7-4

Practice

Double-Angle and Half-Angle Identities

Use a half-angle identity to find the exact value of each function.

1. $\sin 105°$

$$\dfrac{\sqrt{2 + \sqrt{3}}}{2}$$

2. $\tan \dfrac{\pi}{8}$

$$\sqrt{2} - 1$$

3. $\cos \dfrac{5\pi}{8}$

$$\dfrac{\sqrt{2 - \sqrt{2}}}{2}$$

Use the given information to find sin 2θ, cos 2θ, and tan 2θ.

4. $\sin \theta = \dfrac{12}{13}, 0° < \theta < 90°$

$$\dfrac{120}{169}, -\dfrac{119}{169}, -\dfrac{120}{119}$$

5. $\tan \theta = \dfrac{1}{2}, \pi < \theta < \dfrac{3\pi}{2}$

$$\dfrac{4}{5}, \dfrac{3}{5}, \dfrac{4}{3}$$

6. $\sec \theta = -\dfrac{5}{2}, \dfrac{\pi}{2} < \theta < \pi$

$$-\dfrac{4\sqrt{21}}{25}, -\dfrac{17}{25}, \dfrac{4\sqrt{21}}{17}$$

7. $\sin \theta = \dfrac{3}{5}, 0 < \theta < \dfrac{\pi}{2}$

$$\dfrac{24}{25}, \dfrac{7}{25}, \dfrac{24}{7}$$

Verify that each equation is an identity.

8. $1 + \sin 2x = (\sin x + \cos x)^2$

$1 + \sin 2x \overset{?}{=} (\sin x + \cos x)^2$

$1 + \sin 2x \overset{?}{=} \sin^2 x + 2 \sin x \cos x + \cos^2 x$

$1 + \sin 2x \overset{?}{=} 1 + 2 \sin x \cos x$

$1 + \sin 2x = 1 + \sin 2x$

9. $\cos x \sin x = \dfrac{\sin 2x}{2}$

$\cos x \sin x \overset{?}{=} \dfrac{\sin 2x}{2}$

$\cos x \sin x \overset{?}{=} \dfrac{2 \sin x \cos x}{2}$

$\cos x \sin x = \cos x \sin x$

10. *Baseball*   A batter hits a ball with an initial velocity v_0 of 100 feet per second at an angle θ to the horizontal. An outfielder catches the ball 200 feet from home plate. Find θ if the range of a projectile is given by the formula $R = \dfrac{1}{32}v_0{}^2 \sin 2\theta$.
about 20°