A Derivation of the Quantum Phase

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Abstract

The quantum phase has profound effects on quantum mechanics but its physical origin is currently unexplained. This paper derives its general form from two physical axioms: 1) in the limit in which space goes to zero, spacetime reduces to a constant quantity of areatime, and 2) the proper time dimensions of areatime and of spacetime are orthogonal but can be compared to each other according to what will here be called an angular dual bilateral symmetry. The mathematical derivation and the explanation of the physical origin of the quantum phase from these two axioms is straightforward and implies that the quantum phase is intimately related to the quantization of spacetime.

Keywords: Quantum Phase, Quantization of Spacetime, Areatime, Angular Dual Bilateral Symmetry, Proper Time, Imaginary Time, Interpretation of Quantum Mechanics

1 Introduction

The quantum phase appears in canonical non-relativistic quantum mechanics as a consequence of being a solution to Schrödinger's equation, and in the path integral formulation (in a different mathematical form) as an axiom. To this author's knowledge, there exists to date no explanation of its physical origin even though, albeit imaginary and therefore not directly observable, it has profound effects on the theory and its predictions. This paper will present a derivation of its relativistic form $e^{\pm i\omega\tau}$ from two axioms that elucidate its physical origin. It will be seen that according to this derivation the physical origin of the quantum phase is intimately related to the quantization of spacetime.

2 Areatime

Consider any geometric object or volume of space and imagine it could be shrunk in size without changing its shape. One aspect that does change proportionally with size, regardless of inherent composition, is the ratio of its surface area to volume A/V. This ratio becomes larger as the object becomes smaller, and therefore, very small objects are extremely flat in comparison to objects in our every day experience. For example, since for spherical objects the ratio is proportional to the inverse of the radius, a ball of radius 10^{-11}m (roughly the Bohr radius) has an A/V ratio hundred billion times larger than a ball of radius 1 m. So one property that all objects of atomic and nuclear proportion have in common is that, compared to objects on our scale, they are vastly more *two-dimensional*.
We can now carry this observation to its logical conclusion by asking: Is there a scale at which this ratio becomes infinite? This is equivalent to asking whether there exists a scale at which volume vanishes but area does not. Normally, one might assume that once a volume element is shrunk to zero, one is left with 'nothing'. This is indeed what underlies the usual conception of 'point particles', which are by definition

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zero-dimensional. So, the conventional assumption about such a scale is that it marks the transition from *three* spatial dimensions to *zero* spatial dimensions. But if it is true that that there exists a scale at which volume no longer exists but area still does, then this scale must actually mark a transition from three spatial dimensions to *two*. If such a scale exists, one might at first reasonably assume that objects which at that scale are necessarily characterized by zero volume and finite area still exist in spacetime. But there are some great difficulties with this assumption: In order to transition from three to two dimensions, a direction in space has to vanish. By virtue of this fact, that direction is different from all other directions in space. This, however, violates our fundamental assumption of space isotropy. One could attempt to save isotropy by postulating that in principle any direction in space could vanish at that scale, depending on individual circumstances. But then one needs to introduce additional assumptions to specify how in each individual circumstance one direction in space is singled out as the one that vanishes at that scale.

There is a way of postulating a scale at which space vanishes while area is preserved which avoids these difficulties: we assume that at that scale not only space vanishes, but spacetime itself is reduced by one dimension!

This proposition may seem rather unfamiliar, so it deserves elaboration. This author previously formulated a principle which can be used to choose from alternative fundamental assumptions, and which is called the *Principle of Least Speciality.* It says that of alternative fundamental assumptions of which one must be chosen to derive an explanation for a physical phenomenon, that one is to be preferred which assigns to that phenomenon, and closely related aspects of nature, the least special status, unless there exists a logical reason for preferring a more special assumption. Applying this principle to spacetime itself, we may ask whether we should assume that spacetime is special in the sense that as a continuum consisting of a specified combination of length and time dimensions it is unique, or whether we should assume that it is not. We *must* choose from one of these two possibilities. Spacetime certainly seems unique to us, but our perspective is biased, since we are observers who exist in spacetime. Absent any logical reason that explains why spacetime is special in this way, the principle of Least Speciality tells us that we should prefer the assumption that spacetime is not special, i.e. it is no more unique than any other continuum consisting of a different combination of length and time dimensions. So, taking the position that spacetime is not unique, and presuming the existence of a scale at which space but not area vanishes, one can recognize that what is left at that scale is a continuum that consists of two dimensions of length and one dimension of time. A natural name for this continuum is *areatime.* Mathematically, we will express this assumption as follows:

$$\textbf{Axiom I}: \lim_{V \to 0} U_4 = |U_{3max}| \tag{1}$$

Where U_4 symbolizes a quantity of spacetime (the subscript denotes the number of constituent dimensions) and $|U_{3max}|$ is defined as the maximum quantity of areatime below which a volume is zero. 'Areatime' is meant to connote that U_3 has the same metric structure as spacetime, but with two instead of three constituent length dimensions. Note that $|U_{3max}|$ is not an upper limit on areatime, but only a lower limit on spacetime. In other words, axiom I does not preclude the existence of quantities of areatime greater than $|U_{3max}|$, but these do not constitute a limit on spacetime. The absolute value signs on $|U_{3max}|$ are meant to indicate that Axiom I treats it as a scalar quantity without putting constraints on its *shape.* Put differently, it is *not* assumed that in this limit spacetime reduces to a rigid lattice, as would be the case if a volume element anywhere in space reduced to a quantity of areatime of the same shape. On the other hand, since (1) does not refer to any particular volume element, it follows from our fundamental assumption of homogeneity of space that for every volume element spacetime must have the same quantity of areatime as its limit. That is, axiom I requires that $|U_{3max}|$ be a *constant.* To summarize, (1) says that there exists a finite limit in which space vanishes and in which spacetime reduces to a constant quantity of areatime of variable shape.

This axiom sets $|U_{3max}|$ as the limit in which spacetime becomes discontinuous, or quantized. To see this, recall that a function $f(x)$ is continuous at a *iff* $\lim_{x \to a} f(x) = f(a)$. Letting $x = V$, $a = 0$ and $f(x) = U_4(V)$ we see from (1) that $|U_{3max}| \neq U_4(0)$ because the number of constituent dimensions do not match. This contrasts with the assumption that spacetime is continuous, which can be expressed as $\lim_{V \to 0} U_4 = 0$, reflecting a zero amount of a four-dimensional quantity.

Given Axiom I, how might objects which exist in this limit manifest themselves to us? A partial answer to this question will be provided by the axiom presented in the next section.

3 Angular Dual Bilateral Symmetry

Before presenting an axiom that partially answers the question of how phenomena associated with entities in areatime should manifest themselves to us, we need to define a symmetry that arises when certain kinds of translation pairs are superposed. All translation rates here are assumed to vary *sinusoidally*.
Figure 1 depicts the superposition of a continous translation of $x \to 0$ and that of $0 \to y$ at sinusoidally changing rates which are phase shifted by $\pi/2$ e.g. at $(x_{max}, 0)$ the x-velocity is zero while the y-velocity is at a maximum, whereas at $(0, y_{max})$ the x-velocity is maximally negative whereas the y-velocity is zero. The superposition yields a continuous translation in the counter-clockwise angular direction:

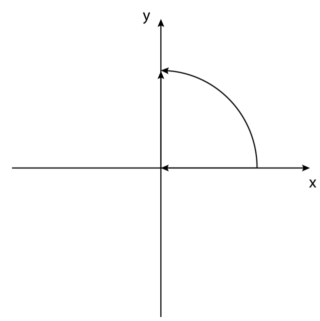

Figure 1: Superposition of one inward and one outward translation with a sinusoidally changing rate

Figure 2 depicts all 8 such possible superpositions of translation pairs along the two axes:

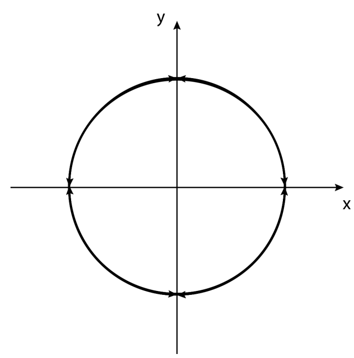

Figure 2: Angular Dual Bilateral Symmetry

Because the resultant pattern is invariant both with respect to a reflection and a rotation of the axes over an angle $\frac{\pi}{2}$, and because the superposed translation is always in an angular direction, we will call this an *angular dual bilateral symmetry*. We are now ready for the second axiom.

> **Axiom II:** *The proper time dimensions of distinct continua are orthogonal but can be compared by matching the shorter time dimension to consecutive intervals of equal magnitude of the longer time dimension according to the angular dual bilateral symmetry* (2)

Axiom II is a concise statement of a process which requires further elaboration, and in which we will symbolize the proper time of spacetime by τ (as usual), and that of areatime by τ_A. *Proper time*, of course, refers to the square root of the metric interval and divided by c, but we will discuss it here in terms of time measured in a rest frame. In the follow-up paper to this work, the rest frame in areatime to which τ_A refers will be shown to belong to something that manifests itself to us as what is called an *actualizable* system in spacetime [3], but because its discussion is otherwise not relevant in this derivation, we will here simply assume that it is possible to coherently define such a frame.

This axiom postulates that τ_A is orthogonal to τ. Thus, we must first clarify how one is to understand the notion of orthogonal time dimensions. Consider two observers O_1 and O_2 who each carry a clock. If over the duration that the clock of O_1 measures a time interval t_1, the clock of O_2 measures a time interval t_2, such that the two quantities can be related to each other by $t_1 = \gamma t_2$, where γ is a real dimensionless factor with the property $0 < \gamma^{|-1|} < \infty$ we could not legitimately claim that for any value $\gamma^{|-1|} \neq 1$ the two observers are subject to distinct time dimensions[1]. Rather, we would say that they are subject to the same time dimension and that the difference in the measured time intervals is the result of the clocks being calibrated differently. We can use the notion of 'aging' instead of the notion of carrying a clock, as long as the clock is always stationary with respect to the observer to whom it belongs, but that does not change anything stated so far. For even if two observers age at different rates such that their rates of aging can be related to one another by a finite factor not equal to one using properly calibrated clocks, we could still not claim that they age along distinct time dimensions. According to the special theory of relativity (SR), observers in spacetime can age at different rates relative to each other (when they are in relative motion) and in spite of this are still considered to age along the 'same' time dimension, since we consider there to exist only one time dimension in spacetime.

The situation is different when $\gamma^{|-1|} = 0$ because in that case the rate of aging of the two observers cannot be related to one another. This means that over the duration that one observer ages, the other does not age at all. We will take this inexpressibility of one observer's time interval in terms of the other's as a plausibility argument to support the idea that when this is the case for two time dimensions, they are *orthogonal*[2]. Thus, we consider τ_A to be orthogonal to τ when an entity that ages according to τ_A does not age relative to another entity that ages according to τ, and vice versa. Since the two time dimensions are constituents of two distinct continua, this means that entities which exist in areatime do not age relative to entities in spacetime, and vice versa.

There is a well-known instance of this in the physical world described by SR and it is instructive to discuss it now. For photons, $\tau = 0$ which means that no matter how long a time interval we as observers in spacetime consider, no time is observed (by us) to pass for such entities over that same interval. This gives rise to an apparent paradox which was pointed out elsewhere and called the photon's *existence paradox*: If such entities do not age, then the instant they come into existence must, to them, be precisely the same instant they go out of existence. For example, the instant a photon is emitted is, to it, the very same instant it is absorbed somewhere else. Therefore, it 'observes' its duration of existence in our spacetime to be precisely equal to zero. But a duration of existence of precisely zero would be intuitively understood to be equivalent to non-existence, which contradicts the known observational consequences associated with the existence of photons. This paradox is resolved by using the Principle of Least Speciality to assume that spacetime is not a unique continuum, which makes it possible to consider that photons exist outside of our spacetime. If they exist outside of our spacetime, our account of their 'observation' of their own existence *in* our spacetime must in fact be consistent with a zero duration of existence. Indeed, the invariance of the speed of light has been derived from this very realization[1]. So SR is quite consistent with the idea that if an entity exists outside of spacetime, it does not age in spacetime. In fact, we can go even further in connecting SR to this framework: since, according to SR, the length of entities traveling at c in their direction of motion is contracted to zero, such entities can at most be characterized by 2 orthogonal length dimensions. Assuming that photons have transverse extent, photons must be 2-dimensional entities. But 2-dimensional entities are exactly those which are presumed to exist in areatime! Hence the framework developed so far in conjunction with SR already makes a definite real-world prediction: *Photons exist in areatime.*

That photons are intrinsically characterized by one fewer dimension than spacetime observers can also be deduced by realizing that the photon energy-momentum vector is really a 3-vector masquerading as a 4-

[1] this is a generalized version of the time dilaton factor, defined as $\gamma^{|-1|} \equiv \gamma$ for $1 < \gamma^{|-1|} \leq \infty$ and $\equiv \gamma^{-1}$ for $0 \leq \gamma^{|-1|} < 1$

[2] It is readily admitted that the notion of 'inexpressibility' is mathematically vague. A mathematically more precise notion could perhaps be constructed as follows: Consider the square of the proper times multiplied by c^2 (i.e. the metrics) as abstract vectors, and take the direct tensor product. This product lives in a 7-dimensional space, but the position of any object in this space can, according to our axioms, only be described using components either in one 4-dimensional or another 3-dimensional subspace. If there is no entity which deviates from this, the two metrics can be considered to be orthogonal. I thank Professor Jim Liu for this suggestion

vector because the energy component is completely dependent on the momentum components. This is exactly analogous to calling the position vector $< x, y, z, r >$ with $r \equiv \sqrt{x^2 + y^2 + z^2}$ a "four-vector" in 4-dimensional Euclidean Space when in fact it exists only in a 3-dimensional subspace. Minkowski spacetime differs from this only in that an entity which actually exists only in a subspace does not progress along the proper time of spacetime. Considering that proper time is proportional to the metric, this must be true by elementary geometric considerations, as the metric of Minkowski spacetime is distinct from the metric of any of its subspaces. It is in this sense that one can think of photons as actually existing 'outside' spacetime.

Having established the meaning of orthogonality between distinct time dimensions, we can now discuss how to compare them to each other, but first let us clarify why this is necessary in the first place. Since we are considering a situation in which observers in spacetime, who do not age along τ_A, wish to describe their observation of effects caused by objects existing in areatime, which do not age along τ, we are faced with the following dilemma: How can an experiment involving 'observations' over a finite duration be meaningfully performed when the observer and the 'observed' age along orthogonal time dimensions? If we can in fact attribute observational consequences over a finite time interval to entities that exist in areatime, then there must be a mechanism by which the passage of time in the system that is being observed (the *time evolution* in the language of QM) is compared to the passage of time for the observer, *even though they do not age relative to one another.* Axiom II outlines this mechanism.

Taking a global view, we can re-express τ_A in terms of $|U_{3max}|$ by

$$\tau_A = (\tau_A)_f - (\tau_A)_i = \frac{|U_{3max}|}{A} \tag{3}$$

where $(\tau_A)_f$ and$(\tau_A)_i$ correspond to the initial and final proper temporal boundaries of $|U_{3max}|$ and A is the area orthogonal to τ_A. Now, we should expect $|U_{3max}|$ to be 'very small', otherwise the effects of the vanishing of spacetime would have been obvious at the scale of our immediate experience. Unless A is also extremely small, we must then expect that the time interval is short, at least compared to time scales of our immediate experience. Keeping in mind that the proper times τ and τ_A are orthogonal, absolute value signs will be used in comparisons between their magnitudes. So, while we already know from the fact that U_3 is a subspace of U_4 that $|\tau_A| \leq |\tau|$, we can from the above considerations strengthen this condition for $|U_{3max}|$ to $|\tau_A| << |\tau|$. This comparison is rendered meaningful by defining an interval on τ to have the same magnitude as τ_A when any point on that interval can be mapped to τ_A in manner that is one-to-one and onto. Combined with our assumption that U_3 has the same metric structure as spacetime, this allows us to assume that the two time dimensions scale the same.

The next part of Axiom II says that τ and τ_A can be compared against each other by matching τ_A to consecutive intervals of equal magnitude of τ. This is only possible if we *identify* the two opposite temporal boundaries of τ_A, which in turn introduces the property of *periodicity* to τ_A:

$$\tau_A \Rightarrow \tau_A + n\frac{|U_{3max}|}{A} \qquad n = 0, 1, 2... \tag{4}$$

The last part of axiom II says we match τ_A to τ according to the angular dual bilateral symmetry, where we take as the 'directions' associated with each time dimension the past and the future and the origin to correspond to the 'present' for both time dimensions. For example, $0 \rightarrow \tau_A$ symbolizes a time translation from the present to the future, whereas $0 \rightarrow -\tau_A$ symbolizes a time translation from the present to the past. But both translations refer to the same time interval, so the present in the first translation must correspond to the past in the second, and the present in the second to the future in the first. With this understanding, the continuous angular translations can be expressed in terms of the superposition of eight pairs of sinusoidal translations in time along orthogonal directions:

$$\begin{array}{rcl} 0 \rightarrow \tau_A & \& & \tau \rightarrow 0 \\ 0 \rightarrow \tau_A & \& & -\tau \rightarrow 0 \\ 0 \rightarrow -\tau_A & \& & \tau \rightarrow 0 \\ 0 \rightarrow -\tau_A & \& & -\tau \rightarrow 0 \\ \tau_A \rightarrow 0 & \& & 0 \rightarrow \tau \\ \tau_A \rightarrow 0 & \& & 0 \rightarrow -\tau \end{array} \tag{5}$$

$$
\begin{array}{ccccccc}
-\tau_A & \rightarrow & 0 & \& & 0 & \rightarrow & \tau \\
-\tau_A & \rightarrow & 0 & \& & 0 & \rightarrow & -\tau
\end{array}
$$

Notice how each time translation pair can be thought of as a transformation of a progression along τ_A to a progression along τ. By itself it is not allowed because then it would consitute a *net transformation*, and this in turn would imply that it is possible to express τ_A in terms of τ, in which case we could no longer consider the time dimensions to be orthogonal and therefore distinct. For the entire angular dual bilateral symmetry the net transformation between the two time dimensions, and therefore their expressibility in terms of each other, is precisely zero because each transformation is canceled by one that is its negative but equal in magnitude, thus satisfying the orthogonality condition.
Since each of the individual superpositions of translation pairs given in (5) amounts to a rotation over an angle $\varphi = \pi/2$, the entire symmetry can be thought of as the sum of eight such rotations, with a net angular displacement equal to zero. Equivalently, it can be thought of as two rotations over an angle $\varphi = 2\pi$ in opposite directions: one clockwise rotation, and one counterclockwise rotation. Fig. 3 shows how this can be visualized:

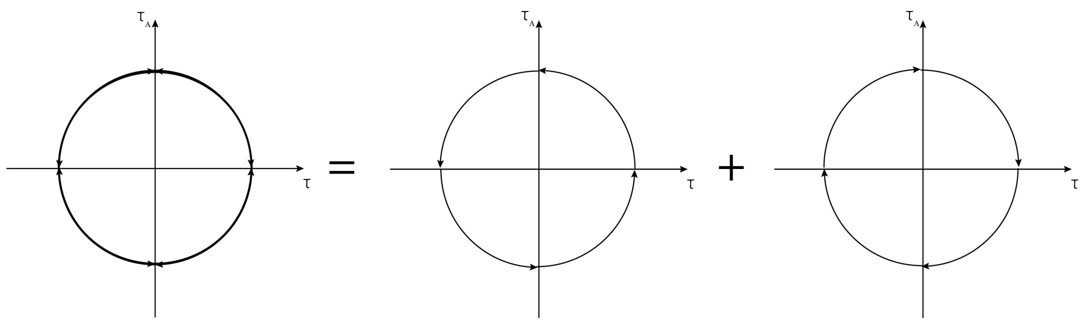

Figure 3: Decomposition of Angular Dual Bilateral Symmetry into 2 opposite rotations

We note that, in group theoretical language, the angular dual bilateral symmetry can be decomposed into two representations of of the $U(1)$ symmetry group which are inverses of one another. These representations are closely related, but not identical, to the known unitary group representations associated with quantum mechanics, as the derivation in section 4 will show. Let us continue, however, with our physical point of view. The duration over which each rotation completes a cycle is proportional to τ_A, so

$$\tau_A \propto \frac{T}{2\pi} \tag{6}$$

Where the period T parameterizes the periodicity of τ_A over the angular distance 2π. Because we defined the relation between the magnitudes of τ_A and τ such that they scale the same, the magnitude (actually the modulus, as we will see momentarily) of the proportionality constant must be equal to one. This means that we must think of the rotations in opposite directions as occurring simultaneously, rather than consecutively, because otherwise for two consecutive full rotations (one clockwise, one counterclockwise) per period, the periodicity would extend over an angular distance of 4π, thereby introducing a scale factor of 2. Since the simultaneous rotations in opposite directions can be thought of as 'time reversed' versions of each other, each value of τ_A maps to *two* values of T: One corresponding to the clockwise rotation and the other to the counterclockwise rotation. These can be distinguished by including a $\pm$sign on the right side of (6), where each sign corresponds to one of the rotations:

$$\tau_A \propto \pm\frac{T}{2\pi} \tag{7}$$

Note that this symmetry really holds only over the duration of a *single period* of simultaneous rotations over 2π because in subsequent periods, τ_A is matched up against subsequent intervals of τ. This has an important implication: If we think of τ_A as the range of φ , and of τ as its domain, then $|\tau_A| << |\tau|$ means that a rotation in both directions over the angle $\varphi = 2\pi$ does not return one to the starting point. In fact, if we take $|\tau|$ to be so much larger than $|\tau_A|$ that by comparison $|\tau| \approx \infty$ (since an observer in spacetime could, in principle, carry on an experiment 'forever'), we can consider the domain of φ to be infinite. *This mathematically defines an imaginary angle!* In other words, the angle that is defined by the relation between τ_A and τ is actually $i\varphi$ where $i \equiv \sqrt{-1}$. *This* is the proportionality constant:

$$\tau_A \ = \pm\frac{T}{i2\pi} = \mp i\frac{T}{2\pi} \tag{8}$$

consistent with the fact that the range of an imaginary angle is itself imaginary. Imaginary numbers were discovered several hundred years ago, and their appearance in equations that model the real world (to wit: Schrödinger's equation) has been one of the main mysteries associated with quantum theory. The framework here suggests a clear mathematical reason for their necessity, but one can also attach to it a physical interpretation distinct from the mathematical reason given above: τ_A does not exist as such in spacetime; if it did, then entities existing in spacetime would age along τ_A, but they actually age along τ, which is real. Thus, τ_A must be modeled as an imaginary quantity of time in spacetime.

4 The Derivation

Having introduced and explicated the principle of dual bilateral symmetry, it is straightforward to translate it into a mathematical form that is readily recognizable as the quantum phase. To begin, we identify each group of 4 translation pairs which sum to a rotation over 2π with a phase. The two phases are then complex conjugates of each other and, as mentioned, must be considered to be simultaneous instead of consecutive, otherwise they introduce a scale factor of 2 between the proper times. Considering for the moment only one of the two phases, for any interval $2\pi n \leq \varphi \leq 2\pi(n+1)$, $n = 0, 1, 2...$, the real angle φ is given by

$$\varphi = \tan^{-1}\left(\frac{\tau_A}{i}\frac{1}{\tau}\right) \tag{9}$$

Where τ_A is divided by i to make it real. The phase amplitude τ_r is given by

$$\tau_r = \sqrt{\tau^2 + \left(\frac{\tau_A}{i}\right)^2} \tag{10}$$

To transform this phase into the quantum phase we must normalize the amplitude and render it dimensionless. This can be accomplished by defining a new angle θ such that

$$\theta \equiv \cot\varphi = \cot\left(\tan^{-1}\left(\frac{\tau_A}{i}\frac{1}{\tau}\right)\right) = i\frac{\tau}{\tau_A} \tag{11}$$

Then we can define a new Argand plane consisting of the orthogonal axes x and iy such that over a single period the real angle θ is given by

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = i\frac{\tau}{\tau_A} \tag{12}$$

From which it follows that

$$\frac{y}{x} = \tan\left(i\frac{\tau}{\tau_A}\right) = \frac{\sin(i\tau/\tau_A)}{\cos(i\tau/\tau_A)} \tag{13}$$

The new amplitude r is then given by

$$r = \sqrt{y^2 + x^2} = \sqrt{\sin^2\left(\frac{i\tau}{\tau_A}\right) + \cos^2\left(\frac{i\tau}{\tau_A}\right)} = 1 \tag{14}$$

which is normalized and dimensionless, as required. Since τ_A is periodic, we can identify it with the inverse of an imaginary angular frequency

$$\tau_A = \mp i\frac{T}{2\pi} = \mp\frac{i}{\omega} \tag{15}$$

Where the $\mp$sign indicates that we are now again considering both phases. Then the imaginary angle $i\theta$, which is now a parameterization of $i\varphi$, is given by

$$\mp i\theta = i\left(i\frac{\tau}{\tau_A}\right) = \mp i\omega\tau \tag{16}$$

Using (16) we can describe the phases mathematically as

$$e^{\left(-\frac{\tau}{\tau_A}\right)} = e^{\mp i\omega\tau} \tag{17}$$

This is the relativistic form of the quantum phase. We can make the connection to quantum theory more explicit by letting $\omega \equiv \frac{mc^2}{\hbar}$and $\tau = \int d\tau$, for then we get

$$e^{\mp i\frac{mc^2}{\hbar}\int d\tau} = e^{\mp i\frac{mc}{\hbar}\int ds} = e^{\pm i\frac{S}{\hbar}} \tag{18}$$

Where $S = -mc\int ds$ is the classical relativistic action of a free particle and the last term in (18) appears in its non-relativistic approximation as an axiom in the path integral formulation of quantum mechanics. Note that both signs of that phase are required in quantum theory because in the final step of calculating probabilities the complex phase must be multiplied by its complex conjugate. From the perspective of this derivation this requirement is due to the fact that only the product of the two conjugate phases leads to the full angular dual bilateral symmetry. This, in turn, ensures that the net phase change is zero, which reflects a net zero transformation of progression along τ_A to progression along τ.

The partial answer to the question posed at the end of section 2 is that any time-dependent description of our observation of phenomena associated with entities that exist in areatime must include phases of the form given in (17) or their mathematical equivalents (including non-relativistic approximations). Connecting this with our knowledge of quantum theory allows us to turn this answer around and generalize our claim about photons: *Any object which must be described by a phase of the form given in (17) (i.e. any quantum object) is our description of the manifestation in spacetime of an entity that actually exists in areatime.*

5 Conclusion

This paper presented a derivation of the quantum phase from two fundamental axioms, one involving a quantization of spacetime in which spacetime in the limit of zero space reduces to areatime and the other involving a conceptualization of the phase in terms of an angular dual bilateral symmetry between the proper time dimension of spacetime and the proper time dimension of areatime. The derivation implies that quantum objects actually exist in areatime.

A less technical, more philosophically oriented overview of how these ideas can fit with standard quantum mechanics is given in [2], whereas the follow-up paper to this one attempts to provide a more detailed derivation of the remaining principles of quantum mechanics from these and a few additional physical assumptions [3].

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