7-4: Double-Angle Identities

12/10/15

Does doubling the angle value double the ratio value?

no!

sin 30° = 0.5 sin 60° = 0.866

cos 20° = 0.940 cos 40° = 0.766

tan 100° = -5.671 tan 200° = 0.364

Double Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$

You can use any of these

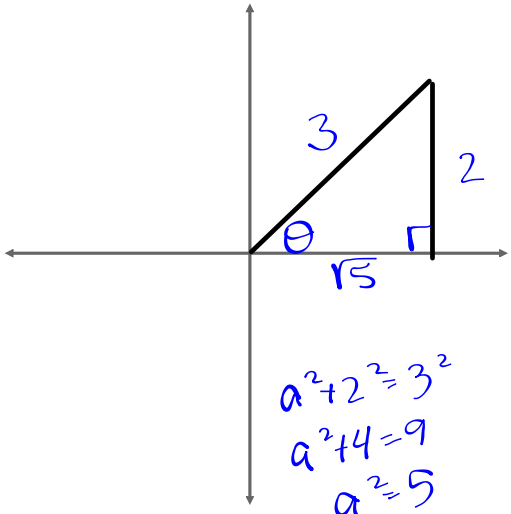
Find the sin 2θ

1.) sin θ = 2/3 and 0° < θ < 90°

$\sin 2\theta = 2 \cdot \sin\theta \cdot \cos\theta$

$\sin 2\theta = 2 \cdot \frac{2}{3} \cdot \frac{\sqrt{5}}{3}$

$\sin 2\theta = \frac{4\sqrt{5}}{9}$



$a^2+2^2=3^2$

$a^2+4=9$

$a^2=5$

$a=\sqrt{5}$

2.) Find the cos 2θ

tan θ = 3/4 and 180° < θ < 270°

(use any cos 2θ identity)

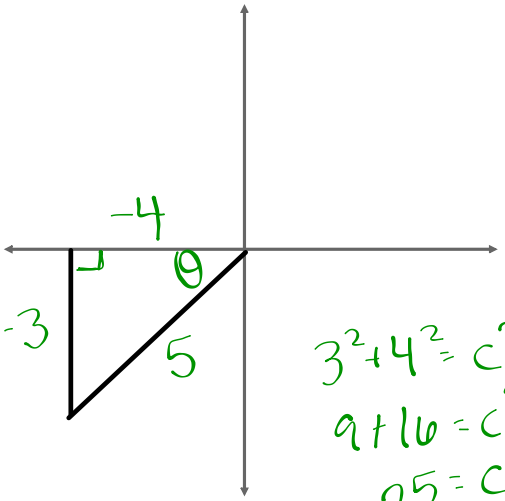
$\cos 2\theta = 1 - 2\cdot\sin^2\theta$

$\cos 2\theta = 1 - 2\cdot\left(\frac{-3}{5}\right)^2$

$= 1 - 2\cdot\frac{9}{25}$

$= \frac{25}{25} - \frac{18}{25}$

$\cos 2\theta = \frac{7}{25}$



$3^2+4^2=c^2$

$9+16=c^2$

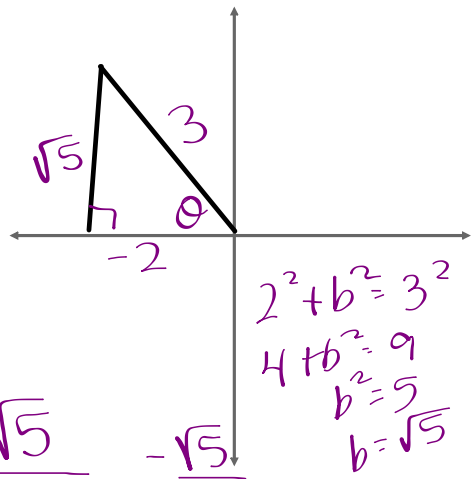
$25=c^2$

$5=c$

3.) Find the tan 2θ

$\cos\theta = -2/3$ and $\pi/2 < \theta < \pi$

$90° < \theta < 180$ (Quad II)



$2^2 + b^2 = 3^2$

$4 + b^2 = 9$

$b^2 = 5$

$b = \sqrt{5}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\tan 2\theta = \frac{2\left(\frac{\sqrt{5}}{-2}\right)}{1-\left(\frac{\sqrt{5}}{-2}\right)^2} = \frac{-\sqrt{5}}{\frac{4}{4}-\frac{5}{4}} = \frac{-\sqrt{5}}{\frac{-1}{4}}$$

$$= -\sqrt{5} \cdot \frac{4}{-1} = \boxed{4\sqrt{5}}$$

Write as a single, simplified trigonometric expression

*use double angle or angle sum/difference identities

1. $2\sin A\cos A$

double angle → sin

$\sin(2A)$

2. $2\cos^2 112.5° - 1$

double angle → cos

$\cos(225°)$

3. $\dfrac{2\tan 67.5°}{1-\tan^2 67.5°}$

double angle → tan

$\tan(135°)$

4. $\sin 75°\cos 15° + \sin 15°\cos 75°$

angle sum → sin

$\sin(90)$

HW: 7.4.1