On an Apparent Resolution of the Catt Question

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Over a number of years there have been some attempts to answer the Catt Question within the context of classical electromagnetic theory. None of the authors of these attempts agree on the answer to the Catt Question, even though they all invoke the very same theory. An attempt at answering the Catt Question appeared in the journal *Physics Education* in 2013, penned by M. Pieraccini and S. Selleri, as a mathematical rendition of their earlier non-mathematical version published in *IEEE Antennas and Propagation Magazine*, 2012. The explanation by these two Authors contains violations of classical electromagnetic theory, although they claim to have satisfactorily answered the Catt Question by means of classical electromagnetic theory. The arguments adduced by Pieraccini and Selleri are therefore invalid.

1 Introduction

In their article [1] "An apparent paradox: Catt's anomaly", the Italian authors Pieraccini and Selleri[*] refer to the Catt Question as "*Catt's Anomaly*". Their earlier paper is titled 'Catt's Anomaly' [2]. Although until 2001 "The Catt Question" was called "The Catt Anomaly", it was in fact *always* a question, to be answered.

The Catt Question [3] pertains to the propagation of a Transverse Electromagnitic (TEM) wave along a transmission line. Upon closure of a switch, the TEM wave (step) travels at the speed of light between the conducting wires of the transmission line, from battery to load, as depicted in Fig. 1.

An electric field **E** appears between the conductors, directed from the top wire to the bottom wire. This electric field is orthogonal to the two parallel wires and moves towards the load; thus there are positive charges on the top conductor and negative charges on the bottom conductor in the region of the transverse electric field. The Catt Question is: *Where does this new charge come from?* [3].

2 Electron current

According to classical electromagnetic theory and circuit theory, electric current in metallic wires is the flow of electrons in the wires (conductors), and a magnetic field is generated around the conducting wires according to the Right-Hand Rule. Since the TEM step travels at the speed of light towards the load, how does the current in the conducting wires keep pace with the TEM wave, if electrons cannot travel at the speed of light? The Authors [1] give the following answer,

> "*The key idea of the explanation of this apparent paradox is related to the great number of electrons in metal. Although each single electron is*
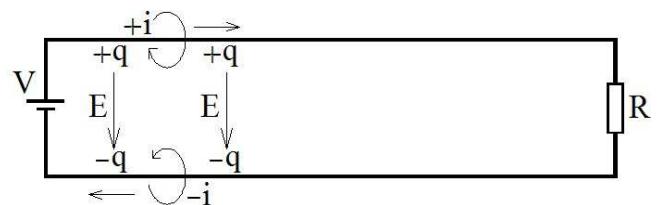

Fig. 1: An electric field points directly from the top conductor to the bottom conductor (from positive charge to negative charge). It is therefore orthogonal to the top and bottom parallel conductors. The transverse electric field travels from battery to load at the speed of light, subject to the dielectric medium between the wires.

> *not able to travel at the speed of light, a great number of slow electrons are able to produce a current as fast as an electromagnetic wave travelling at the speed of light in the conductor.*"

What do they mean by "current"? They say here that electrons "*produce a current*". However, the Authors actually assume the classical electron flow along wires as the meaning of electric current in wires, and claim that this current travels along the conductors at the speed of light even though the drift speed of electrons in the wires is a snail's pace (e.g. 2mm/s in 1.0mm copper wire [1]). Strangely, the flow of electrons, although very slow, produces an electron current that is "*as fast as an electromagnetic wave travelling at the speed of light in the conductor*" [1]: after all, a current of electrons is an electron current. This impossible duality occurs, they say, because the free electron density in the conductors is very high, and they derive an equation for electron drift "velocity".

Electron drift velocity in a wire is proportional to the vector electric field $\mathbf{E}_w$ in the wire, which supposedly causes the electron drift,

$$\mathbf{v} = -\mu \mathbf{E}_w \tag{0}$$

and so the electron drift velocity and the electric field in the wire are collinear but point in opposite directions. The constant of proportionality μ is called the *mobility*.

[*]Massimiliano Pieraccini, Associate Professor, Department of Electronics and Telecommunications, University of Florence; Stefano Selleri, Assistant Professor, University of Florence.

The Authors begin with the following equation for electron current,

$$I = \pi a^2 v q N, \qquad (1)$$

where a is the radius of the conductors, v "*the drift velocity of the charges (in practice electrons, and the speed is much lower than the speed of light)*" [1]*, q the elementary charge, and N the free electron density in the conductors.

Since the current, they say, travels at the speed of light, in time $\Delta t = \Delta x / c$ they obtain a passage of charge ΔQ along the top conductor, given by,

$$\Delta Q = I \Delta t = I \frac{\Delta x}{c}, \qquad (2)$$

where Δx is the distance travelled by the TEM step in time Δt. This charge ΔQ the Authors call "*an imbalance of charge*" [1] because they say it is confined to a leading volume element of length Δx in the top conducting wire, and induces equal but opposite polarity charge on the bottom conducting wire.

Using a cylindrical Gaussian surface they next apply Gauss' Law to calculate the magnitude E of the electric field $\mathbf{E}$ due to ΔQ in the top conductor,

$$\frac{\Delta Q}{\epsilon_0} = (2\pi a \Delta x) E, \qquad (3)$$

where ϵ_0 is the permitivity of free space. Substituting ΔQ from equation (2) and I from equation (1) the Authors obtain,

$$v = \frac{2c\epsilon_0 E}{qNa}. \qquad (4)$$

From equation (4) they conclude,

"*The notable point of this result is that the necessary speed decreases with the number of electrons per volume unit N. Therefore, a great number of slow electrons are able to generate enough unbalanced charge to follow an electromagnetic wave travelling at much higher speed.*"

Thus electrons flow slowly in the conducting wires but the electron current in the wires is nevertheless flowing along the conductors at the speed of light.

Although equation (4) follows from equations (1), (2) and (3) by purely mathematical operations, the transverse electric field $\mathbf{E}$ cannot drive electrons along the inside or outside of the wires. Equations (1), (2) and (4) imply flow of electrons along the wires, but the transverse electric field at equation (3) is orthogonal to the parallel axes of the top and bottom wires. According to classical electrodynamics, free electrons in a metallic conductor flow in the direction opposite to the direction of the electric field, according to equation (0), not orthogonal to the electric field ($\mathbf{E} \neq \mathbf{E}_w$). The Authors confound battery EMF† with the transverse electric field, and so

make the transverse electric field the battery EMF to drive electrons along the wires; at equation (4).

Then they introduce the "skin effect" [1]:

"*Up to this point, the current has been considered constant in the wire section, but in reality the current flow tends to be bound to the portion of the conductor closer to the surface.*"

The equation for current in the wires they then give as,

$$I = 2\pi a \delta v q N, \qquad (1b)$$

where δ is the skin depth, which is frequency dependent. With the "skin effect" they still argue that electrons flowing along the wire is electric current, orthogonal to the electric field they calculated at equation (3), and continue to make that transverse electric field the driver of the electrons in the conducting wires. Using equations (1b), (2) and (3) they then obtain the electron drift speed,

$$v = \frac{2c\epsilon_0 E}{qN\delta} \qquad (5)$$

although the 2 in the numerator should not in fact appear.

3 Conclusion

Pieraccini and Selleri have not answered the Catt Question. On the one hand they treat current in the conducting wires as electron current but on the other hand they invoke the transverse electric field between the conducting wires to drive this electron current at the electron drift speed. Their analysis violates the classical electromagnetic theory they use in their attempt to prove that what they call "*Catt's Anomaly*" is merely an "*apparent paradox*" [1]. The real paradox here is their claim that very slowly flowing electrons in the wires of a transmission line produce an electron current in those wires that travels at the speed of light, driven by an elecric field orthogonal to those wires.

"*If I have promised to deliver one dozen eggs to Oxford, one hour from now, Oxford being 100 miles away, there is no point in despatching ten dozen eggs in a vehicle which travels at only ten miles/h*" [4].

References

1. Pieraccini M. and Selleri S. An apparent paradox: Catt's anomaly. *Physics Education*, 2013, v. 48 (6), 718–722.

2. Pieraccini M. and Selleri S. Catt's anomaly. *IEEE Antennas Propag. Mag.*, 2012, v. 54, no. 6, 242–244.

3. Catt I. The Catt Question. http://www.ivorcatt.co.uk/x54c1.htm

4. Catt I. The death of electric current. *Wireless World*, December 1982, 79–81. Accessed from http://www.ivorcatt.co.uk/x18j100.pdf

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*The Authors confound velocity with speed; the latter denoted by $|\mathbf{v}| = v$

†What EMF is, is another question.