CHAPTER

10 The Pythagorean Theorem

Content Summary

One of the best-remembered theorems of geometry is the Pythagorean Theorem. This chapter begins by reviewing the Pythagorean Theorem and then considering its converse. It then examines shortcuts in the case of special triangles. Finally, the Pythagorean Theorem is used to calculate distances on the coordinate plane, making it possible to write an equation for a circle.

The Pythagorean Theorem and Its Converse

Many people identify the Pythagorean Theorem as $a^2 + b^2 = c^2$, without recalling what *a, b,* and *c* stand for. The theorem actually says that if *a, b,* and *c* are the lengths of sides of a right triangle whose hypotenuse has length *c,* then $a^2 + b^2 = c^2$.

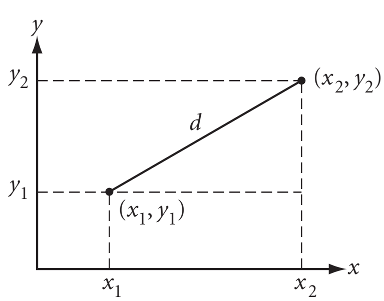
The converse of the Pythagorean Theorem is also true: If *a, b,* and *c* are the lengths of sides of a triangle, and if $a^2 + b^2 = c^2$, then the triangle is a right triangle whose hypotenuse has length *c.*

Applications to Triangles

Discovering Geometry illustrates how to apply the Pythagorean Theorem to solve real-world problems. It also applies the theorem to find the relative lengths of sides of two special right triangles: the 45°-45°-90° triangle, which is half of a square; and the 30°-60°-90° triangle, which is half of an equilateral triangle.

Applications to Circles

From algebra, students know how to write equations to represent lines and parabolas. Coupled with the conjectures from Chapter 9, the Pythagorean Theorem can be used to develop equations for circles. The restatement of the Pythagorean Theorem in coordinate geometry is the distance formula, which tells how to find the distance between two points whose coordinates are known. From this formula, the book derives the equation of a circle.



$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

(continued)

Summary Problem

Why is the general equation of a circle $(x - h)^2 + (y - k)^2 = r^2$?

Questions you might ask in your role as student to your student:

- What numbers do h, k, and r represent?
- What points satisfy the equation if h and k are both 0?
- What can you say about the distance from any point on a circle to the center?
- How does this equation compare with the distance formula?
- Why does the sum of the squares on the left equal the square on the right?

Sample Answers

Encourage your student to relate this equation to the distance formula shown above. Looking at the diagram, imagine that the point (x_1, y_1) is the center of a circle, and all the points on the circle are a distance d away from it. If you rename the center (h, k) and rename the distance r, you can see that the circle is all the points (x, y) that are this distance away from (h, k).

If the center is at the point (0, 0), the points satisfying the equation are all a distance r from the origin—that is, they all lie on a circle centered at the origin and having radius r.

The equation of that circle is $x^2 + y^2 = r^2$, which is the Pythagorean Theorem. Try having your student use a compass to draw a circle centered at the origin with a certain radius (for example, 5 or 10), and then ask him or her to identify a few points on that circle and substitute them into the equation.

Chapter 10 • Review Exercises

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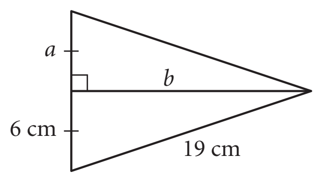
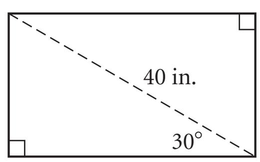
Round solutions to the nearest tenth of a unit.

1. *(Lesson 10.1)* Find each missing length.



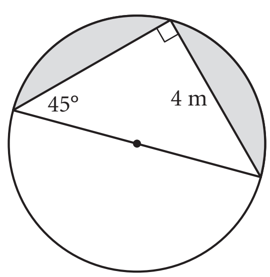
2. *(Lesson 10.2)* Is a triangle with sides measuring 3 ft., 4 ft., and 5 ft. a right triangle? Justify your answer.

3. *(Lessons 10.3, 10.4)* A 40-in. TV has a diagonal equal to 40 in., as shown here. Find the dimensions of the TV.



4. *(Lesson 10.5)* Find the equation of the circle with diameter endpoints (–2, 2) and (4, 2).

5. *(Lesson 10.6)* Find the area of the shaded region.



SOLUTIONS TO CHAPTER 10 REVIEW EXERCISES

1. $a = 6$ cm — Congruent segments.

$6^2 + b^2 = 19^2$ — Pythagorean Theorem.

$36 + b^2 = 361$

$b^2 = 325$

$b \approx 18.0$ cm

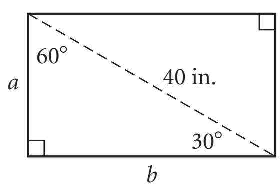
2. If sides with lengths 3, 4, and 5 form a right triangle, the Pythagorean Theorem should make a true statement:

$3^2 + 4^2 = 5^2$

$25 = 25$

Therefore, it is a right triangle.

3. Use the Triangle Sum Conjecture to find the other angle of the right triangle. The triangle is a 30°-60°-90° triangle, so you can use the shortcut.



$a = \frac{1}{2}(40) = 20$ in.

$b = a\sqrt{3} = 20\sqrt{3} \approx 34.6$ in.

4. Using the given diameter, the center is at (1, 2), and the radius is 3. Enter these values into the circle equation.

$(x - 1)^2 + (y - 2)^2 = 9$

5. The triangle is a 45°-45°-90° triangle, therefore the diameter is $4\sqrt{2}$ m.

$r = \frac{1}{2}(4\sqrt{2}) = 2\sqrt{2}$ m

Area of semicircle $= \frac{1}{2}$ area of circle $= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2\sqrt{2})^2 = \frac{1}{2}\pi(4 \cdot 2) = 4\pi \text{ m}^2$

Area of triangle $= \frac{1}{2}bh = \frac{1}{2}(4)(4) = 8 \text{ m}^2$

Area of shaded region = area of semicircle − area of triangle $= 4\pi - 8 \approx 4.6 \text{ m}^2$