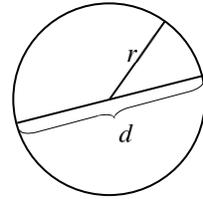


Students have found the area and perimeter of several polygons. Next they consider what happens to the area as more and more sides are added to a polygon. By exploring the area of a polygon with many sides, they learn that the limit of a polygon is a circle. They extend what they know about the perimeter and area of polygons to circles, and find the relationships for the circumference ( $C$ ) and area ( $A$ ) of circles.



$$C = \pi d \text{ or } 2\pi r, \quad A = \pi r^2$$

“ $C$ ” is the circumference of the circle (a circle’s perimeter), “ $d$ ” is the diameter, and “ $r$ ” is the radius. “ $\pi$ ,” which is in both formulas, is by definition the ratio  $\frac{\text{circumference}}{\text{diameter}}$ , and it is always a constant for any size circle.

Using these formulas, along with ratios, students are able to find the perimeter and area of shapes containing parts of circles.

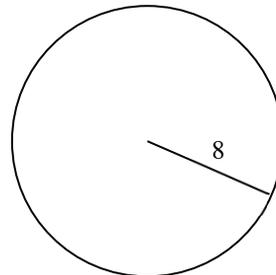
See the Math Notes boxes in Lessons 7.1.2, 8.3.2, and 8.3.3.

**Example 1**

The circle at right has a radius of 8 cm. What are the circumference and the area of the circle?

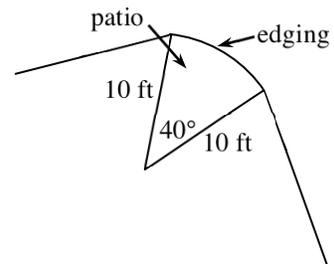
Using the formulas,

$C = 2\pi r$	$A = \pi r^2$
$= 2\pi(8)$	$= \pi(8)^2$
$= 16\pi$	$= 64\pi$
$\approx 50.27 \text{ cm}$	$\approx 201.06 \text{ sq. cm}$



**Example 2**

Hermione has a small space on her corner lot that she would like to turn into a patio. To do this, she needs to do two things. First, she must know the length of the curved part, where she will put some decorative edging. Second, with the edging in place, she will need to purchase concrete to cover the patio. The concrete is sold in bags. Each bag will fill 2.5 square feet to the required depth of four inches. How much edging and concrete should Hermione buy?



The edging is a portion of the circumference of a circle with the center at point  $O$  and a radius of 10 feet. We can determine the exact fraction of the circle by looking at the measure of the central angle. Since the angle measures  $40^\circ$ , and there are  $360^\circ$  in the whole circle, this portion is  $\frac{40^\circ}{360^\circ} = \frac{1}{9}$  of the circle. If we find the circumference and area of the whole circle, then we can take  $\frac{1}{9}$  of each of those measurements to find the portion needed.

$$\begin{aligned} C &= \frac{1}{9}(2\pi r) & A &= \frac{1}{9}\pi r^2 \\ &= \frac{1}{9}(2 \cdot \pi \cdot 10) & &= \frac{1}{9} \cdot \pi \cdot (10)^2 \\ &= \frac{20\pi}{9} \approx 6.98 \text{ feet} & &= \frac{100\pi}{9} \approx 34.91 \text{ square feet} \end{aligned}$$

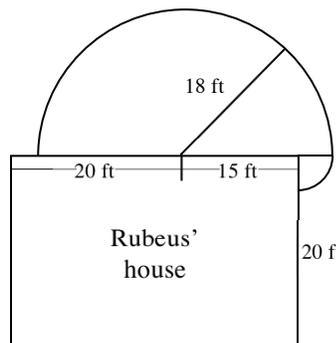
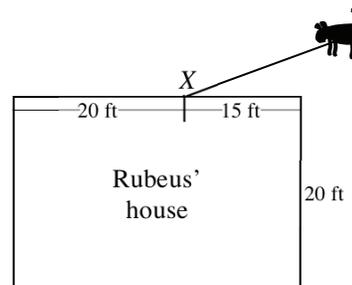
Hermione should buy 7 feet of edging, and she should buy 14 bags of concrete ( $34.91 \div 2.5 \approx 13.96$  bags). Concrete is sold in full bags only.

### Example 3

Rubeus' dog Fluffy is tethered to the side of his house at point  $X$ . If Fluffy's rope is 18 feet long, how much area does Fluffy have to run in?

Because Fluffy is tethered to a point by a rope, he can only go where the rope can reach. Assuming that there are no obstacles, this area would be circular. Since Fluffy is blocked by the house, the area will only be a portion of a circle.

From point  $X$ , Fluffy can reach 18 feet to the left and right of point  $X$ . This initial piece is a semicircle. But, to the right of point  $X$ , the rope will bend around the corner of the house, adding a little more area for Fluffy. This smaller piece is a quarter of a circle with a radius of 3 feet.



Semicircle:

$$\begin{aligned} A &= \frac{1}{2}\pi r^2 \\ &= \frac{18^2\pi}{2} \\ &= \frac{324\pi}{2} \\ &= 162\pi \approx 508.94 \end{aligned}$$

Quarter circle:

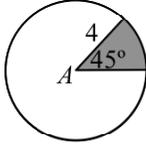
$$\begin{aligned} A &= \frac{1}{4}\pi r^2 \\ &= \frac{3^2\pi}{4} \\ &= \frac{9\pi}{4} \\ &\approx 7.07 \end{aligned}$$

Fluffy has a total of  $508.94 + 7.07 \approx 516$  square feet in which to run.

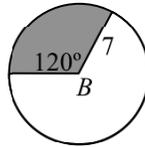
## Problems

Find the area of the shaded sector in each circle below. Points  $A$ ,  $B$ , and  $C$  are the centers.

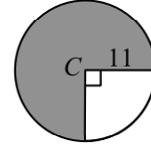
1.



2.

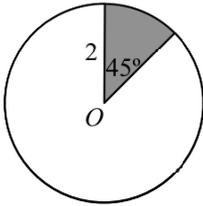


3.

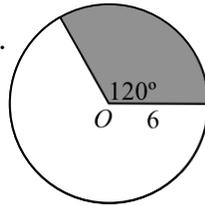


Calculate the area of the following shaded sectors. Point  $O$  is the center of each circle.

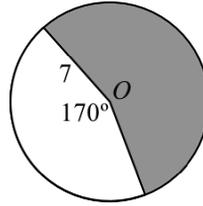
4.



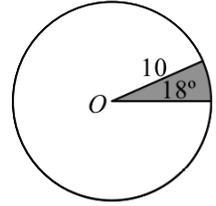
5.



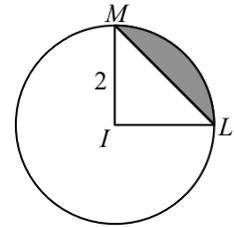
6.



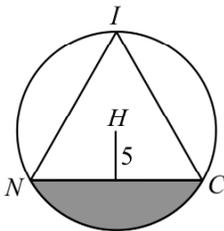
7.



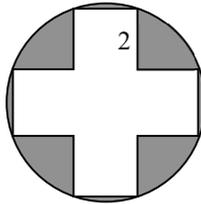
8. The shaded region in the figure is called a segment of the circle. It can be found by subtracting the area of  $\triangle MIL$  from the sector  $MIL$ . Find the area of the segment of the circle.



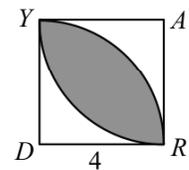
9.



10.

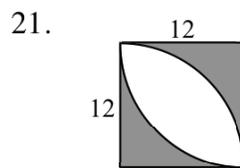
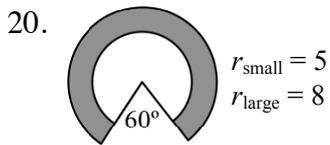
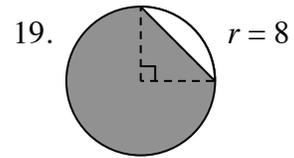
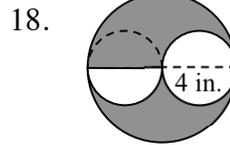
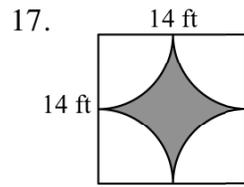
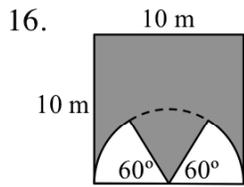


11.  $YARD$  is a square;  $A$  and  $D$  are the centers of the arcs.

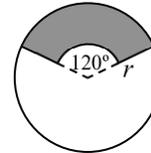


12. Find the area of a circular garden if the diameter of the garden is 30 feet.
13. Find the area of a circle inscribed in a square whose diagonal is 8 feet long.
14. The area of a  $60^\circ$  sector of a circle is  $10\pi \text{ m}^2$ . Find the radius of the circle.
15. The area of a sector of a circle with a radius of 5 mm is  $10\pi \text{ mm}^2$ . Find the measure of its central angle.

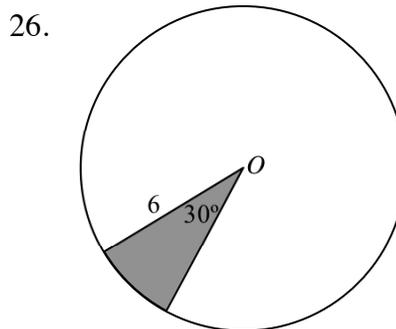
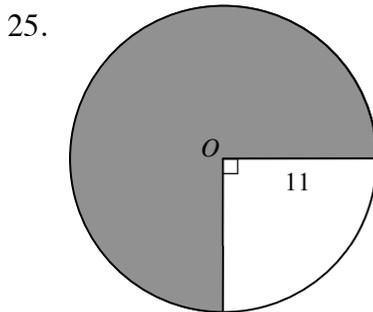
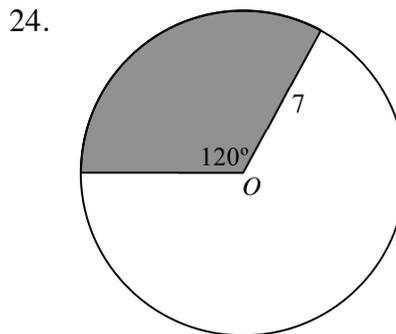
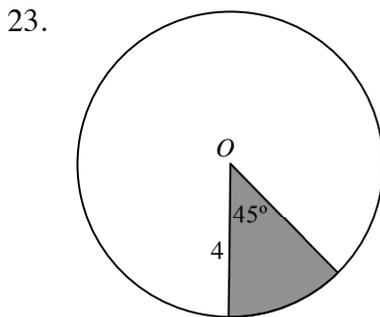
Find the area of each shaded region.



22. Find the length of the radius. The shaded area is  $12\pi \text{ cm}^2$ .

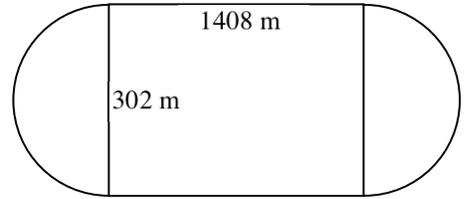


Find the area of the shaded sector in each circle below. In each case, point  $O$  is the center.

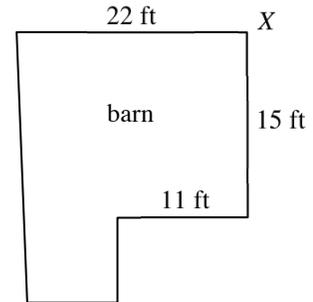


27. Find the arc length of the shaded sector in problem 1.
28. Find the arc length of the shaded sector in problem 2.
29. Find the arc length of the shaded sector in problem 3.
30. Find the arc length of the shaded sector in problem 4.

31. Kennedy and Tess are constructing a racetrack for their horses. The track encloses a field that is rectangular, with two semicircles at each end. A fence must surround this field. How much fencing will Kennedy and Tess need?



32. Rubeus has moved his dog Fluffy to a corner of his barn because he wants him to have more room to run. If Fluffy is tethered at point  $X$  on the barn with a 20 foot rope, how much area does Fluffy have to explore?



### Answers

- |  |  |   |
|--|--|---|
| 1. $2\pi$ units <sup>2</sup>                                       | 2. $\frac{49}{3}\pi$ units <sup>2</sup>        | 3. $\frac{363\pi}{4}$ units <sup>2</sup>              |
| 4. $\frac{\pi}{2}$ units <sup>2</sup>                              | 5. $12\pi$ units <sup>2</sup>                  | 6. $\frac{931\pi}{36}$ units <sup>2</sup>             |
| 7. $5\pi$ units <sup>2</sup>                                       | 8. $\pi - 2$ units <sup>2</sup>                | 9. $\frac{100}{3}\pi - 25\sqrt{3}$ units <sup>2</sup> |
| 10. $10\pi - 20$ units <sup>2</sup>                                | 11. $8\pi - 16$ units <sup>2</sup>             | 12. $225\pi$ ft <sup>2</sup>                          |
| 13. $8\pi$ ft <sup>2</sup>   | 14. $2\sqrt{15}$ m                             | 15. $144^\circ$                                       |
| 16. $100 - \frac{25}{3}\pi$ m <sup>2</sup>                         | 17. $196 - 49\pi$ ft <sup>2</sup>              | 18. $10\pi$ in. <sup>2</sup>                          |
| 19. $48\pi + 32$ units <sup>2</sup>                                | 20. $\frac{65}{2}\pi$ units <sup>2</sup>       | 21. $\approx 61.8$ units <sup>2</sup>                 |
| 22. 6 cm   | 23. $2\pi \approx 6.28$ sq. units              | 24. $\frac{49}{3}\pi \approx 51.31$ sq. units         |
| 25. $\frac{363\pi}{4} \approx 285.10$ sq. units                    | 26. $3\pi \approx 9.42$ sq. units              | 27. $\pi + 8 \approx 11.14$ units                     |
| 28. $\frac{14\pi}{3} + 14 \approx 28.66$ units                     | 29. $\frac{33\pi}{2} + 22 \approx 73.84$ units | 30. $\pi + 12 \approx 15.14$ units                    |
| 31. $2816 + 302\pi \approx 3764.76$ meters of fencing              |  |   |
| 32. $200\pi + 100\pi + \frac{25\pi}{4} \approx 962.11$ square feet |  |   |