Modern Control Systems

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Lecture 1: Introduction

MMAE 543: Modern Control Systems

This course is on **Modern** Control Systems

- Techniques Developed in the Last 50 years
- Computational Methods
  - No Root Locus
  - No Bode Plots
  - No pretty pictures

Classical Control Systems is a different Course

- MAE 506 or undergrad

We focus on State-Space Methods

- In the time-domain
- We use large state-space matrices

$$\frac{d}{dt}\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\\x_4(t)\end{bmatrix} = \begin{bmatrix}-1 & 1.2 & -1 & .8\\1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\\x_4(t)\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 1\\0 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}u_1(t)\\u_2(t)\end{bmatrix}$$

- We require Matlab
  - Need robust control toolbox (Available in MMAE computer lab)

Linear Systems

Linear systems have the form

$$\dot{x}(t) = Ax(t)$$

where

- $A \in \mathbb{R}^{n\times n}$ ($\mathbb{R}^{n\times n}$ is the set of real matrices of size n by n)
- If the system is nonlinear, linearize it : $f(x) = \nabla_x f(x)|_{x=x_0} x$.
  - More on this later.

Finite Dimensional

- $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$
- There are no delays
  - If you have delays, use a Padé approximation
  - Adds additional states
- No Partial-Differential Equations
  - If you have a PDE, perform a finite-difference approximation.

MMAE 543: Modern Control Systems

A Computational Class

- Divides modern from classical
- Solutions by hand are too complicated
- A new definition of solution
  - Design an algorithm
- We have classes of solutions
  - Convex Optimization
  - Linear Matrix Inequalities

A Mathematical Class

- We will use mathematical shorthand
  - $\forall$ is "for all"
  - $X \cap Y$ means "intersection of X and Y"
  - $X \cup Y$ means "union of X and Y"
  - $x \in Y$ means x is an element of Y
- We will use proofs.
  - Based on definitions
  - Wording is important

Proof Example

Begin with a theorem statement

Theorem 1.

$$\lim_{x\to\infty} \frac{1}{1+x} = 0$$

To prove such a things, we need to understand what it means.

Definition 2.

We say z is the limit, $z = \lim_{x\to\infty} f(x)$, if for any $\delta > 0$, there exists an $\epsilon > 0$ such that for any $x > \epsilon$, $\|z - f(x)\| \le \delta$

Proof Example

With a definition, we know how to proceed:
Given an arbitrary δ, find the $\epsilon(\delta) > 0$ and we are done.

Proof.

- Let $f(x) = \frac{1}{1+x}$ and $z = 0$.
- Choose $\epsilon = \frac{1}{\delta} - 1$.
- Then for any $x \geq \epsilon > \frac{1}{\delta} - 1$,

$$\|1 + x\| = \|1 + x\| > \frac{1}{\|\delta\|}$$

- We conclude

$$\|z - f(z)\| = \left\| \frac{1}{1+x} \right\| = \frac{1}{\|1 + x\|} < \delta.$$

□

We will cover the basics of normed linear spaces, convergence, linear operators, etc.

Goal: Not too hot, not too cold

- Adjust water temperature to within acceptable tolerance
- Don't burn yourself!
- Don't take forever.

Human-in-the-Loop Control: How to do it?

- Too cold - rotate knob clockwise
- Too hot - rotate knob counterclockwise
- Much too cold - rotate faster
- Much too hot - rotate faster

If we go too fast, we overshoot

- Abstract the system using block diagrams

What is a Control **System**?

It is a System.
Lets start with the simpler question:
What is a System?

Definition 3.

A **System** is anything with inputs and outputs

For **Optimal Control**, we have two sets of inputs and outputs.

- Inputs: Actuators and Disturbances
- Outputs: Sensor Measurements and Regulated Output

Examples of Systems

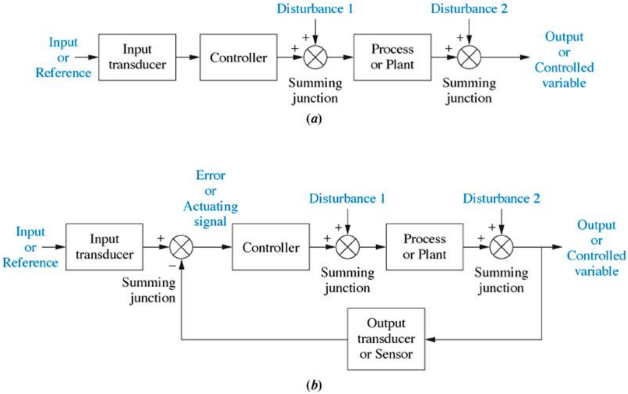
College

Definition 4.

The System to be controlled is called the **Plant**.

Definition 5.

A **Control System** is a system which modifies the inputs to the *plant* to produce a desired output.



Fundamentals of Control

Any controller must have one fundamental part: The Actuator

Definition 6.

The **Actuator** is the mechanism by which the controller affects the input to the plant.

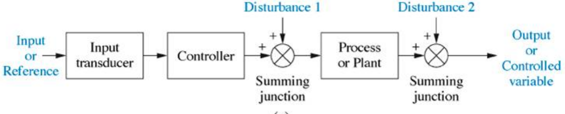
Examples:

- Ailerons, Rudder
- Force Transducers: Servos/Motors
  - Servos
  - Motors
- Furnace/Boiler

The first basic type of control is **Open Loop**.

Definition 7.

An **Open Loop Controller** has actuation, but no measurement.



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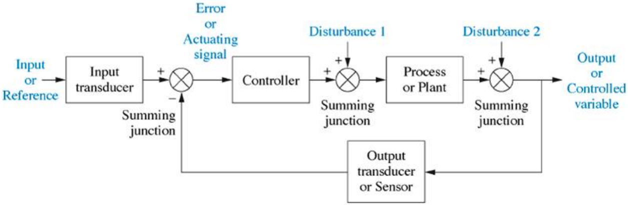

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The Two Basic types of Control

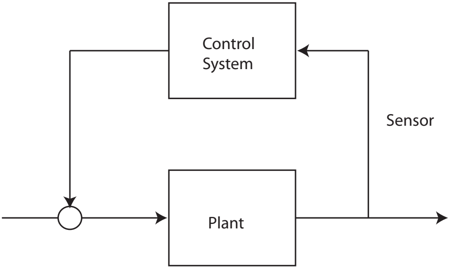
The second basic type of control is **Closed Loop**.

Definition 8.

The **Sensor** is the mechanism by which the controller detects the outputs of the plant.

Definition 9.

A **Closed Loop Controller** uses *Sensors* in addition to Actuators.



Lets go through some detailed examples.

History of Feedback Control Systems

Egyptian Water Clocks 1200BC

Time left is given by the amount of water left in the pot.
Problem: Measurement is limited to time left and by amount of water in pot.
Solution: Measure the amount of water that comes out of the pot.

History of Water Clocks

Time passed is amount of water in pot.
Problem: Water flow varies by amount of water in the top pot.
Solution: Maintain a constant water level in top pot.

History of Water Clocks

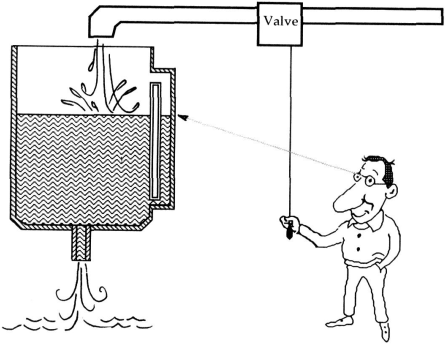


FIGURE 1.1. Level Control System. A sight tube and operator's eye form a sensor: a device which converts information into electrical signal.

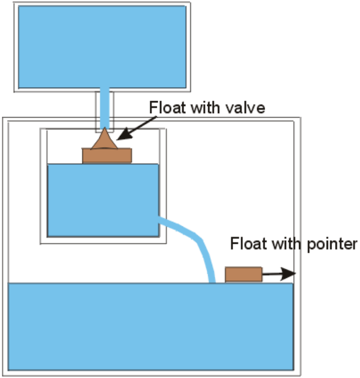
Problem: Manually refilling the top pot is labor intensive and inaccurate.
Solution: Design a control System (Inputs, Outputs?).

History of Water Clocks

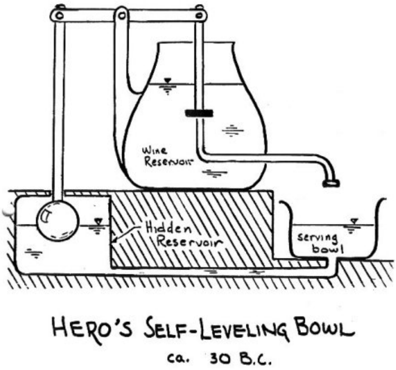
Ctesibius c. 220-285 BC

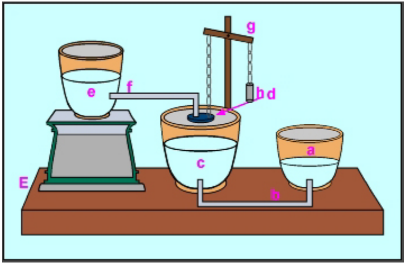
Father of pneumatics

- Dirt Poor
- Created most accurate clock until Huygens (1657 AD)
- Overshadowed by better-known student Heron (Hero) of Alexandria



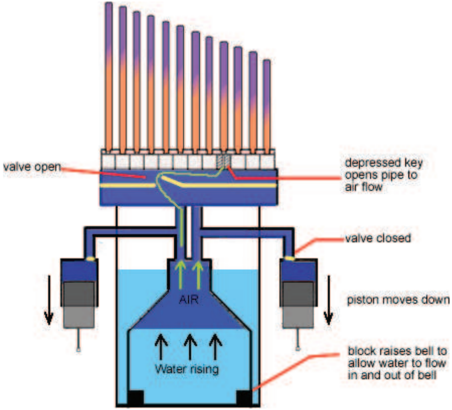
As any good student, Hero used Ctesibius' water clock to perform party tricks.





The self-replenishing wine bowl. (Inputs, Outputs?)

Ctesibius himself applied the principle of pneumatic control to create a pipe organ.



The Industrial Revolution

More Serious Applications

In addition to Wine bowls, Heron also developed the steam engine.

Unfortunately, the result was not applied and was unregulated.

The Modern Aeolipile

Modern (Relatively) Steam Engines

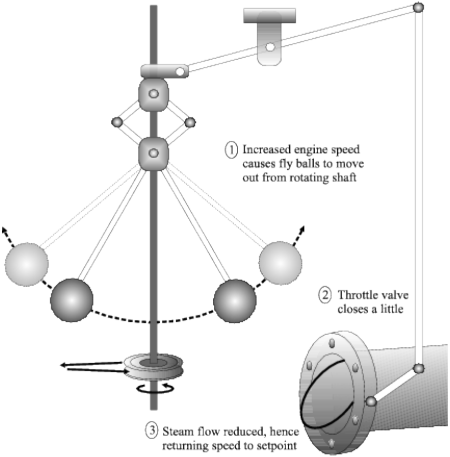
The Flyball Governor

Problem: To be useful, steam engines must rotate a piston at a *fixed speed*.

The Flyball Governor

Flyballs are attached to rotating piston

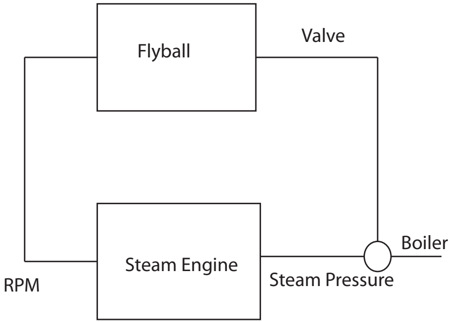
- Faster rotation = More centrifugal force.
- Centrifugal force lifts the flyballs, which move a lever which releases steam.
- Release of steam reduces pressure.
- Reduced pressure decreases engine/piston speed.



Identify the inputs and outputs

The Flyball Governor

Block Diagram Representation



The Flyball is a feedback controller for the steam engine.

The Flyball Governor

The Flyball Governor in Operation

Stuart-Turner No9 Steam Engine