Coping with a Smart Jammer in Wireless Networks: A Stackelberg Game Approach

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***Abstract*— Jamming defense is an important yet challenging problem. In this paper, we study the jamming defense problem in the presence of a smart jammer, who can quickly learn the transmission power of the user and adaptively adjust its transmission power to maximize the damaging effect. We consider both the single-channel model and the multi-channel model. By modeling the problem as a Stackelberg game, we compute the optimal transmission power for the user to maximize its utility, in the presence of a smart jammer. For the single-channel model, we prove the existence and uniqueness of the Stackelberg Equilibrium (SE) by giving closed-form expressions for the SE strategies of both the user and the player. For the multi-channel model, we prove the existence of the SE. We design algorithms for computing the jammer's best response strategy and approximating the user's optimal strategy. Finally, we validate our theoretical analysis through extensive simulations.**

***Index Terms*—Jamming, Stackelberg Game.**

I. Introduction

WIRELESS networks are highly vulnerable to jamming attacks, since jamming attacks are easy to launch. Attacks of this kind usually aim at the physical layer and are realized by means of a high transmission power signal that corrupts a communication channel, as shown in Fig. 1. We are interested in defending against smart jammers, who can quickly learn the transmission pattern of the users and adjust their jamming strategies so as to exacerbate the damage. Since jammers need to consider transmission cost, transmitting with the maximum power may not be the optimal strategy. As a first step along this line, we study the battle between a single user (a transmitter-receiver pair) and a single smart jammer (a malicious transmitter). This problem arises, for example, in military operations, where one radio station transmits data to another in a hostile environment. In this paper, we aim to derive the optimal power control for the user in the presence of a smart jammer.

Game theory is a natural tool to model and address this problem. Jamming defense can be considered a game, where both the user and the jammer are players. Previous works [1, 2] have been done on this topic by proving the existence of Nash Equilibria and computing a Nash Equilibrium. A *Nash Equilibrium* (NE) is the status where no player has an incentive to change its strategy unilaterally so as to increase its own utility. However, Nash Equilibrium is not the best solution to the problem studied in this paper, because the rationality of Nash Equilibrium is based on the assumption that all players take actions simultaneously. In our model, the jammer is intelligent in the sense that it can quickly learn the user's transmission power and adjust its transmission power accordingly. Stackelberg game serves the purpose of modeling this scenario. In this game, players, including one *leader* and one *follower*, are in a hierarchical structure. The leader takes actions first, and then the follower takes actions accordingly. Similar to the Nash Equilibrium in the standard game, there is *Stackelberg Equilibrium* in this game. Different from the Nash Equilibrium, Stackelberg Equilibrium is the optimal strategy of the leader, given the fact that the follower would take actions according to the leader's strategy, together with the optimal strategy of the follower corresponding to the leader's optimal strategy.

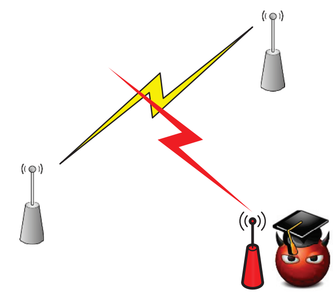

Fig. 1. Jamming in wireless networks.

To the best of our knowledge, this paper is the first to study the power control problem in the presence of a smart jammer. As an initial step, we consider a single user and a single jammer (the more challenging scenario with multi users/jammers is a subject of future research). Both the user and the jammer can adjust their transmission power levels. We consider both the single-channel model and the multi-channel model. We model the power control problem with a smart jammer as a Stackelberg game [14], called *Power Control with Smart Jammer* (PCSJ) game. In this game, the user is the *leader* and the jammer is the *follower*. The user is aware of the jammer's existence and has the knowledge of jammer's intelligence, based on which the user chooses an optimal strategy so as to maximize its own utility, while

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the jammer plays its best response strategy given the user's strategy. For the single-channel model, we derive closed-form expressions for the jammer's best response strategy and the user's optimal strategy, which together constitute the unique Stackelberg Equilibrium (SE). For the multi-channel model, we design an algorithm for computing the jammer's best response strategy, given the user's strategy. We also develop two algorithms to approximate the user's optimal strategy and thus the SE strategies.

The rest of this paper is organized as follows: In Section II, we briefly describe the related works. In Section III, we introduce the system model and the Stackelberg game formulation. In Section IV, we study the PCSJ game under the single-channel model. In Section V, we study the PCSJ game under the multi-channel model. In Section VI, we present numerical results. We conclude this paper in Section VII.

II. Related Work

Due to the importance of jamming defense, wireless network jamming has been extensively studied in the past few years. Many jamming defense mechanisms have been proposed on both the physical layer [8–10, 17, 18] and the MAC layer [11, 12] to detect jamming, as well as to avoid it. Spread spectrum technologies have been shown to be very effective to avoid jamming. With enough bandwidth or widely spread signals, it becomes harder to detect the start of a packet quickly enough in order to jam it.

Since jamming activities can be considered as a player (the jammer) playing against another player (the user), game theory is an appropriate tool to deal with this kind of problem. Many previous works have studied jamming defense with game theory formulations [1, 2, 7, 13, 16, 20]. In [1], Altman *et al.* studied the jamming game in wireless networks with transmission cost. In this game, both the user and the jammer take the power allocation on channels as their strategies. The utility of the user is the weighted capacity minus transmission cost. The utility of the jammer is the negative of the user's weighted capacity minus transmission cost. The authors proved the existence and uniqueness of Nash Equilibrium. In addition, they provided analytical expressions for the equilibrium strategies. In [2], the same group of authors extended the jamming problem to the case with several jammers. The difference from [1] is that they did not consider transmission cost and they considered SINR and −SINR as the utility values for the user and the jammers, respectively. They showed that the jammers equalize the quality of the best sub-carriers for transmitter on as low level as their power constraint allows, meanwhile the user distributes its power among these jamming sub-carriers. In [13], Sagduyu *et al.* considered the power-controlled MAC game, which includes two types of players, selfish and malicious transmitters. Each type of user has a different utility function depending on throughput reward and energy cost. They also considered the case where the transmitters have incomplete information regarding other transmitter's types, modeled as probabilistic beliefs. They derived the Bayesian Nash Equilibrium strategies for different degrees of uncertainty, and characterized the resulting equilibrium throughput of selfish nodes.

The jamming problems have also been studied in cognitive radio networks [7, 16, 20]. The anti-jamming game in this scenario is often modeled as a (stochastic) zero-sum game, where the sum of the utility values of the jammer(s) and the secondary user is zero. In [20], Zhu *et al.* assumed the transition between idle and busy states of the channel to be Markovian. They considered a single secondary user and a single jammer in the cognitive radio system. The strategy of the user is the channel selected to transmit on, while the strategy of the jammer is the channel selected to jam. The utility of the user is 1 if the selected channel is not occupied by the primary user and not jammed by the jammer. They considered mixed strategies and proved the conditions for the uniqueness of the Nash Equilibrium. They also showed that the secondary user can either improve its sensing capability to confuse the jammer or choose to communicate under states where the available channels are less prone to jamming, in order to improve its utility value. In [7], Li and Han studied the problem of defending primary user emulation attack, which is similar to the jamming attack in wireless networks. There is only one jammer and one or multiple secondary users in their models. The strategy of each secondary user is the channel selected to transmit on, while the strategy of the jammer is the channel selected to jam. The utility of each secondary user is a reward if it senses a channel and the jammer is not jamming. They computed the unique Nash Equilibrium and analyzed the efficiency. In [16], Wu *et al.* first investigated the case where a secondary user can access only one channel at a time and then extended to the scenario where secondary users can access all the channels simultaneously. For the former case, the secondary user uses channel hopping as its defense strategy. The utility of the secondary user is equal to a communication gain, if the transmission is successful, minus cost and a significant loss when jammed. They found an approximation to the Nash Equilibrium by letting the user and jammers iteratively update their strategies against each other. For the latter case, the secondary user could allocate power to several channels. The utility of the secondary user is equal to the total number of successful transmissions. They showed that the defense strategy from the Nash Equilibrium is optimal.

In all the previous works on jamming defense, the authors assumed that the users and the jammers take actions simultaneously. In this paper, we study the power control problem in the presence of a smart jammer, which has more power compared to the jammer model studied before. To the best of our knowledge, we are the first to address this problem.

III. System Model and Game Formulation

In this section, we present the system model and formulate the problem to be studied.

A. System Model

Our system consists of a *user* (i.e., a transmitter-receiver pair), and a *jammer* (i.e., a malicious transmitter), as illustrated in Fig. 1. The user (jammer, respectively) has control over its own transmission power. This problem arises, for example, in military operations, where one radio station transmits data to another radio station in a hostile environment. We consider two models in this paper: single-channel model and multi-channel model.

Single-channel model: Let P denote the transmission power of the user and J denote the transmission power of the jammer. In addition, we assume that the user and the jammer transmit with cost E and C *per unit power*. As in [2, 13], we adopt SINR as the reward of the user in our model. Hence, the *utility of the user* is

$$u_s(P,J) = \frac{\alpha P}{N+\beta J} - EP, \tag{1}$$

and the *utility of the jammer* is

$$v_s(P,J) = -\frac{\alpha P}{N+\beta J} - CJ, \tag{2}$$

where N is the background noise on the channel, and $\alpha > 0$ and $\beta > 0$ are fading channel gains of the user and the jammer, respectively. Note that we have omitted power constraint in this model. If power constraint is added, we can still derive closed-form expressions for the SE. However, the corresponding analysis is more complicated, involving ad-hoc discussions of many cases. Hence, we choose to concentrate on this model, which allows us to to emphasize the main contributions without excessive ad-hoc analysis.

Multi-channel model: We assume that there are n available channels. Let $\alpha_i \in (0,1]$ and $\beta_i \in (0,1]$ denote the fading channel gains of the user and the jammer on channel i, respectively. Let $\hat{P} > 0$ and $\hat{J} > 0$ denote the *total transmission power* of the user and the jammer, respectively. Let P_i and J_i denote the transmission power allocated to channel i by the user and the jammer, respectively. Let $\boldsymbol{P} = (P_1, P_2, \ldots, P_n)$ and $\boldsymbol{J} = (J_1, J_2, \ldots, J_n)$ denote the *transmission power vectors* of the user and the jammer, respectively. $\boldsymbol{P}$ is feasible if $\sum_{i=1}^n P_i \leq \hat{P}$, and $\boldsymbol{J}$ is feasible if $\sum_{i=1}^n J_i \leq \hat{J}$. Let $\mathcal{P} = \{(P_1, P_2, \ldots, P_n) | P_i \geq 0, \sum_{i=1}^n P_i \leq \hat{P}\}$ and $\mathcal{J} = \{(J_1, J_2, \ldots, J_n) | J_i \geq 0, \sum_{i=1}^n J_i \leq \hat{J}\}$ denote the sets of feasible power vectors of the user and the jammer, respectively. Similarly to the single-channel model, we assume that the user and the jammer transmit with cost E and C per unit power. The *utility of the user* is

$$u_m(\boldsymbol{P},\boldsymbol{J}) = \sum_{i=1}^{n} \frac{\alpha_i P_i}{N_i + \beta_i J_i} - E\sum_{i=1}^{n} P_i. \tag{3}$$

The *utility of the jammer* is

$$v_m(\boldsymbol{P},\boldsymbol{J}) = -\sum_{i=1}^{n} \frac{\alpha_i P_i}{N_i + \beta_i J_i} - C\sum_{i=1}^{n} J_i. \tag{4}$$

In this paper, we deal with a *smart* jammer, who can quickly learn the user's transmission power and adjust its transmission power accordingly to maximize its utility. The user's transmission power can be accurately learned using physical carrier sensing and location knowledge. We are interested in determining the transmission power of the user such that its utility is maximized, in the presence of a smart jammer. We call this problem *the power control problem with a smart jammer*.

TABLE I
A UTILITY MATRIX: THE FIRST NUMBER IN EACH CELL IS THE UTILITY OF PLAYER A, WHILE THE SECOND IS THE UTILITY OF PLAYER B.

		Player B	
		L	R
Player A	U	3,2	**6,5**
	D	4,3	8,2

B. Basics of Stackelberg Game

In game theory, Stackelberg game [4] is a tool to model the scenario where hierarchy of actions exists between two types of players: one is *leader*, and the other is *follower*. The leader makes its move first. After the leader chooses a strategy, the follower always chooses a *best response strategy* that maximizes its utility. Knowing this reaction from the follower, the leader strategically chooses a strategy to maximize its utility. This optimal strategy of the leader, together with the corresponding best response strategy of the follower, constitutes a Stackelberg Equilibrium (SE) of the game.

We illustrate these concepts using a simple example given in Table I. Note that this example is just for the illustration of an SE and not an instance of the problem studied. Assume that Player A is the leader, and Player B is the follower. If A plays strategy U, B would play strategy R, as it gives player B a utility of 5 (as opposed to a utility of 2 should B play strategy L). This leads to a utility of 6 for player A. If A plays strategy D, B would play strategy L, as it gives player B a utility of 3 (as opposed to a utility of 2 should B play strategy R). This leads to a utility of 4 for player A. Hence A would play strategy U, since doing so would result in a utility of 6 compared to 4 by playing strategy D. As explained before, B would play R if A plays U. Therefore the Stackelberg Equilibrium of this game is (U, R).

C. Stackelberg Game Formulation

In our model, the jammer is smart and can adjust its transmission power based on the user's transmission power. Based on this fact, we model the power control problem in the presence of a smart jammer as a Stackelberg game, called *Power Control with Smart Jammer* (PCSJ) game. In this game, both the user and the jammer are *players*, of which the user is the leader, and the jammer is the follower. The *strategy* of each player is its transmission power. The *utility* of the user (resp. the jammer) is defined in (1) (resp. (2)) for the single-channel model and (3) (resp. (4)) for the multi-channel model.

IV. PCSJ UNDER SINGLE-CHANNEL MODEL

In this section, we study the PCSJ game under the single-channel model. First, we compute the best response strategy of the jammer, for a given strategy of the user. Then we compute the optimal strategy of the user, based on the knowledge of the best response strategy of the jammer.

A. Jammer's Best Response Strategy

Assume that the user's strategy P is given. Then the jammer's best response strategy can be computed by solving the following optimization problem.

$$\max_{J\geq 0} v_s(P,J) = -\frac{\alpha P}{N+\beta J} - CJ. \tag{5}$$

Thus we have the following lemma.

Lemma 1. *Let P be a given strategy of the user. Then the corresponding optimal strategy of the jammer is*

$$J(P) = \begin{cases} 0, & P \leq \frac{CN^2}{\alpha\beta}, \\ \frac{\sqrt{\frac{\alpha\beta P}{C}}-N}{\beta}, & P > \frac{CN^2}{\alpha\beta}. \end{cases} \tag{6}$$

Proof: To find the maximum value of $v_s(P,J)$, we differentiate $v_s(P,J)$ with respect to J and set the resulting derivative equal to 0,

$$0 = \frac{\partial v_s(P,J)}{\partial J} = \frac{\alpha\beta P}{(N+\beta J)^2} - C. \tag{7}$$

Considering the constraint $J \geq 0$, we have the optimal strategy of the jammer in (6). ■

B. User's Optimal Strategy

The user is aware of the existence of the jammer and knows that the jammer will play its best response strategy to maximize its own utility. Therefore, the user can derive the jammer's strategy based on Lemma 1. To compute the user's optimal strategy, we solve the following optimization problem.

$$\max_{P\geq 0} u_s(P,J(P)) = \frac{\alpha P}{N+\beta J(P)} - EP, \tag{8}$$

where $J(P)$ is given in (6).

The optimal strategy of the user is given in the following Lemma.

Lemma 2. *The optimal strategy of the user is*

$$P^{SE} = \begin{cases} \frac{\alpha C}{4\beta E^2}, & E \leq \frac{\alpha}{2N}, \\ \frac{CN^2}{\alpha\beta}, & \frac{\alpha}{2N} < E \leq \frac{\alpha}{N}, \\ 0, & E > \frac{\alpha}{N}. \end{cases} \tag{9}$$

Proof: Plugging (6) into the objective function (8), we have

$$u_s(P,J(P)) = \begin{cases} (\frac{\alpha}{N} - E)P, & P \leq \frac{CN^2}{\alpha\beta}, \\ \sqrt{\frac{\alpha CP}{\beta}} - EP, & P > \frac{CN^2}{\alpha\beta}. \end{cases} \tag{10}$$

Hence $u_s(P,J(P))$ is a linear function in P for $0 \leq P \leq \frac{CN^2}{\alpha\beta}$, and is a strictly concave function in P for $P > \frac{CN^2}{\alpha\beta}$. Note that the derivative of $u_s(P,J(P))$ with respect to P in the range $P > \frac{CN^2}{\alpha\beta}$ is given by

$$\frac{\partial u_s(P,J(P))}{\partial P} = \frac{1}{2}\sqrt{\frac{\alpha C}{\beta P}} - E. \tag{11}$$

Setting equation (11) to 0, we obtain $P = \frac{\alpha C}{4\beta E^2}$.

To compute the maximum value of (10), we consider three disjoint cases.

Case-1: $E \leq \frac{\alpha}{2N}$. In this case, we can verify that $\frac{CN^2}{\alpha\beta} \leq \frac{\alpha C}{4\beta E^2}$. As illustrated in Fig. 2(a), $u_s(P,J(P))$ achieves its maximum value of $\frac{\alpha C}{4\beta E}$ when $P = \frac{\alpha C}{4\beta E^2}$.

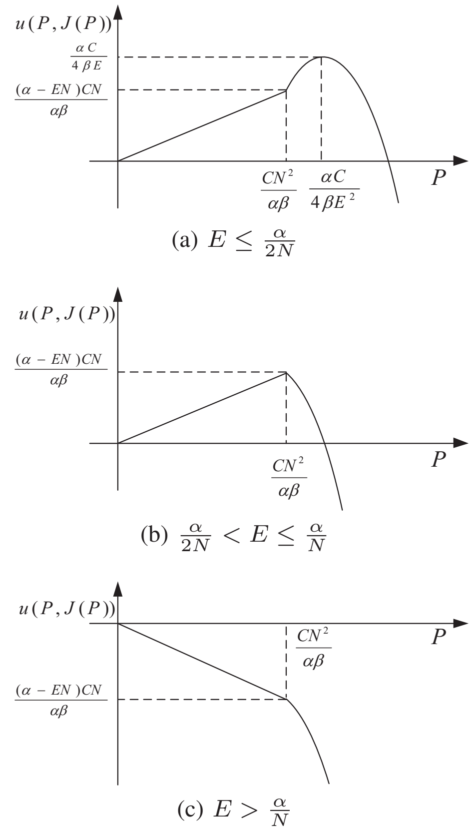


Fig. 2. User's utility function for different values of E.

Case-2: $\frac{\alpha}{2N} < E \leq \frac{\alpha}{N}$. In this case, we can verify that $\frac{CN^2}{\alpha\beta} > \frac{\alpha C}{4\beta E^2}$. As illustrated in Fig. 2(b), $u_s(P,J(P))$ achieves its maximum value of $\frac{(\alpha-EN)CN}{\alpha\beta}$ when $P = \frac{CN^2}{\alpha\beta}$.

Case-3: $E > \frac{\alpha}{N}$. In this case, we also have $\frac{CN^2}{\alpha\beta} > \frac{\alpha C}{4\beta E^2}$. As illustrated in Fig. 2(c), $u_s(P,J(P))$ achieves its maximum value of 0 when $P = 0$.

This proves the lemma. ■

Lemmas 1 and 2 lead to the following theorem.

Theorem 1. *The strategy pair (P^{SE}, J^{SE}) is the Stackelberg Equilibrium of the PCSJ game, where*

$$P^{SE} = \begin{cases} \frac{\alpha C}{4\beta E^2}, & E \leq \frac{\alpha}{2N}, \\ \frac{CN^2}{\alpha\beta}, & \frac{\alpha}{2N} < E \leq \frac{\alpha}{N}, \\ 0, & E > \frac{\alpha}{N}, \end{cases}$$

and

$$J^{SE} = \begin{cases} \frac{\frac{\alpha}{2E}-N}{\beta}, & E \leq \frac{\alpha}{2N}, \\ 0, & E > \frac{\alpha}{2N}. \end{cases}$$

Remark. Note that the user needs to have the knowledge of β to compute P^{SE}. This can be achieved as follows: The user randomly selects its initial transmission power $P^{[0]} > 0$. It then keeps increasing its transmission power to $P^{[i]}$ until the received jamming signal is non-zero. For example, it can set $P^{[i]} = 2^i \times P^{[0]}$ for $i > 0$. The received jamming signal can be measured by taking advantage of the delay in jamming's decision making. According to (6), we have $\beta J(P^{[i]}) =$

$\sqrt{\frac{\alpha\beta P^{[i]}}{C}} - N$. Hence we have $\beta = \frac{C\left(\beta J(P^{[i]})+N\right)^2}{\alpha P^{[i]}}$, where $\beta J(P^{[i]})$ is the received jamming signal.

V. PCSJ UNDER MULTI-CHANNEL MODEL

In this section, we study the PCSJ game under the multi-channel model.

A. Jammer's Best Response Strategy

Given the user's strategy $\boldsymbol{P}$, the problem of power allocation for the jammer can be formulated as a convex optimization problem as follows.

$$\max_{\boldsymbol{J}\in\mathcal{J}} v_m(\boldsymbol{P},\boldsymbol{J}) = -\sum_{i=1}^{n} \frac{\alpha_i P_i}{N_i + \beta_i J_i} - C\sum_{i=1}^{n} J_i. \tag{12}$$

In the following, Theorem 2 guarantees the existence of the jammer's best response strategy, and Theorem 3 computes the jammer's best response strategy, when the user's strategy is given.

Theorem 2. *Let $\boldsymbol{P}$ be user's strategy. There exists a unique $\boldsymbol{J}(\boldsymbol{P})$ such that $v_m(\boldsymbol{P},\boldsymbol{J}(\boldsymbol{P}))$ is maximized.*

Proof: Since $v_m(\boldsymbol{P},\cdot)$ is a continuous function on the compact set $\mathcal{J}$, it can achieve its maximum value at some $\boldsymbol{J}\in\mathcal{J}$ [15].

Without loss of generality, we assume that $P_i > 0$ for $1 \le i \le k$ and $P_i = 0$ for $k < i \le n$. It is obvious that, in any optimal solution of the optimization problem (12), we have $J_i(\boldsymbol{P}) = 0$ for $k < i \le n$. Otherwise, we can increase the value of $v_m(\boldsymbol{P},\boldsymbol{J})$ by setting $J_i(\boldsymbol{P}) = 0$, contradicting the optimality of $\boldsymbol{J}$. The optimization problem then becomes

$$\max_{\boldsymbol{J}} v_m(\boldsymbol{P},\boldsymbol{J}) = -\sum_{i=1}^{k} \frac{\alpha_i P_i}{N_i + \beta_i J_i} - C\sum_{i=1}^{k} J_i \tag{13}$$

s.t.

$$\sum_{i=1}^{k} J_i \le \hat{J}, J_i \ge 0, \text{for all } i\in[1,k].$$

The first order partial derivative of $v_m(\boldsymbol{P},\boldsymbol{J})$ with respective to J_i, for $i\in[1,k]$, is

$$\frac{\partial v_m(\boldsymbol{P},\boldsymbol{J})}{\partial J_i} = \frac{\alpha_i\beta_i P_i}{\left(N_i+\beta_i J_i\right)^2} - C, \tag{14}$$

and the second order partial derivatives of $v_m(\boldsymbol{P},\boldsymbol{J})$ are

$$\frac{\partial^2 v_m(\boldsymbol{P},\boldsymbol{J})}{\partial J_i \partial J_j} = \begin{cases} -\frac{2\alpha_i\beta_i^2 P_i}{(N_i+\beta_i J_i)^3}, & i=j,\\ 0, & i\neq j. \end{cases} \tag{15}$$

The Hessian matrix is negative definite [3]: $\bigtriangledown^2 v_m(\boldsymbol{P},\boldsymbol{J}) \prec 0$, implying that the objective function (13) is strictly concave, and there is a unique solution to the optimization problem. ■

Theorem 3. *Let $\boldsymbol{P}$ be user's strategy. We define $\pi(\lambda) = \sum_{i=1}^{k}\left[\frac{\sqrt{\frac{\alpha_i\beta_i P_i}{C+\lambda}}-N_i}{\beta_i}\right]^+$ for $\lambda\in[0,\infty)$, where $[x]^+ = \max\{x,0\}$. Then the best response strategy of the jammer is $\boldsymbol{J}(\boldsymbol{P}) = (J_1(\boldsymbol{P}), J_2(\boldsymbol{P}),\dots,J_n(\boldsymbol{P}))$, where*

$$J_i(\boldsymbol{P}) = \begin{cases} \left[\frac{\sqrt{\frac{\alpha_i\beta_i P_i}{C+\lambda_0}}-N_i}{\beta_i}\right]^+, & 1\le i\le k,\\ 0, & k+1\le i\le n, \end{cases} \tag{16}$$

and

$$\lambda_0 = \begin{cases} 0, & \pi(0) < \hat{J},\\ \textit{the unique root of } \pi(\lambda)=\hat{J}, & \textit{otherwise}. \end{cases} \tag{17}$$

In addition, $\boldsymbol{J}(\boldsymbol{P})$ can be computed in $O(n\log n)$ time. □

Proof: We convert the optimization problem (13) into a standard form of convex optimization problem [3]:

$$\min_{\boldsymbol{J}} f(\boldsymbol{J}) = \sum_{i=1}^{k} \frac{\alpha_i P_i}{N_i + \beta_i J_i} + C\sum_{i=1}^{k} J_i \tag{18}$$

s.t.

$$\sum_{i=1}^{k} J_i - \hat{J} \le 0,$$
$$-J_i \le 0, \forall i\in[1,k].$$

The first order partial derivative of $f(\boldsymbol{J})$ with respective to J_i, for $i\in[1,k]$, is

$$\frac{\partial f(\boldsymbol{J})}{\partial J_i} = -\frac{\alpha_i\beta_i P_i}{\left(N_i+\beta_i J_i\right)^2} + C, \tag{19}$$

and the second order partial derivatives of $f(\boldsymbol{J})$ are

$$\frac{\partial^2 f(\boldsymbol{J})}{\partial J_i \partial J_j} = \begin{cases} \frac{2\alpha_i\beta_i^2 P_i}{(N_i+\beta_i J_i)^3}, & i=j,\\ 0, & i\neq j. \end{cases} \tag{20}$$

The Hessian matrix is positive definite [3]: $\bigtriangledown^2 f(\boldsymbol{J}) \succ 0$, implying that the objective function (18) is strictly convex.

Since the constraints of the optimization problem are also convex, we know that the Karush-Kuhn-Tucker (KKT) conditions [3] are necessary and sufficient for optimality.

We define the Lagrangian as

$$L_J(\boldsymbol{J},\lambda) = v_m(\boldsymbol{P},\boldsymbol{J}) + \lambda_0\left(\sum_{i=1}^{k} J_i - \hat{J}\right) + \sum_{i=1}^{k}\lambda_i J_i, \tag{21}$$

where $\lambda_i \ge 0$, $0\le i\le k$, are the Lagrange multipliers. The KKT conditions for the optimal solution of (18) are given by

$$\begin{aligned} \frac{\partial L_J(\boldsymbol{J},\lambda)}{\partial J_i} &= 0, \forall i\in[1,k], & (22)\\ \sum_{i=1}^{k} J_i - \hat{J} &\le 0, & (23)\\ -J_i &\le 0, \forall i\in[1,k], & (24)\\ \lambda_i &\ge 0, \forall i\in[0,k], & (25)\\ \lambda_0\left(\sum_{i=1}^{k} J_i - \hat{J}\right) &= 0, & (26)\\ -\lambda_i J_i &= 0, \forall i\in[1,k]. & (27) \end{aligned}$$

Combining (22), (23), and (27), we have (16) and (17).

Our algorithm for computing $\boldsymbol{J}(\boldsymbol{P})$ is given in Algorithm 1. Lines 1–8 compute the value of λ_0 satisfying (17). Line 9 computes $\boldsymbol{J}(\boldsymbol{P})$ according to (16). When $\pi(0) < \hat{J}$, Line 2 of Algorithm 1 computes $\lambda_0 = 0$, which is consistent with the first case in (17). When $\pi(0) \geq \hat{J} > 0$, we have $\left[\frac{\sqrt{\frac{\alpha_i\beta_i P_i}{C+\lambda}} - N_i}{\beta_i}\right]^+ > 0$ for at least one $i \in [1, k]$. Line 4 computes the values $\{\lambda_0^i\}_{i=1}^k$, such that $\frac{\sqrt{\frac{\alpha_i\beta_i P_i}{C+\lambda}} - N_i}{\beta_i} > 0$ if and only if $\lambda < \lambda_0^i$. Line 5 sorts these values such that $\lambda_0^{i_1} \leq \lambda_0^{i_2} \leq \cdots \leq \lambda_0^{i_k}$. Hence $\pi(\lambda) = \sum_{j=l}^k \frac{\sqrt{\frac{\alpha_{i_j}\beta_{i_j} P_{i_j}}{C+\lambda}} - N_{i_j}}{\beta_{i_j}}$ for $\lambda \in [\lambda_0^{i_{l-1}}, \lambda_0^{i_l})$. This also implies that $\pi(\lambda_0^{i_k}) = 0$, and $\pi(\lambda)$ is strictly decreasing for $\lambda \in [0, \lambda_0^{i_k}]$. Hence there is a unique $\lambda_0 \in (0, \lambda_0^{i_k})$ such that $\pi(\lambda_0) = \hat{J}$. Lines 6 and 7 compute this value.

Line 5 in Algorithm 1 takes $O(k \log k)$ time. The rest of the algorithm takes $O(k)$ time. Since $k \leq n$, the running time of Algorithm 1 is $O(n \log n)$.

This completes the proof. ∎

Algorithm 1: Computation of $\boldsymbol{J}(\boldsymbol{P})$

input : The power vector $\boldsymbol{P}$ of the user

1 **if** $\pi(0) < \hat{J}$ **then**

2 $\quad \lambda_0 \leftarrow 0$;

3 **else**

4 $\quad \lambda_0^i \leftarrow \frac{\alpha_i\beta_i P_i}{N_i^2} - C$, for $i \leftarrow 1$ to k ;

5 $\quad$ Sort $\{\lambda_0^i\}_{i=1}^k$ such that $\lambda_0^{i_1} \leq \lambda_0^{i_2} \leq \cdots \leq \lambda_0^{i_k}$;

6 $\quad$ Find $r \in [1, k]$ such that $\pi(\lambda_0^{i_{r-1}}) \geq \hat{J} > \pi(\lambda_0^{i_r})$, where $\lambda_0^{i_0} = 0$;

7 $\quad \lambda_0 \leftarrow \left(\sum_{j=r}^k \sqrt{\frac{\alpha_{i_j} P_{i_j}}{\beta_{i_j}}} \Big/ \left(\hat{J} + \sum_{j=r}^k \frac{N_{i_j}}{\beta_{i_j}}\right)\right)^2 - C$;

8 **end**

9 $J_i(\boldsymbol{P}) \leftarrow \begin{cases} \left[\frac{\sqrt{\frac{\alpha_i\beta_i P_i}{C+\lambda_0}} - N_i}{\beta_i}\right]^+, & 1 \leq i \leq k, \\ 0, & k+1 \leq i \leq n, \end{cases}$;

10 **return** $\boldsymbol{J}(\boldsymbol{P})$;

B. User's Optimal Strategy

We first rigorously prove the existence of the user's optimal strategy. This implies the existence of SEs of the PCSJ game. We then design algorithms for approximating the user's optimal strategy.

Lemma 3. *Let $\{\boldsymbol{P}^{[\kappa]}\}$ be a sequence in $\mathcal{P}$ converging to a point $\bar{\boldsymbol{P}}$ in $\mathcal{P}$. Then the sequence $\{\boldsymbol{J}(\boldsymbol{P}^{[\kappa]})\}$ converges to $\boldsymbol{J}(\bar{\boldsymbol{P}})$.*

Proof: To the contrary, assume that $\{\boldsymbol{J}(\boldsymbol{P}^{[\kappa]})\}$ does not converge to $\boldsymbol{J}(\bar{\boldsymbol{P}})$. Since $\{\boldsymbol{J}(\boldsymbol{P}^{[\kappa]})\}$ is contained in the compact set $\mathcal{P}$, it must have a sub-sequence $\{\boldsymbol{J}(\boldsymbol{P}^{[s_\kappa]})\}$ converging to a point $\boldsymbol{J}' \neq \boldsymbol{J}(\bar{\boldsymbol{P}})$. Clearly $\{\boldsymbol{P}^{[s_\kappa]}\}$ converges to $\bar{\boldsymbol{P}}$ since $\{\boldsymbol{P}^{[\kappa]}\}$ converges to $\bar{\boldsymbol{P}}$. Hence $\{(\boldsymbol{P}^{[s_\kappa]}, \boldsymbol{J}(\boldsymbol{P}^{[s_\kappa]}))\}$ converges to $(\bar{\boldsymbol{P}}, \boldsymbol{J}')$. Without loss of generality, we assume that $\{(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\boldsymbol{P}^{[\kappa]}))\}$ converges to $(\bar{\boldsymbol{P}}, \boldsymbol{J}')$.

Since $\boldsymbol{J}(\bar{\boldsymbol{P}})$ is the unique optimal strategy of the jammer for the strategy $\bar{\boldsymbol{P}}$ of the user, we have

$$v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}(\bar{\boldsymbol{P}})) - v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}') > 0. \tag{28}$$

Define

$$3\epsilon = v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}(\bar{\boldsymbol{P}})) - v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}'). \tag{29}$$

Since $v_m(\boldsymbol{P}, \boldsymbol{J})$ is a continuous function on $\mathcal{P} \times \mathcal{J}$, it is continuous at points $(\bar{\boldsymbol{P}}, \boldsymbol{J}(\bar{\boldsymbol{P}}))$ and $(\bar{\boldsymbol{P}}, \boldsymbol{J}')$. Since $\{(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\bar{\boldsymbol{P}}))\}$ converges to $(\bar{\boldsymbol{P}}, \boldsymbol{J}(\bar{\boldsymbol{P}}))$, and $\{(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\boldsymbol{P}^{[\kappa]}))\}$ converges to $(\bar{\boldsymbol{P}}, \boldsymbol{J}')$, there exists an integer K such that

$$|v_m(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\bar{\boldsymbol{P}})) - v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}(\bar{\boldsymbol{P}}))| < \epsilon, \text{ when } \kappa \geq K, \tag{30}$$

and

$$|v_m(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\boldsymbol{P}^{[\kappa]})) - v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}')| < \epsilon, \text{ when } \kappa \geq K. \tag{31}$$

Therefore, for all $\kappa \geq K$, we have (using (30), (29), and (31))

$$\begin{aligned} v_m(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\bar{\boldsymbol{P}})) &> v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}(\bar{\boldsymbol{P}})) - \epsilon && (32)\\ &= (v_m(\bar{\boldsymbol{P}}, \boldsymbol{J}') + 3\epsilon) - \epsilon && (33)\\ &> v_m(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\boldsymbol{P}^{[\kappa]})) - \epsilon + 3\epsilon - \epsilon && (34)\\ &= v_m(\boldsymbol{P}^{[\kappa]}, \boldsymbol{J}(\boldsymbol{P}^{[\kappa]})) + \epsilon. && (35) \end{aligned}$$

This is in contradiction with the assumption that $\boldsymbol{J}(\boldsymbol{P}^{[\kappa]})$ is the best response strategy of the jammer. This proves the lemma. ∎

Lemma 4. *$u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ is a continuous function in $\boldsymbol{P}$.*

Proof: By (3), $u_m(\boldsymbol{P}, \boldsymbol{J})$ is continuous in the variables $(\boldsymbol{P}, \boldsymbol{J})$. From Lemma 3, $\boldsymbol{J}(\boldsymbol{P})$ is continuous in $\boldsymbol{P}$. Hence $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ is continuous in $\boldsymbol{P}$. ∎

Theorem 4. *There exists $\boldsymbol{P}^{SE} \in \mathcal{P}$ such that $(\boldsymbol{P}^{SE}, \boldsymbol{J}(\boldsymbol{P}^{SE}))$ is a Stackelberg Equilibrium of the PCSJ game.*

Proof: We know that $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ is a continuous function in $\boldsymbol{P}$. Since the set $\mathcal{P}$ is compact, $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ achieves its maximum at some point $\boldsymbol{P}^{SE} \in \mathcal{P}$ [15]. This proves the theorem. ∎

Based on the analytical results of the jammer's best response strategy given user's strategy, the user can optimize its strategy $\boldsymbol{P}$ to maximize its utility $u_m(\boldsymbol{P}, \boldsymbol{J})$, being aware that its decision will affect the jammer's strategy. From the user's prospective, its objective is to solve the following optimization problem.

$$\max_{\boldsymbol{P} \in \mathcal{P}} u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P})) = \sum_{i=1}^n \frac{\alpha_i P_i}{N_i + \beta_i J_i(\boldsymbol{P})} - E \sum_{i=1}^n P_i, \tag{36}$$

where $J_i(\boldsymbol{P})$ is derived from Theorem 3.

Although Theorem 4 proves the existence of an SE of PCSJ, computing an SE is challenging. The reason is that the objective function in (36) is not concave.

Non-Concavity of $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$: Define $g(\boldsymbol{P}) = u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$. We use an example to show that there exists $\boldsymbol{P}^{[1]}$ and $\boldsymbol{P}^{[2]}$ such that

$$\frac{g(\boldsymbol{P}^{[1]}) + g(\boldsymbol{P}^{[2]})}{2} > g\left(\frac{\boldsymbol{P}^{[1]} + \boldsymbol{P}^{[2]}}{2}\right).$$

In this example, $n = 2$, $\alpha_1 = \alpha_2 = 0.6$, $\beta_1 = 0.5$, $\beta_2 = 0.2$, $N_1 = N_2 = 0.2$, $\hat{P} = 10$, $\hat{J} = 4$, $E = 0.1$, and $C = 1$. We set $\boldsymbol{P}^{[1]} = (4,3)$ and $\boldsymbol{P}^{[2]} = (5,4)$. Using Algorithm 1 and the definition of $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$, we have $g(\boldsymbol{P}^{[1]}) = 4.49089$, $g(\boldsymbol{P}^{[2]}) = 5.57603$, and $g\left(\frac{\boldsymbol{P}^{[1]}+\boldsymbol{P}^{[2]}}{2}\right) = 4.93331$. Hence we show that $\frac{g(\boldsymbol{P}^{[1]})+g(\boldsymbol{P}^{[2]})}{2} > g\left(\frac{\boldsymbol{P}^{[1]}+\boldsymbol{P}^{[2]}}{2}\right)$.

Algorithm 2: SE-SA

input : Algorithm parameters I, T, σ, and δ
1 Randomly initialize $\boldsymbol{P}$, $\boldsymbol{P}_{best} \leftarrow \boldsymbol{P}$;
2 **repeat**
3 **for** $i \leftarrow 1$ **to** I **do**
4 $\boldsymbol{P}_{new} \leftarrow neighbor(\boldsymbol{P})$;
5 Randomly select r from $(0,1)$;
6 **if** $u_m(\boldsymbol{P}_{new}, \boldsymbol{J}(\boldsymbol{P}_{new})) \geq u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ **or** $r \leq e^{(u_m(\boldsymbol{P}_{new}, \boldsymbol{J}(\boldsymbol{P}_{new})) - u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P})))/T}$ **then**
7 $\boldsymbol{P} \leftarrow \boldsymbol{P}_{new}$;
8 **if** $u_m(\boldsymbol{P}_{new}, \boldsymbol{J}(\boldsymbol{P}_{new})) > u_m(\boldsymbol{P}_{best}, \boldsymbol{J}(\boldsymbol{P}_{best}))$ **then**
9 $\boldsymbol{P}_{best} \leftarrow \boldsymbol{P}_{new}$;
10 **end**
11 **end**
12 **end**
13 $T \leftarrow \sigma T$;
14 **until** $T \leq 1$;

We propose two algorithms to approximate the optimal strategy of the user. The first is simulated annealing [6], denoted by SE-SA and presented in Algorithm 2. The second is a mesh-based hill-climbing algorithm, denoted by SE-MESH and presented in Algorithm 3.

Since simulated annealing has been widely used in the literature, we do not give detailed description of SE-SA, and refer the readers to [5]. The algorithm finds a global optimal solution with probability 1 when the number of iterations goes to infinity [5]. The algorithm parameters are as follows. $T > 0$ is the initial temperature. $\sigma \in (0,1)$ is the annealing parameter. I is the number of iterations to be performed at each temperature. δ is the parameter used for generating perturbations. For any feasible power vector $P \in \mathcal{P}$, the function $neighbor(\boldsymbol{P})$ generates a perturbation $P' \in \mathcal{P}$ in the following way. For each i, let $P'_i = [P_i + \delta_i]^+$, where δ_i is a random number uniformly drawn from $[-\delta, \delta]$. If $\sum_{i=1}^{n} P'_i > \hat{P}$, set $P'_i = \frac{\hat{P} P'_i}{\sum_{i=1}^{n} P'_i}$.

In SE-MESH, we first narrow down the searching space $\mathcal{P}$ to the points $(\delta_1\epsilon, \delta_2\epsilon, \ldots, \delta_n\epsilon)$ on a mesh with space ϵ between lines. We then select the top t points that have highest values of $u(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$. Starting from each of these t points, we apply a searching strategy similar to that used in SE-SA (Lines 3–12) except that only the point resulting in a higher $u(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ is accepted in SE-MESH. In addition, for each point $\boldsymbol{P}$, the searching process terminates if we could not find a neighbor yielding higher $u(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ after I iterations.

Remark 1. Similar to the single-channel model, the user needs to know the value of β_i for $1 \leq i \leq n$ to compute $\boldsymbol{P}^{SE}$. To achieve this, the user can simply use the method in Section IV for each channel and compute β_i.

Algorithm 3: SE-MESH

input : Algorithm parameters ϵ, t, I, and δ
1 $\boldsymbol{P}_{best} \leftarrow \boldsymbol{0}$;
2 Let $\mathcal{P}' \leftarrow \{(\delta_1\epsilon, \delta_2\epsilon, \ldots, \delta_n\epsilon) | \delta_i \in \mathbb{Z}^*, 1 \leq i \leq n\} \cap \mathcal{P}$, where $\mathbb{Z}^*$ is the set of nonnegative integers;
3 Compute $u(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ for each $\boldsymbol{P} \in \mathcal{P}'$;
4 Let $\mathcal{P}'[1], \mathcal{P}'[2], \ldots, \mathcal{P}'[t]$ denote the top t power transmission vectors with highest $u(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$;
5 **for** $i \leftarrow 1$ *to* t **do**
6 $\boldsymbol{P} \leftarrow \mathcal{P}'[i]$;
7 **if** $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P})) > u_m(\boldsymbol{P}_{best}, \boldsymbol{J}(\boldsymbol{P}_{best}))$ **then**
8 $\boldsymbol{P}_{best} \leftarrow \boldsymbol{P}$;
9 **end**
10 $cnt \leftarrow 0$;
11 **while** $cnt < I$ **do**
12 $\boldsymbol{P}_{new} \leftarrow neighbor(\boldsymbol{P})$;
13 **if** $u_m(\boldsymbol{P}_{new}, \boldsymbol{J}(\boldsymbol{P}_{new})) \geq u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ **then**
14 $\boldsymbol{P} \leftarrow \boldsymbol{P}_{new}$;
15 $cnt \leftarrow 0$;
16 **if** $u_m(\boldsymbol{P}_{new}, \boldsymbol{J}(\boldsymbol{P}_{new})) > u_m(\boldsymbol{P}_{best}, \boldsymbol{J}(\boldsymbol{P}_{best}))$ **then**
17 $\boldsymbol{P}_{best} \leftarrow \boldsymbol{P}_{new}$;
18 **end**
19 **else**
20 $cnt \leftarrow cnt + 1$;
21 **end**
22 **end**
23 **end**

Remark 2. Since $\mathcal{P}$ is compact and $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ is a continuous function on $\mathcal{P}$ by Lemma 4, $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ is uniformly continuous on $\mathcal{P}$. Therefore, there exists a Lipschitz constant $L > 0$, such that $|u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P})) - u_m(\boldsymbol{P}', \boldsymbol{J}(\boldsymbol{P}'))| \leq L||\boldsymbol{P} - \boldsymbol{P}'||$ [15]. Therefore, as ϵ approaches zero, the solution computed by SE-MESH converges to the optimal solution. More importantly, we have $u_m(\boldsymbol{P}_\epsilon, \boldsymbol{J}(\boldsymbol{P}_\epsilon)) \geq u_m(\boldsymbol{P}_{opt}, \boldsymbol{J}(\boldsymbol{P}_{opt})) - \epsilon n L$, where $\boldsymbol{P}_\epsilon$ is the transmit power computed by SE-MESH, $\boldsymbol{P}_{opt}$ is optimal transmission power of the user, and n is the dimension of $\boldsymbol{P}$ vector.

VI. SIMULATIONS

In this section, we validate the theoretical insights of the PCSJ game through extensive simulations.

A. Simulation Setup

For the single-channel model, five variables determine the players' strategies and their utility values, which are N, α, β, C, and E. Among these five variables, only α and β, i.e., fading channel gains of the user and the jammer, may vary significantly due to the change of players' physical locations. Hence, we explore the relations of user and jammer's utility values with respect to different values of α and β. We set α and β to be in the range of $[0.1, 0.9]$. Moreover, let $C = E = 1$ (as in [1]), and $N = 0.2$.

For the multi-channel model, we have $n \in [2, 12]$ and $\hat{P} = \hat{J} = 10$. We assume that α_i is randomly distributed over $(0, 1]$

and β_i is randomly distributed over $(0, 0.5]$, for all $1 \leq i \leq n$. Same as the single-channel model, we have $C = E = 1$ and $N_i = 0.2$ for all $1 \leq i \leq n$. For the parameters of SE-SA, we set $I = 1000$, $T = 100$, $\sigma = 0.6$, and $\delta = 0.25$. For the parameters of SE-MESH, we set $\epsilon = 1$, $t = 100$, $I = 1000$, and $\delta = 0.25$.

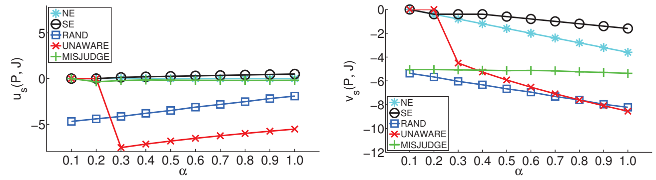
We compare the SE of the PCSJ game with the following scenarios:

- Power Control with Standard Jammer (NE) [19]: The jammer set its power without knowing the user's. Thus both the user and the jammer set their power simultaneously.
- Random Power Control (RAND): Both the user and the jammer randomly set their power, regardless of the existence of the other, as long as the power allocation is feasible.
- Power Control While Being Unaware of Jammer's Existence (UNAWARE): The user maximizes its utility, without the knowledge of the smart jammer's existence. The smart jammer still maximizes it utility with its intelligence.
- Power Control with Misjudgement (MISJUDGE): The user assumes the intelligence of the jammer, while the jammer is just a regular one using random transmission power.

B. Result Analysis

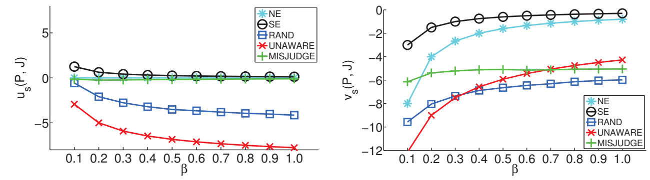
Figs. 3 and 4 show the results of the single-channel model. Specifically, Figs. 3(a) and 3(b) show the impact of α on the players' utility values with $\beta = 0.5$, for different scenarios. We observe that SE leads to the highest utility values for the user. The fact that the utility at SE is higher than that at NE is consistent with the results in [19]. Recall that α is the fading channel gain of the user. Therefore the larger α is, the closer the transmitter is from the receiver. Hence, as α increases, user's utility increases, as shown in Fig. 3(a), while jammer's utility decreases, as shown in Fig. 3(b). For the user, both NE and MISJUDGE result in higher utility than both RAND and UNAWARE. It is because the user prepares for the worst case where the jammer has intelligence. In RAND, the user randomly sets its power, which results in a negative utility when $\alpha = 0.1, 0.2$ even without the jammer. Therefore, the utility of the user in RAND is lower than that in UNAWARE when $\alpha = 0.1, 0.2$. However, when $\alpha > 0.2$, the user's utility in RAND is always higher than that in UNAWARE, due to the unawareness of the jammer's existence in UNAWARE. For the jammer, SE leads to the highest utility. Again, the higher utility in SE compared to NE is consistent with the results in [19]. Compare to RAND, jammer's utility is higher in UNAWARE where jammer has intelligence. In addition, MISJUDGE is higher than RAND, because the user assumes the existence of the jammer in MISJUDGE, but sets power randomly in RAND. Another observation is that MISJUDGE results in higher utility than UNAWARE when $\alpha \geq 0.4$. It is because good channel condition (i.e. large value of α) makes the user transmit with the maximum power in UNAWARE.

Figs. 4(a) and 4(b) show the impact of β on the players' utility values with $\alpha = 0.5$, for different scenarios. Again,



(a) User's utility (b) Jammer's utility

Fig. 3. Impact of α on players' utility values.



(a) User's utility (b) Jammer's utility

Fig. 4. Impact of β on players' utility values.

SE leads to the highest utility values for the user. Note that the jammer's SE utility increases while the user's SE utility decreases due to the fact that the jammer's influence on the receiver gets stronger as β increases. We also have the similar observations about the relationship among different scenarios as in Figs. 3 and 4.

Figs. 5 through 8 show the results of the multi-channel model. For SE-SA, three parameters I, T, and σ need to be decided. Fig. 5 shows $u_m(\boldsymbol{P}, \boldsymbol{J}(\boldsymbol{P}))$ as a function of the number of iterations with our parameter settings. Plugging I, T, and σ into Algorithm 2, we know that there are $\lceil \log_\sigma \frac{1}{T} \rceil * I = 10000$ iterations in total. We observe that the algorithm stops making improvement after 3000 iterations.

Fig. 6 shows the comparison between SE-SA and SE-MESH. In particular, Fig. 6(a) shows the user's utility and Fig. 6(b) shows the running time. Although SE-MESH performs a little better than SE-SA, the running time of SE-MESH grows exponentially in $1/\epsilon$. Actually, the running time of SE-MESH is dominated by the number of mesh points we evaluate in Line 3 in Algorithm 3, which is $\Theta((n+1)^{\hat{P}/\epsilon})$. To have a better idea on how this scales, we plot it as a function of n and ϵ in Fig. 7. As we can see, when $n = 12$, the number of mesh points is more than 10^{100} for $\epsilon = 0.1$, which is beyond the computability of current PC machines. Hence SE-SA is the recommended approach.

Fig. 8 shows the impact of n on players' utility values under the multi-channel model. We observe that SE has the best performance for the user, followed by RAND at the second and UNAWARE at the bottom. In general, the user's utility increases when there are more channels. The reason is that the user has a better chance to allocate power to channels with better channel gains, i.e., α_i. Another observation is that the jammer has lower utility values in UNAWARE than SE when there are less than 7 channels, while higher utility values when there are more than 7 channels. This is because the number of channel does not affect the jammer's utility as much in UNAWARE as it does in SE. The user will always only use the channel(s) with the best channel gain(s) if it is unaware

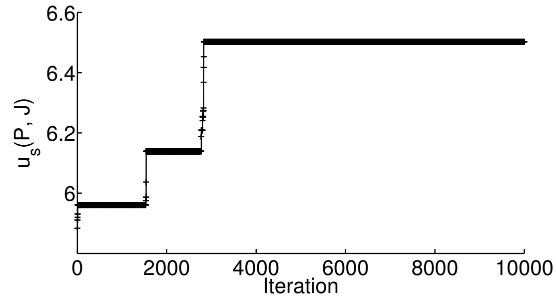
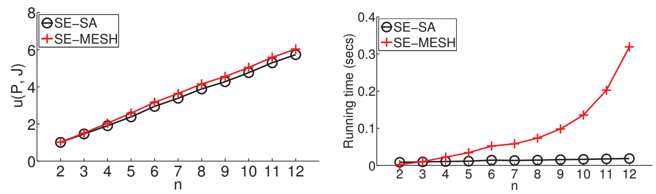


Fig. 5. Convergence of the simulated annealing algorithm.



(a) User's utility (b) Running time

Fig. 6. Comparison between SE-SA and SE-MESH.

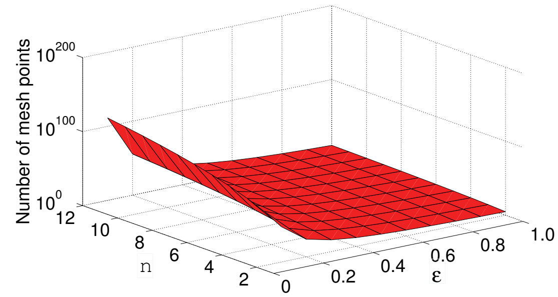
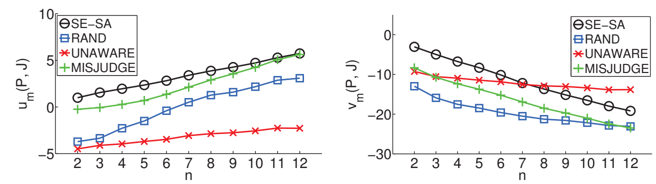


Fig. 7. Scalability of SE-MESH in log-scale.



(a) User's utility (b) Jammer's utility

Fig. 8. Impact of n on players' utility values.

of the smart jammer's existence. In contrast, the user has more flexible where there are more channels in SE, with the knowledge of the jammer's intelligence. Regarding the user's utility in MISJUDGE compared to other scenarios, we have the similar observations as in the single-channel model. For the jammer, it has higher utility in UNAWARE than it does in MISJUDGE. It is because unlike the single-channel model, where the user can transmit with the maximum power at one channel, the user allocates power equally to channels with the same condition under the multi-channel in UNAWARE. The even power distribution allows the jammer to attack the channels with better channel conditions for the jammer and thus to improve its utility. This advantage of the jammer is enhanced when the number of channels increases.

VII. Conclusions

In this paper, we have studied the problem of optimal power control in the presence of a smart jammer, who can quickly learn the transmission power of the user and adjust its transmission power to maximize the damaging effect. We have considered both the single-channel and the multi-channel models. We modeled the problem as a Stackelberg game, called PCSJ game. For the single-channel model, we proved the existence and the uniqueness by giving the closed-form expressions for the Stackelberg Equilibrium (SE). For the multi-channel model, we proved the existence and designed algorithms for approximating an SE.

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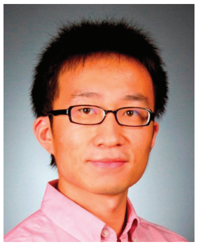

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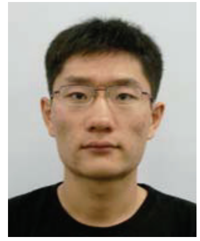

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