PYTHAGOREAN THEOREM

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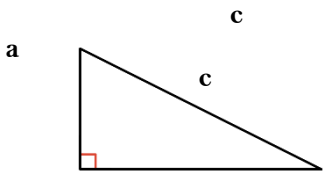
In any right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse,

$$a^2 + b^2 = c^2$$

$$(Leg)^2 + (Other\ Leg)^2 = (Hypotenuse)^2$$



The equation $a^2 + b^2 = c^2$ is called the **Pythagorean equation**.

Example: **One leg of a right triangle is 8 ft. If the hypotenuse is 17 ft, find the length of the other leg.**

$$x^2 + 8^2 = 17^2$$
$$x^2 + 64 = 289$$
$$x^2 - 225 = 0$$
$$(x + 15)(x - 15) = 0$$
$$x = 15\ or\ x = -15$$

$x = -15$ is not a valid since the length of a side of a triangle cannot be negative.
The length of the other leg is 15 ft.

Example: **One leg of a right triangle is 1 cm more than the other leg. If the hypotenuse is 5 cm, find the length of the legs.**

$$x^2 + (x + 1)^2 = 5^2$$
$$x^2 + (x + 1)(x + 1) = 25$$
$$x^2 + x^2 + x + x + 1 = 25$$
$$2x^2 + 2x - 24 = 0$$
$$2(x^2 + x - 12) = 0$$
$$2(x + 4)(x - 3) = 0$$
$$2 \neq 0\,,\ x + 4 = 0,\ x - 3 = 0$$
$$x = -4\ \ or\ \ x = 3$$

$x = -4$ is not a valid answer, so the only valid answer is $x = 3$.
If one of the legs is $x = 3$, the other leg is $x + 1 = 3 + 1 = 4$.
The lengths of the other legs are 3 cm and 4 cm.

EXERCISES:

(1) One leg of a right triangle is 24 inches. If the hypotenuse is 25 inches, find the length of the other leg.

(2) One leg of a right triangles is 2 meters less than the other leg. If the hypotenuse is 10 meters, find the length of the legs.

(3) One leg of a right triangle is 5 feet. If the other leg is 1 ft. less than the hypotenuse, find that that leg and the hypotenuse.

Answers

1.) 7 in
2.) 6 m, 8 m
3.) 12 ft, 13 ft