# Using *Mathematica* for matrices

## Matrices

Matrices are entered in "row form", such that

```
In[195]:= aa = {{2, 1}, {-1, 2}}
Out[195]= {{2, 1}, {-1, 2}}
```

gives the following matrix (the // and "MatrixForm" displays the result so it looks like a matrix)

```
In[196]:= aa // MatrixForm
Out[196]//MatrixForm=
```

$$\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

Picking out components now requires two indices, which are in standard "row, column" order:

```
aa[[1, 2]]
1
```

```
In[197]:= bb = {{3, 2}, {-1, -1}}; bb // MatrixForm
Out[197]//MatrixForm=
```

$$\begin{pmatrix} 3 & 2 \\ -1 & -1 \end{pmatrix}$$

There are some canned matrices, in particular the identity
(the argument of IdentityMatrix giving the linear dimension):

```
id = IdentityMatrix[3]; id // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Another predefined set of matrices are the Pauli matrices:

```
{PauliMatrix[1] // MatrixForm,
 PauliMatrix[2] // MatrixForm, PauliMatrix[3] // MatrixForm}
```

$$\left\{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right\}$$

There's a special command to create a diagonal matrix:

```
DiagonalMatrix[{1, 2, 3}] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Matrix multiplication is written with a Dot (and is not commutative, as we know)

In[199]:=
```
aa.bb // MatrixForm
bb.aa // MatrixForm
```

Out[199]//MatrixForm=

$$\begin{pmatrix} 5 & 3 \\ -5 & -4 \end{pmatrix}$$

Out[200]//MatrixForm=

$$\begin{pmatrix} 4 & 7 \\ -1 & -3 \end{pmatrix}$$

Whereas a product simply multiplies the corresponding elements, one by one:

In[201]:=
```
aa bb // MatrixForm
```

Out[201]//MatrixForm=

$$\begin{pmatrix} 6 & 2 \\ 1 & -2 \end{pmatrix}$$

Addition and subtraction and multiplication by scalars work:

```
aa + bb // MatrixForm
aa - bb // MatrixForm
3 aa // MatrixForm
```

$$\begin{pmatrix} 5 & 3 \\ -2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 \\ 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 3 \\ -3 & 6 \end{pmatrix}$$

Multiplication works with any shape matrices, as long as they are conformable. Here's a vector, which, although it's entered as a row-like vector:

```
v5 = {3, 1}
```

```
{3, 1}
```

is treated like a column vector under matrix multiply:

```
aa.v5
```

```
{7, -1}
```

It is displayed like a column.

```
aa.v5 // MatrixForm
```

$$\begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

However, one can also multiply from the left, in which case the vector is treated as a row

```
v5.aa
```

```
{5, 5}
```

Transpose transposes:

```
Transpose[aa] // MatrixForm
```

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

Conjugate conjugates, element by element:

```
cc = {{2 + I, 3 I}, {-3 I, 4}}; cc // MatrixForm
```

$$\begin{pmatrix} 2 + i & 3 i \\ -3 i & 4 \end{pmatrix}$$

```
Conjugate[cc] // MatrixForm
```

$$\begin{pmatrix} 2 - i & -3 i \\ 3 i & 4 \end{pmatrix}$$

To hermitian conjugate use the ConjugateTranspose[ ] function

```
ConjugateTranspose[cc] // MatrixForm
```

$$\begin{pmatrix} 2 - i & 3 i \\ -3 i & 4 \end{pmatrix}$$

or you can make a “dagger” which does the same thing by typing “escape ct escape”

```
cc† // MatrixForm
```

$$\begin{pmatrix} 2 - i & 3 i \\ -3 i & 4 \end{pmatrix}$$

## Functions of Matrices

Recall the matrix aa:

```
aa // MatrixForm
```

$$\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$

Trace

```
Tr[aa]
```

4

Determinant

```
Det[aa]
```

5

Inverse

```
aainv = Inverse[aa]; aainv // MatrixForm
```

$$\begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}$$

Check that inverse “works”

```
aainv.aa // MatrixForm
aa.aainv // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

There's no problem with moving to larger matrices, which would be painful by hand:

```
dd = {{1, 2, 3, 4, 5}, {2, 3, 7, 8, 9}, {-3, 0, 6, 4, 2},
  {6, 2, 4, 5, 1}, {-1, -2, 5, 2, 3}}; dd // MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 7 & 8 & 9 \\ -3 & 0 & 6 & 4 & 2 \\ 6 & 2 & 4 & 5 & 1 \\ -1 & -2 & 5 & 2 & 3 \end{pmatrix}$$

```
Inverse[dd] // MatrixForm
```

$$\begin{pmatrix} \frac{44}{35} & -\frac{4}{5} & -\frac{3}{35} & \frac{8}{35} & \frac{2}{7} \\ \frac{351}{35} & -\frac{31}{5} & \frac{23}{35} & \frac{32}{35} & \frac{8}{7} \\ \frac{621}{70} & -\frac{28}{5} & \frac{19}{35} & \frac{31}{35} & \frac{19}{14} \\ -\frac{179}{14} & 8 & -\frac{4}{7} & -\frac{8}{7} & -\frac{27}{14} \\ \frac{59}{70} & -\frac{2}{5} & -\frac{4}{35} & -\frac{1}{35} & \frac{3}{14} \end{pmatrix}$$

Rank---it does the row reduction for you:

```
MatrixRank[dd]
```

5

The rank is the same for the transpose, as it should be:

```
MatrixRank[Transpose[dd]]
```

5

*Mathematica* does row reduction for you. Technically this gives the "reduced row echelon form", with as many off-diagonal zeroes as possible.

```
RowReduce[dd] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

A simple example discussed in lecture notes:

```
ee = {{2, 2}, {1, 1}}; ee // MatrixForm
```

$$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$$

```
MatrixRank[ee]
```

1

```
RowReduce[ee] // MatrixForm
```

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

## More complicated functions of matrices

*Mathematica* has a built in function for exponentiating a matrix

```
aa = {{1, 1}, {0, 2}}; MatrixExp[aa] // MatrixForm
```

$$\begin{pmatrix} e & -e + e^2 \\ 0 & e^2 \end{pmatrix}$$

Note that this is different from exponentiating in the usual way, which simply exponentiates each element.

```
E^aa // MatrixForm
```

$$\begin{pmatrix} e & e \\ 1 & e^2 \end{pmatrix}$$

There's also a function for taking powers of matrices (which works for all complex powers too)

```
MatrixPower[aa, 10] // MatrixForm
```

$$\begin{pmatrix} 1 & 1023 \\ 0 & 1024 \end{pmatrix}$$

```
MatrixPower[aa, -2] // MatrixForm
```

$$\begin{pmatrix} 1 & -\frac{3}{4} \\ 0 & \frac{1}{4} \end{pmatrix}$$

```
MatrixPower[aa, I]
```

$\{\{1, -1 + 2^{i}\}, \{0, 2^{i}\}\}$