted them. Allowing students to explain their responses in class helped them to make connections with other students and their ideas.

Fig. 9 Jane struggled with the first version of this problem. Weeks later, she improved with a different version.

Making Sour Dough	How to make **Story**
Cups of flour: 2c	2 cups of mashed potatoa flakes
Cups of salt: c	w = how many cups of water
Cups of water: ¾ c	3/4 a cup of milk
2c + c + ¾ c = 60 cups	2c + w + 3/4 c = 60 cups
2c + c + 3/4c = 60	3c + 3/4 c = 60
	3 3/4 c = 60

(a)

Fixing Your Car	Story
Time (hours): h	I got my car repaired + they driged by hour.
Cost of Mike's Mechanics: 15h + 75	Mike Mechanics cost 15 dollars a hour plus the parts which are 75 $.
Cost of Bubba's Body Shop: 25h	Bubba's Body Shop cost 25 dollars a hour
15h + 75 = 25h	Either way its going to take
−15h −15	the same amount of hours.
75 = 10h	
10 10 h = 7.5	The hours they will spent on my car will take 7.5 hours.

a. Write a story that matches the expressions and equation shown above.
b. Solve the equation.
c. Interpret the answer.

(b)

POSITIVE PERCEPTION OF HOMEWORK

I feel that these tasks had a positive impact on my students' perception of homework. A careful selection of homework tasks can help students practice, understand, and explore mathematical concepts. If students are confused about what a question is asking or how to begin, they are not as likely to persevere through the problem.

Teachers and students evaluate homework questions differently, and they cannot be labeled as simply easy or difficult. The appropriateness of the level of demand of the problem is important when considering what students should gain from completing a task. Questions such as those in Find and Fix the Problem were very popular and successful with students. They had clear expectations, and students were able to determine answers that they thought made sense and were acceptable to them.

I found the benefits of using this style of homework to include the following:

- Improved class discussions: Students were able to explain the mathematical concepts with more confidence and use better vocabulary in the classroom setting.
- Teacher insights: Explanations for and justification of the homework problems made it easier for me to determine current levels of understanding as well as notice common misconceptions shared by the class.

This small change in my homework approach helped me gain information about students' perception of homework in a different way. Although the process did not determine why some students continu-

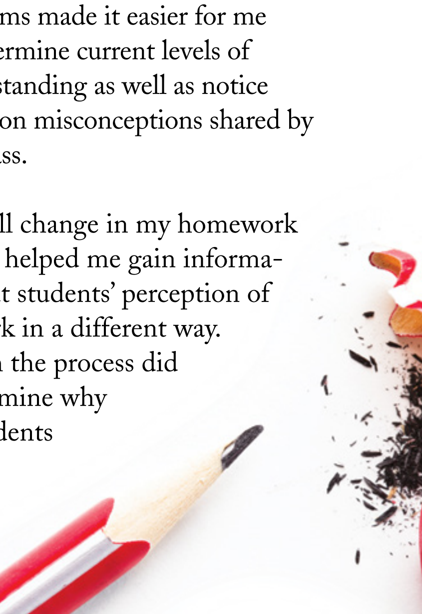


Fig. 10 Some students simply repeated classroom phrases, whereas others went a little deeper.

6) In your own words, explain what it means when a solution to an equation results in an inequality, such as $3 \neq 4$. It means that there is no solution to the problem.

(a) Sharon's work

6) In your own words, explain what it means when a solution to an equation results in an inequality, such as $3 \neq 4$. No solution if variables cancel and the answer is false.

(b) Amy's work

6) In your own words, explain what it means when a solution to an equation results in an inequality, such as $3 \neq 4$.

When a solution to an equation results in an inequality, such as $3 \neq 4$ because the numbers on each side of the equation does not come together to equal each other out.

(c) Kim's work

6) In your own words, explain what it means when a solution to an equation results in an inequality, such as $3 \neq 4$. That means no solution, no solution means 3 and 4 can never equal each other.

(d) Connor's work

ally fail to complete their homework, it did help me see why certain students skip the occasional problem. In working with new homework practices, I learned how important it is for homework to contain an appropriate level of demand. Skills-practice problems are beneficial to students' math knowledge, but good cognitive problems require students to invoke deeper levels of thinking. I plan to continue to adjust my homework structure to include skills practice and cognitive thought. Problems with an appropriate level of demand and timely feedback allow students to learn from their homework and be confident that the work they do outside of class is meaningful.

REFERENCES

Dettmers, S., Ulrich Trautwein, Oliver Lüdtke, Mareike Kunter, and Jurgen Baumert. 2010. "Homework Works If Homework Quality Is High: Using Multilevel Modeling to Predict the Development of Achievement in Mathematics." *Journal of Educational Psychology* 102 (May): 467–82.

Friedlander, Alex, and Abraham Arcavi. 2012. "Practicing Algebraic Skills: A Conceptual Approach." *The Mathematics Teacher* 105 (April): 608–14.

Lange, Karin E., Julie L. Booth, and Kristie J. Newton. 2014. "Learning Algebra from Worked Examples." *The Mathematics Teacher* 107 (March): 534–40.

Smith, Margaret Schwan, and Mary Kay Stein. 1998. "Selecting and Creating Mathematical Tasks: From Research to Practice." *Mathematics Teaching in the Middle School* 3 (February): 344–50.

Trautwein, Ulrich. 2007. "The Homework-Achievement Relation Reconsidered: Differentiating Homework Time, Homework Frequency, and Homework Effort." *Learning and Instruction* 17 (3): 372–88.

Wieman, Rob, and Fran Arbaugh. 2014. "Making Homework More Meaningful." *Mathematics Teaching in the Middle School* 20 (October): 160–65.

Lee Walk, lwalk@cumberland.k12.il.us, teaches eighth-grade math and science at Cumberland Middle School in Toledo, Illinois. He particularly enjoys using logic puzzles and riddles when teaching mathematics. **Marshall Lassak**, mblassak@eiu.edu, teaches at Eastern Illinois University in Charleston. He is interested in improving the teaching of mathematics through effective action research experiences.