trapezoid: The Trapezoidal Distribution

Jeremy Thoms Hetzel

December 28, 2012

1 Introduction

The trapezoidal distribution is defined by minimum, lower mode, upper mode, and maximum parameters. The generalized trapezoidal distribution adds three more parameters: the growth rate, decay rate, and boundary ratio parameters. Van Dorp and Kotz[2] and van Dorp and colleagues[1] formally describe the generalized trapezoidal distribution, representing the minimum, lower mode, upper mode, maximum, growth rate, decay rate, and boundary ratio with parameters $a, b, c, d, m, n,$ and α, respectively.

The probability density function of the generalized trapezoidal distribution with parameters $a, b, c, d, m, n,$ and α is given by:

$$f_X(x \mid \theta) = \mathcal{C}(\Theta) \times \begin{cases} \alpha \left(\frac{x-\alpha}{b-\alpha}\right)^{m-1}, & \text{for } a \leq x < b \\ (1-\alpha)\left(\frac{x-b}{c-b}\right) + \alpha, & \text{for } b \leq x < c \\ \left(\frac{d-x}{d-c}\right)^{n-1}, & \text{for } c \leq x \leq d \end{cases}$$

with the normalizing constant $\mathcal{C}(\Theta)$ defined as:

$$\mathcal{C}(\Theta) = \frac{2mn}{2\alpha\,(b-a)\,n + (\alpha+1)\,(c-b)\,mn + 2\,(d-c)\,m}$$

and where the parameter vector $\Theta = \{a, b, c, d, m, n, \alpha\}, a \leq b \leq c \leq d,$ and $m, n, \alpha > 0$.

The trapezoid package provides functions for the probability density function (`dtrapezoid`), cumulative distribution function (`ptrapezoid`), quantile function (`qtrapezoid`), and random generation (`rtrapezoid`). The parameters $a, b, c, d, m, n,$ and α are specified by the arguments `min`, `mode1`, `mode2`, `max`, `n1`, `n3`, and `alpha`, respectively. The argument names were chosen to avoid conflicts with names that commonly have specific meaning in R functions, such as `c` and `n`.

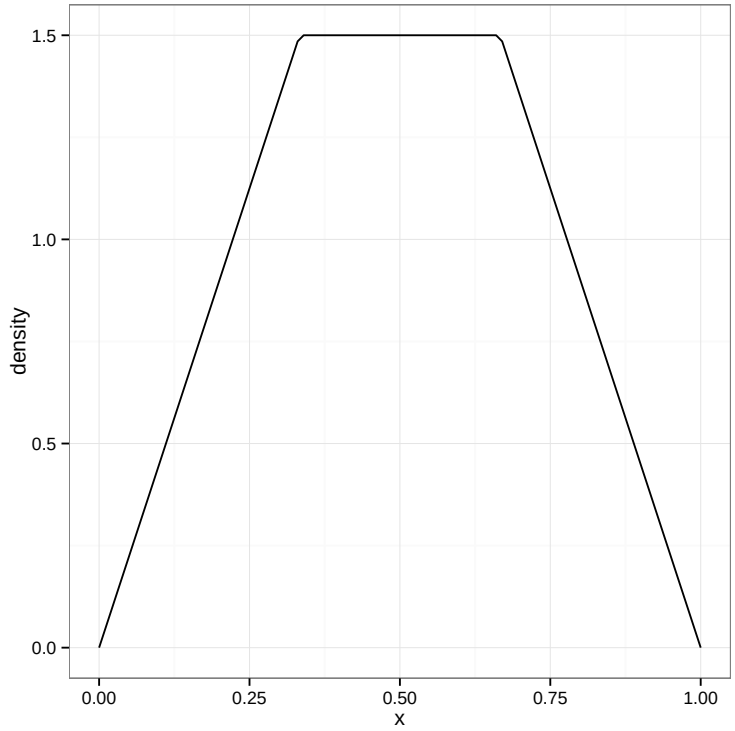
2 Examples

2.1 Trapezoid

The generalized trapezoidal distribution simplifies to the trapezoidal distribution when $m = n = 2$ and $\alpha = 1$.

```
> # plyr and ggplot2 are required for these examples
> require(trapezoid)
> require(plyr)
> require(ggplot2)
> # Trapezoid
> x <- seq(from = 0, to = 1, by = 0.01)
> density <- dtrapezoid(x, min = 0, mode1 = 1/3, mode2 = 2/3, max = 1,
   n1 = 2, n3 = 2, alpha = 1)
> trapezoid <- ggplot(data = data.frame(x, density),
   aes(x = x, y = density)) + geom_line() + theme_bw()
> print(trapezoid)
```

Figure 1: A trapezoidal distribution.

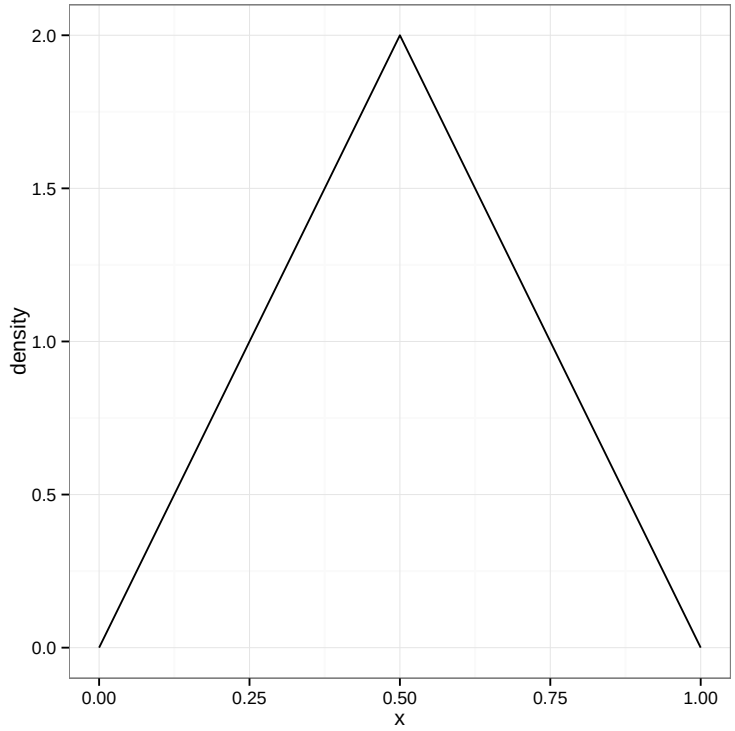


2.2 Triangle

The trapezoidal distribution further simplifies to the triangular distribution when $b = c$.

```
> # Triangle
> x <- seq(from = 0, to = 1, by = 0.01)
> density <- dtrapezoid(x, min = 0, mode1 = 1/2, mode2 = 1/2, max = 1,
   n1 = 2, n3 = 2, alpha = 1)
> triangle <- ggplot(data = data.frame(x, density),
   aes(x = x, y = density)) + geom_line() + theme_bw()
> print(triangle)
```

Figure 2: A triangular distribution.



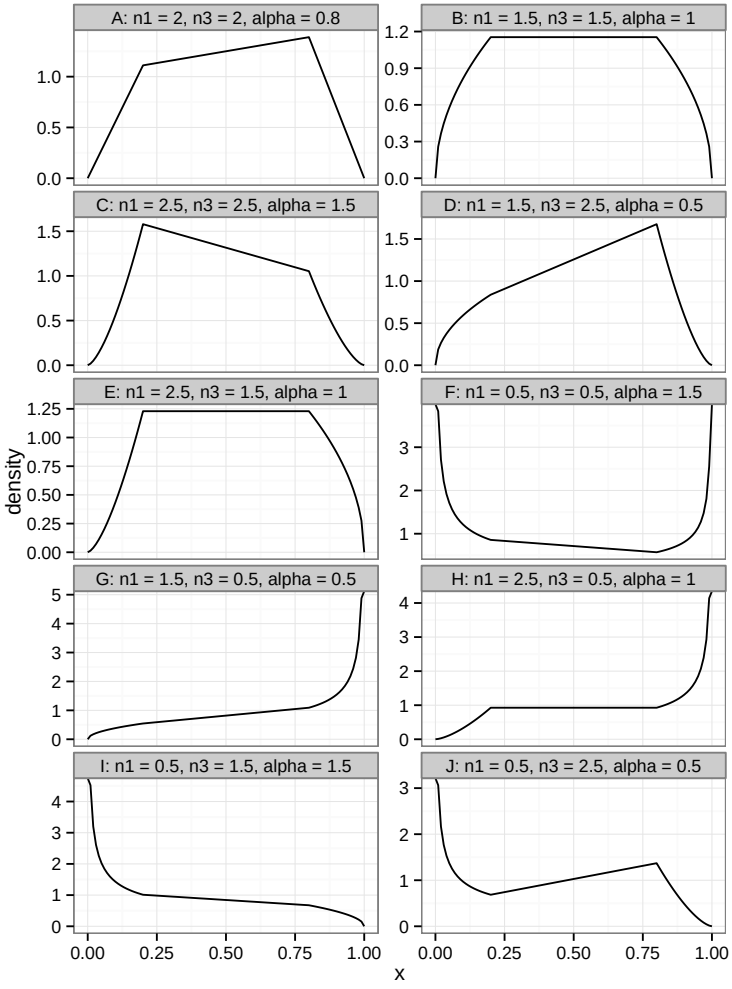
2.3 Generalized trapezoidal distribution

Parameters m, n, and α control the growth rate, decay rate, and boundary ratio, respectively, of the distribution. In the trapezoid package, these parameters are controlled by the `n1`, `n3`, and `alpha` arguments. To demonstrate the effects of these three parameters, van Dorp and Kotz[2] generated eight distributions with varying parameter values. The distributions are approximately re-generated here.

```
> # Generalized trapezoidal distributions
> x <- seq(from = 0, to = 1, by = 0.01)
> # Create a lists of arguments, varying n1, n3, and alpha
> arguments <- list()
> arguments[['A']] <- list(x = x, n1 = 2, n3 = 2, alpha = 0.8)
> arguments[['B']] <- list(x = x, n1 = 1.5, n3 = 1.5, alpha = 1)
> arguments[['C']] <- list(x = x, n1 = 2.5, n3 = 2.5, alpha = 1.5)
```

```
> arguments[['D']] <- list(x = x, n1 = 1.5, n3 = 2.5, alpha = 0.5)
> arguments[['E']] <- list(x = x, n1 = 2.5, n3 = 1.5, alpha = 1)
> arguments[['F']] <- list(x = x, n1 = 0.5, n3 = 0.5, alpha = 1.5)
> arguments[['G']] <- list(x = x, n1 = 1.5, n3 = 0.5, alpha = 0.5)
> arguments[['H']] <- list(x = x, n1 = 2.5, n3 = 0.5, alpha = 1)
> arguments[['I']] <- list(x = x, n1 = 0.5, n3 = 1.5, alpha = 1.5)
> arguments[['J']] <- list(x = x, n1 = 0.5, n3 = 2.5, alpha = 0.5)
> # Calculate the distributions
> plot.data <- ldply(arguments, function(z)
 {
   x <- z$x
   density <- dtrapezoid(x = z$x, min = 0, mode1 = 0.2, mode2 = 0.8,
     max = 1, n1 = z$n1, n3 = z$n3, alpha = z$alpha)
   args <- paste("n1 = ", z$n1, ", n3 = ", z$n3, ", alpha = ", z$alpha,
     sep="", collapse="")
   out <- data.frame(x, density, args)
 })
> # Create labels for later use in displaying the arguments on the plots
> plot.data$label <- paste(plot.data$.id, ": ", plot.data$args, sep="")
> # Create plots
> generalizedTrapezoids <- ggplot(data = plot.data,
   aes(x = x, y = density)) + geom_line() + theme_bw() +
   facet_wrap(~label, ncol = 2, scales = "free_y")
> print(generalizedTrapezoids)
```

Figure 3: Examples of generalized trapezoidal distributions.



References

[1] J. R. van Dorp, S. C. Rambaud, J. G. Perez, and R. H. Pleguezuelo. An Elicitation Procedure for the Generalized Trapezoidal Distribution with a Uniform Central Stage. *Decision Analysis*, 4(3):156–166, September 2007.

[2] J René van Dorp and Samuel Kotz. Generalized Trapezoidal Distributions. *Metrika*, 58(1):85–97, 2003.