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Compressible Fluid Flow

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To my wife Jane and my late mother Eileen for all their loving
encouragement and help.

P.H.O.

To my loving wife, Elizabeth and our four children, Peter, Laleah,
Emma, and Mather.

W.E.C.

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PREFACE

Compressible flow occurs in many devices encountered in mechanical and aerospace engineering practice and a knowledge of the effects of compressibility on a flow is therefore required by many professional mechanical and aerospace engineers. Most conventional course sequences in fluid mechanics and thermodynamics deal with some aspects of compressible fluid flow but the treatment is usually relatively superficial. For this reason, many mechanical and aerospace engineering schools offer a course dealing with compressible fluid flow at the senior undergraduate or at the graduate level. The purpose of such courses is to expand and extend the coverage given in previous fluid mechanics and thermodynamics courses. The present book is intended to provide the background material for such courses. The book also lays the foundation for more advanced courses on specialized aspects of the subject such as hypersonic flow, thus complementing the more advanced books in this area such as those by Anderson.

The widespread use of computer software for the analysis of engineering problems has, in many ways, increased the need to understand the assumptions and the theory on which such analyses are based. Such an understanding is required to interpret the computer results and to judge whether a particular piece of software will give results that are of adequate accuracy for the application being considered. Therefore, while computer methods are discussed in this book and some computer software is outlined, the major emphasis is on developing an understanding of the material and of the assumptions conventionally used in analyzing compressible fluid flows. Compared to available textbooks on the subject, then, the present book is, it is hoped, distinguished by its attempt to develop a thorough understanding of the theory and of the assumptions on which this theory is based, by its attempt to develop in the student a fascination with the phenomena involved in compressible flow, and by the breadth of its coverage.

Contents

Our goal in writing this new text was to provide students with a clear explanation of the physical phenomena encountered in compressible flow, to develop in them an awareness of practical situations in which compressibility effects are likely to be important, to provide a thorough explanation of the assumptions used in the analysis of compressible flows, to provide a broad coverage of the subject, and to provide a firm foundation for the study of more advanced and specialized aspects of the subject. We have also tried to adopt

an approach that will develop in the student a fascination with the phenomena involved in compressible flow.

The first seven chapters of the book deal with the fundamental aspects of the subject. They review some background material and discuss the analysis of isentropic flows, of normal and oblique shock waves, and of expansion waves. The next three chapters discuss the application of this material to the study of nozzle characteristics, of friction effects, and of heat exchange effects. Chapters dealing with the analysis of generalized one-dimensional flow, with simple numerical methods, and with two-dimensional flows are then given. The last three chapters in the book are interrelated and provide an introduction to hypersonic flow, to high temperature gas effects, and to low density flows. Some discussion of experimental methods is incorporated into the book, mainly to illustrate the theoretical material being discussed. However, because a number of shadowgraph and Schlieren photographs are given in the text, a separate discussion of these and other related methods of flow visualization is provided in an appendix.

The first ten chapters together with selected material from the remaining chapters provide the basis of the typical undergraduate course in compressible fluid flow. A graduate level course will typically cover more of the material on two-dimensional flows and high temperature gas effects.

The subject of compressible fluid flow involves so many interesting physical phenomena, involves so many intriguing mathematical complexities, and utilizes so many interesting and novel numerical techniques that the difficulty faced in preparing a book on the subject is not that of deciding what material to include but rather in deciding what material to omit. In preparing this book an attempt has been made to give a coverage that is broad enough to meet the needs of most instructors but at the same time to try to avoid losing the students' interest by going too deeply into the specialized areas of the subject.

Acknowledgments

This book is based on courses on gas dynamics and compressible fluid flow taught by one of the authors (P.H.O.) at the University of Cape Town and at Queen's University and by the other author (W.E.C.) at Carleton University. The students in these courses, by their questions, comments, advice, and encouragement, have had a major influence on the way in which the material is presented in this book and their help is very gratefully acknowledged. The indirect influence of the authors' own professors on this book must also be gratefully acknowledged. In particular, R. Stegen at the University of Cape Town and I. I. Glass at the University of Toronto introduced one of the authors (P.H.O.) to the exciting nature of the subject, as did W. Gilbert to the other author (W.E.C.). The authors would also like to gratefully acknowledge the advice and help they received from colleagues at Queen's University, at the National Research Council of Canada, and at Carleton University. Jane Paul undertook the tedious job of typing and checking most of the text and

her effort and encouragement is also gratefully acknowledged, as is the help given by Michael Bishop, Wayne Turnbull, and Laleah Carscallen.

The authors would also like to express their gratitude to the following reviewers of the text for their contributions to the development of this textbook: Kavesh Tagavi, University of Kentucky; C. H. Marston, Villanova University; Jay Khadadadi, Auburn University; John Lloyd, Michigan State University; and Jechel Jagoda, Georgia Institute of Technology. The authors would also like to thank John Lloyd and John D. Anderson for their advice and encouragement during the early stages of the development of this textbook.

Solutions Manual

A manual that contains complete solutions to all of the problems in this textbook is available. In writing this manual, we have used the same problem-solving methodology as adopted in the worked examples in the book. The solution manual also provides summaries of the major equations developed in each chapter.

Software

An interactive computer program, **COMPROP**, for the calculation of the properties of various compressible flows was developed by A. J. Ghajar of the School of Mechanical and Aerospace Engineering at Oklahoma State University, and L. M. Tam of the Mechanical Engineering Department of the University of Macau, to support this textbook. The program has modules for Isentropic Flow, Normal Shock Waves, Oblique Shock Waves, Fanno Flow, and Rayleigh Flow. The use of this software is described in Appendix A. The software is available free of charge to adopters of this book via the McGraw-Hill Web Page. The web address to access the program is: <http://www.mhcollege.com/engineering/mecheng.html>.

Patrick H. Oosthuizen
William E. Carscallen

NOMENCLATURE

The following is a list of definitions of the main symbols used in this book:

A	area
A^*	area at section where $M = 1$
A_e	exit plane area
A_{throat}	throat area
a	local speed of sound
a^*	speed of sound at section where $M = 1$
a_0	stagnation speed of sound
C	a constant
C_D	drag coefficient
C_L	lift coefficient
C_p	pressure coefficient
c	speed of light
c_m	mean molecular speed
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
D	diameter
D_H	hydraulic diameter of a duct
e	internal energy
F	force on system in direction indicated by subscript
F_D	drag force
F_μ	friction force
f	friction factor
$\bar{f}$	mean friction factor over length of duct considered
f_D	Darcy friction factor
h	enthalpy
h_f	enthalpy of formation

I	impulse function	Re	Reynolds number
Kn	Knudsen number	r	recovery factor or relaxation parameter
K_P	equilibrium constant	S	V/c
k	thermal conductivity	s	entropy
L	velocity component parallel to oblique shock wave or size of system	s^*	entropy at section where $M = 1$
l^*	length of duct required to produce choking	s_0	stagnation entropy
M	Mach number	T	temperature
M_{crit}	critical Mach number	T_0	stagnation temperature
M_{des}	design Mach number	T_∞	freestream temperature
M_{div}	divergence Mach number	T_{wa}	adiabatic wall temperature
M_e	exit plane Mach number	T^*	temperature at section where $M = 1$
M_N	component of Mach number normal to wave	t	time
M_R	reflected shock Mach number	U_S	shock velocity
M_S	shock Mach number	U_{SR}	reflected shock velocity
m	molar mass	u	velocity component in x direction
$\dot{m}$	mass flow rate	V	velocity in one-dimensional flow
N	velocity component normal to oblique shock wave	V^*	velocity at section where $M = 1$
Nu	Nusselt number	V_e	exit plane velocity
n	index of refraction	$\hat{V}$	maximum velocity
P	perimeter	v	velocity component in y direction
p	pressure	w	rate at which work is done per unit mass flow rate
p^*	pressure at section where $M = 1$	W	rate at which work is done
p_0	stagnation pressure	x	coordinate direction
p_b	back pressure	y	coordinate normal to x
p_{berit}	back pressure at which sonic flow first occurs	Z	compressibility factor, $p/R\rho T$
p_e	exit plane pressure	α	Mach angle
Pr	Prandtl number	β	shock angle
q	heat transfer rate per unit mass flow rate or per unit area	γ	specific heat ratio
$\bar{Q}$	heat transfer rate	δ	turning angle
R	gas constant for gas being considered	δ_{max}	maximum oblique shock wave turning angle
$\mathcal{R}$	universal gas constant		

θ	Prandtl–Meyer angle
θ_{vib}	vibrational excitation temperature
λ	mean free path
μ	coefficient of viscosity
ν	kinematic viscosity or Prandtl–Meyer angle
ρ	density
ρ^*	density at section where $M = 1$
ρ_0	stagnation density
ρ_e	exit plane density
τ_w	wall shear stress
ψ	dimensionless stream function
Φ	potential function
Φ_p	perturbation potential function
ω	vorticity

Compressible Fluid Flow