GEO

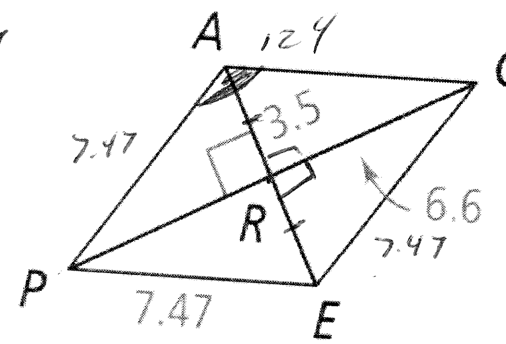
B/W

HW 11/21
P 379
1–7 all
9–23 odd
47

1) PACE is a parallelogram and m∠PAC = 124. Complete the following. (Pearson CC Geometry)

a. AC = PE = 7.47

b. CE = $(3.5)^2 + (6.6)^2 = (CE)^2 = 7.47$



c. PA = CE = 7.47

d. RE = AR = 3.5

e. CP = 2(CR) = 2(6.6) = 13.2

f. m∠CEP = m∠PAC = 124°

g. m∠EPA = $\frac{360-(124*2)}{2} = 56°$ OR ▱ consec ∠s are suppl. so 180 − 124 = 56°

h. m∠ECA = m∠EPA = 56°

i. m∠ACR = ???

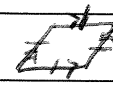
Lecture 6.4: Prop of Rhombuses, Rectangles, and Squares
Source: Pearson CC Geometry

KEY

SWBAT: To define, classify and use properties of rhombuses, rectangles, and squares.

VOCABULARY

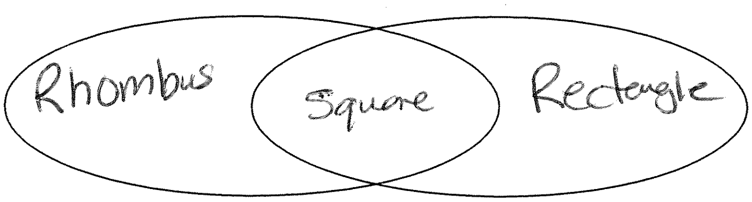
Rhombus: a ▱ w/ 4 ≅ sides



Rectangle: a ▱ w/ 4 right ∠s

Square: a ▱ w/ 4 ≅ sides and 4 rt ∠s

Special Parallelograms

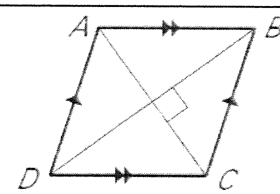


a sq is always a rectangle.
a sq is always a rhombus
a rhombus is sometimes a sq / sometimes a rect.
a rectangle is sometimes a sq / sometimes a rhombus

THEOREM 6-13:

A parallelogram is a rhombus if and only if its diagonals are $\perp$.

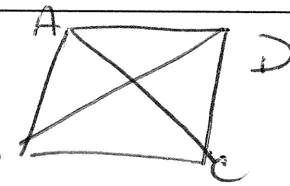
▱*ABCD* is a rhombus if and only if $\overline{AC}$ ⊥ $\overline{BD}$.



PROVE IT!

Given: ABCD is a rhombus

Prove: $\overline{AC} \perp \overline{BD}$



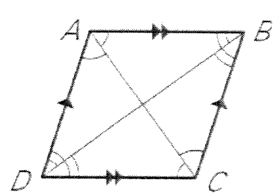
Stmts	Reasons
(1) $\overline{AD} \cong \overline{DC} \cong \overline{CB} \cong \overline{BA}$	(1) def of rhombus
(2) A & C are equidistant from B & D; B & D are equidistant from A & C	(2) Def. of ≅
(3) A & C are on the ⊥ bisector of $\overline{BD}$, B & D are on ⊥ bisector of $\overline{AC}$	(3) Convers. ⊥ bisector Thm
(4) $\overline{AC} \perp \overline{BD}$	(4) Thru 2 pts there is 1 unique line ⊥ to a given line.

THEOREM 6-14:

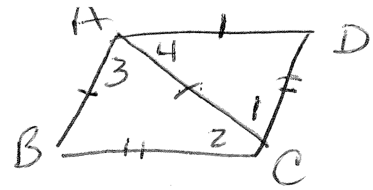
A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

▱*ABCD* is a rhombus if and only if $\overline{AC}$ bisects ∠ A and ∠ C and

$\overline{BD}$ bisects ∠ B and ∠ D.



PROVE IT! (Proof of Theorem 6-14)

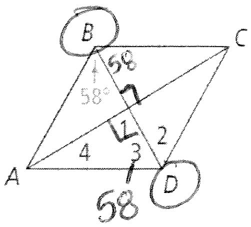


Given: ABCD is a rhombus

Prove: $\overline{AC}$ bisects ∠BAD and ∠BCD

Statements	Reasons
1. ABCD is a rhombus	① given
2. $\overline{AB} \cong \overline{AD}$; $\overline{CB} \cong \overline{CD}$	② Def of rhombus
3. $\overline{AC} \cong \overline{AC}$	③ reflex Prop
4. $\triangle ABC \cong \triangle ADC$	④ SSS
5. $\angle 3 \cong \angle 4$, $\angle 1 \cong \angle 2$	⑤ CPCTC
6. $\overline{AC}$ bisects ∠BAD ; ∠BCD	⑥ Def of bisector

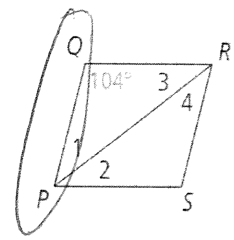
EX 1: What are the measures of the numbered angles in rhombus ABCD? ⊥ diagonals ; bisect opp ∠s



$m\angle 1 = 90°$

$m\angle 2 = m\angle 3 = 58°$

$m\angle 4 = 180 - 90 - 58 = 32°$



$m\angle P = 180 - 104$

$= 76°$

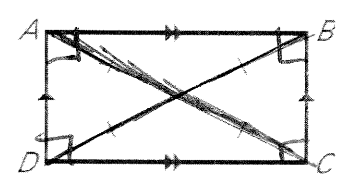
$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4$

$= \frac{76}{2} = 38°$

THEOREM 6-15:

A parallelogram is a rectangle if and only if its diagonals are ≅.

▱ABCD is a rectangle if and only if $\overline{AC}$ ≅ $\overline{BD}$

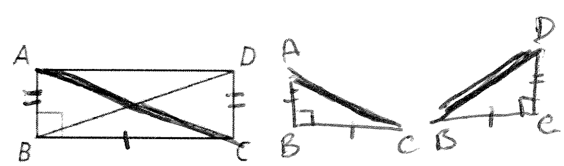


PROVE IT!

Developing Proof Complete the flow proof of Theorem 6-15.

Given: *ABCD* is a rectangle.

Prove: $\overline{AC} \cong \overline{BD}$



ABCD is a rectangle.
a. given

ABCD is a ▱.
b. def of rectangle

e. $\overline{AB} \cong \overline{DC}$
Opposite sides of a ▱ are ≅.

$\overline{BC} \cong \overline{BC}$
c. Reflex. Prop

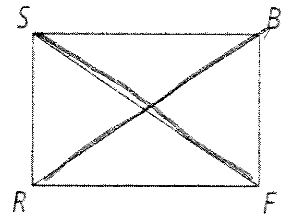
∠ABC and ∠DCB are right ∠s.
d. Def of rectangle

∠ABC ≅ ∠DCB
g. all rt ∠s are ≅

f. $\triangle ABC \cong \triangle DCB$
SAS

$\overline{AC} \cong \overline{BD}$
h. CPCTC

Ex 2: In rectangle RSBF, SF = 2x + 15 and RB = 5x – 12 . What is the length of a diagonal?



$2x+15=5x-12$

$27=3x$

$9=x$

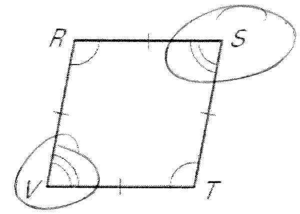
$2(9)+15$

$18+15$

$\boxed{33}$ = SF = RB

Ex 3: For any rhombus *RSTV*, decide whether the statement is ***always or sometimes true.***
Draw a sketch to help explain your reasoning.

a. $\angle S \cong \angle V$

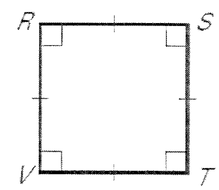


Solution
By definition, a rhombus is a parallelogram with four congruent sides.

We know by theorem that opposite angles of a parallelogram are $\cong$. So, $\angle S \cong \angle V$.

The statement is always true.

b. $\angle T \cong \angle V$

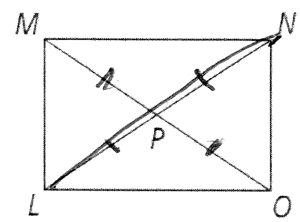


Solution
If rhombus *RSTV* is a square, then all four angles are congruent right angles.

So $\angle T \cong \angle V$ if *RSTV* is a square. Because not all rhombuses are also square, the statement is sometimes true.

PRACTICE:

1. If LN = 4x – 17 and MO = 2x + 13, what are the lengths of the diagonals of rectangle LMNO? What type of triangle is ΔPMN? Justify your answer.



$4x-17=2x+13$

$2x=30$

$x=15$

$2(15)+13$

$\boxed{43 = MO = LN}$

ΔPMN is isosceles Δ b/c MP = PN

2. For any square *CDEF*, is it always or sometimes true that $\overline{CD} \cong \overline{DE}$? Explain your reasoning.

C D F E

always b/c a square has 4 ≅ sides

3. A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

Square

Ex 4: List the quadrilaterals that have the given property. Choose among parallelogram (PG), rhombus (Rh), rectangle (REC) and square (SQ)

All sides are ≅. Rh, SQ

Opposite sides are ∥. PG, Rh, Rec, SQ

All ∠s are right ∠s. Rec, SQ

Diagonals bisect each other. PG, Rh, REC, SQ

Diagonals are ⊥. Rh, SQ

Opposite sides are ≅. PG, Rh, Rec, SQ

Opposite ∠s are ≅. PG, Rh, Rec, SQ

Consecutive ∠s are supplementary. PG, Rh, Rec, Sq.

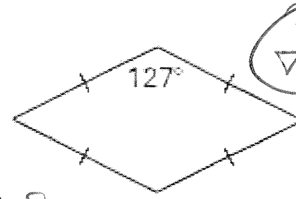
Diagonals are ≅. Rec, SQ

Each diagonal bisects opposite ∠s. Rh, Sq.

PRACTICE:

1. **Classify the special quadrilateral. *Explain* your reasoning.**

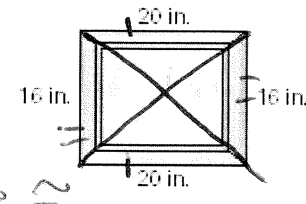
sq, Rh



The quadrilateral has four congruent Sides.

One of the angles is not a right ∠, so the rhombus is not also a Square. By the definition of Rhombus, the quadrilateral is a Rhombus.

2. **You are building a frame for a painting. The measurements of the frame are shown at the right.**



a. The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain.*

no. all we know ~~is it~~ is it is a ▱ because opp sides are ≅ but we don't know all ∠s are rt ∠'s.

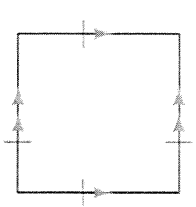
b. You measure the diagonals of the frame. The diagonals are about 25.6 inches. What can you conclude about the shape of the frame?

Since the diagonals are ≅, then it is either a sq. or a rectangle. Since 4 sides are not ≅, it is a rectangle.

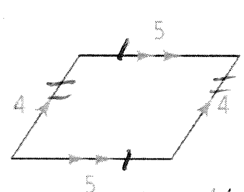
c. Suppose the diagonals of the frame are not congruent. Could the frame still be a rectangle? *Explain.*

no. Diagonals of a rectangle are always ≅.

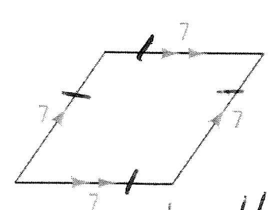
3. **Determine the most precise name for each quadrilateral**



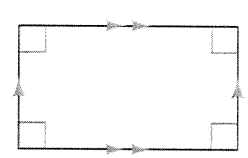
opp sides ∥
4 ≅ sides
so Rhombus
(don't know if rt ∠s)



opp sides ∥
opp sides ≅
so ▱



opp sides ∥
4 ≅ sides
so Rhombus



opp sides ∥
4 rt ∠s
4 sides not ≅
rectangle

HOMEWORK: p379 1-7all, 9-23 odd, and 47

Lecture 6.4: Prop of Rhombuses, Rectangles, and Squares
Source: Pearson CC Geometry

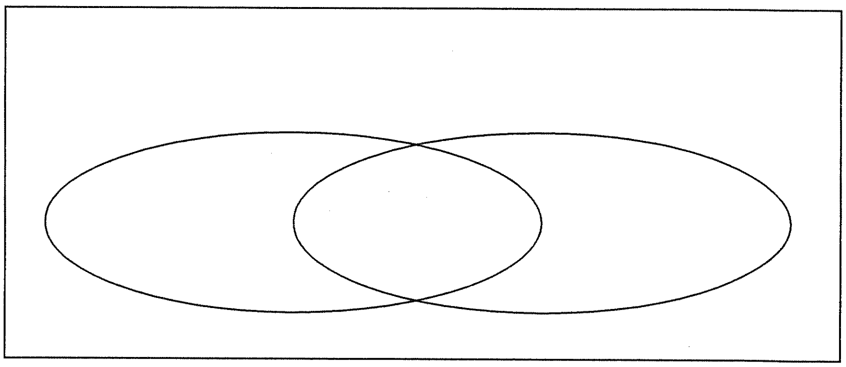
SWBAT: To define, classify and use properties of rhombuses, rectangles, and squares.

VOCABULARY

Rhombus____

Rectangle____

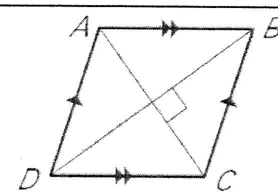
Square____



THEOREM 6-13:

A parallelogram is a rhombus if and only if its diagonals are________________.

$\square ABCD$ is a rhombus if and only if ________ $\perp$ ________.



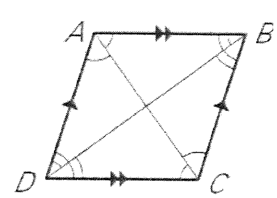
PROVE IT!
Given: ABCD is a rhombus
Prove:

THEOREM 6-14:

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

$\square ABCD$ is a rhombus if and only if $\overline{AC}$ bisects $\angle$ _______ and $\angle$ _______ and

$\overline{BD}$ bisects $\angle$ ________ and $\angle$ ________.

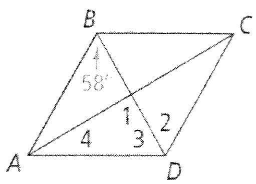


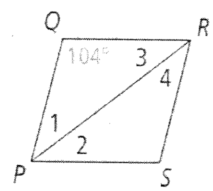
PROVE IT! (Proof of Theorem 6-14)

Given: ABCD is a rhombus
Prove:

Statements	Reasons
1.	
2.	
3.	
4.	
5.	
6.	

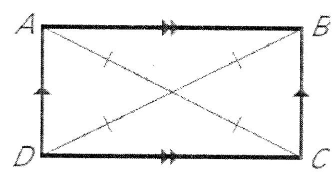
EX 1: What are the measures of the numbered angles in rhombus ABCD?





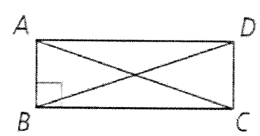
THEOREM 6-15:
A parallelogram is a rectangle if and only if its diagonals are __________.

▱*ABCD* is a rectangle if and only if ____________ ≅ ____________



PROVE IT!

Developing Proof Complete the flow proof of Theorem 6-15.
Given: *ABCD* is a rectangle.
Prove: $\overline{AC} \cong \overline{BD}$



ABCD is a ▱.
b. ?

e. ?
Opposite sides of a ▱ are ≅.

ABCD is a rectangle.
a. ?

$\overline{BC} \cong \overline{BC}$
c. ?

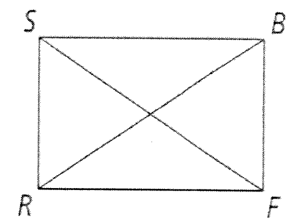
f. ?
SAS

$\overline{AC} \cong \overline{BD}$
h. ?

∠*ABC* and ∠*DCB* are right ∠s.
d. ?

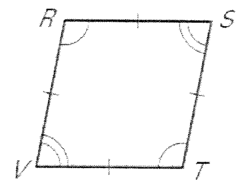
∠*ABC* ≅ ∠*DCB*
g. ?

Ex 2: In rectangle RSBF, SF = 2x + 15 and RB = 5x – 12 . What is the length of a diagonal?



Ex 3: For any rhombus *RSTV*, decide whether the statement is ***always or sometimes true*.**
Draw a sketch to help explain your reasoning.

a. $\angle S \cong \angle V$



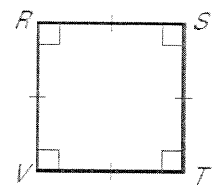
Solution

By definition, a rhombus is a parallelogram with four congruent ____________.

We know by theorem that opposite angles of a parallelogram are ____________________. So, $\angle S \cong \angle V$.

The statement is _____________ true.

b. $\angle T \cong \angle V$



Solution

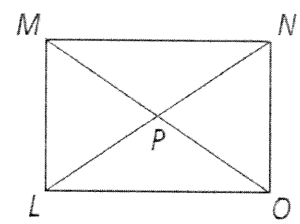
If rhombus *RSTV* is a______________, then all four angles are congruent right angles.

So $\angle T \cong \angle V$ if *RSTV* is a ___________. Because not all rhombuses are also

___________, the statement is _____________ true.

PRACTICE:

1. If LN = 4x – 17 and MO = 2x + 13, what are the lengths of the diagonals of rectangle LMNO? What type of triangle is ΔPMN? Justify your answer.



2. For any square *CDEF*, is it always or sometimes true that $\overline{CD} \cong \overline{DE}$? Explain your reasoning.

3. A quadrilateral has four congruent sides and four congruent angles. Classify the quadrilateral.

Ex 4: List the quadrilaterals that have the given property. Choose among parallelogram (PG), rhombus (Rh), rectangle (REC) and square (SQ)

All sides are $\cong$.

Opposite sides are $\cong$.

Opposite sides are $\parallel$.

Opposite $\angle$s are $\cong$.

All $\angle$s are right $\angle$s.

Consecutive $\angle$s are supplementary.

Diagonals bisect each other.

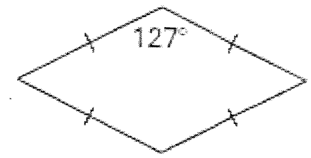
Diagonals are $\cong$.

Diagonals are $\perp$.

Each diagonal bisects opposite $\angle$s.

PRACTICE:

1. Classify the special quadrilateral. *Explain* your reasoning.

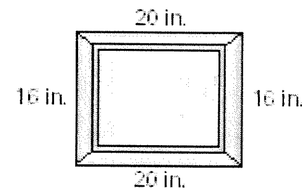


The quadrilateral has four congruent _______________.

One of the angles is not a _________________, so the rhombus is not also a

______________. By the definition of Rhombus, the quadrilateral is a ________________.

2. You are building a frame for a painting. The measurements of the frame are shown at the right.

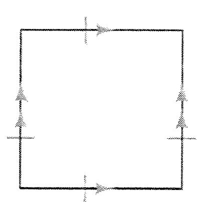


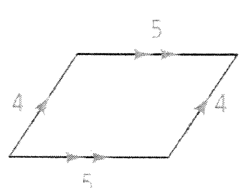
a. The frame must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain.*

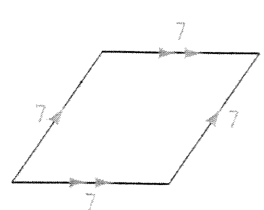
b. You measure the diagonals of the frame. The diagonals are about 25.6 inches. What can you conclude about the shape of the frame?

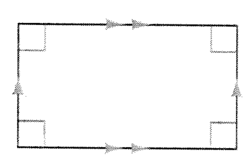
c. Suppose the diagonals of the frame are not congruent. Could the frame still be a rectangle? *Explain.*

3. Determine the most precise name for each quadrilateral









HOMEWORK: p379 1-7all, 9-23 odd, and 47