

Learning Targets:

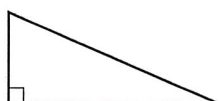
- Solve problems using the areas of triangles and composite figures.
- Use coordinates to compute perimeters and areas of figures.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Group Presentation, Identify a Subtask, Use Manipulatives, Quickwrite

Lisa asks Rashid to help her understand how to determine the area of triangular-shaped tabletops. Lisa and Rashid start the investigation with right and isosceles triangles.

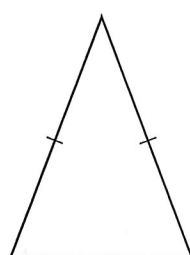
1. Use the given triangles below to create quadrilaterals. Using what you know about these quadrilaterals, explain why the formula for the area of a triangle is $\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height}$.

a.



*right Δ is half of a rectangle

b.



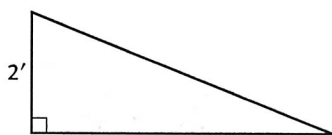
*isosceles Δ is half of a parallelogram

2. **Reason abstractly.** Compare and contrast the steps required to find the area of a right triangle and an isosceles triangle whose side lengths are given.

- both use same formula
- have to use Pythagorean Theorem to find height in isosceles Δ

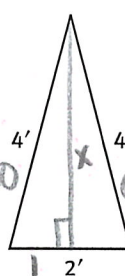
3. Find the area and subsequent charge for each tabletop below. *\$8.50*

a.



$$a = \frac{1}{2} \cdot 2 \cdot 5 = \boxed{5 \text{ ft.}^2} \cdot 8.5 = \$42.50$$

b.

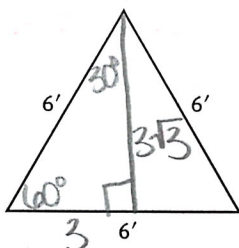


$$x^2 + 1^2 = 4^2$$

$$x = \sqrt{15}$$

$$a = \frac{1}{2} \cdot 2 \cdot \sqrt{15} = \boxed{\sqrt{15} \text{ ft.}^2} \cdot 8.50 = \boxed{\$32.92}$$

c.



$$a = \frac{1}{2} \cdot 6 \cdot 3\sqrt{3} = \boxed{9\sqrt{3} \text{ ft.}^2} \cdot 8.50 = \boxed{\$32.50}$$

My Notes

MATH TIP

An isosceles triangle has two congruent sides and two congruent angles.

ACTIVITY 30

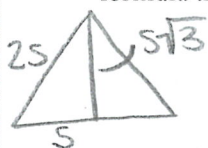
continued

Lesson 30-2

Areas of Triangles

My Notes

4. **Reason abstractly.** Rashid draws several tabletops in the shape of an equilateral triangle, each having different side lengths. Derive an area formula that will work for all equilateral triangles with side length s .



$$\text{area} = \frac{1}{2} \cdot 2s \cdot s\sqrt{3} \\ = s^2\sqrt{3}$$

5. Use the formula created in Item 4 to find the area of an equilateral triangle with side length 6 feet. Compare your answer to the area you found in Item 3c.

$$s = 3$$

$$\text{area} = 3^2\sqrt{3} \text{ * SAME} \\ = 9\sqrt{3}$$

(b) a) 2 rectangles
2 triangles

b) both - Jalen found the sum of each of the three pieces - Bree took the total area of rectangle it lies in & subtracted areas that aren't part of table

$$\begin{aligned} A_{r_1} &= 2 \cdot 5 = 10 \text{ units} \\ A_{r_2} &= 4 \cdot 14 = 56 \text{ units} \\ A_{t_1} &= \frac{1}{2} \cdot 7 \cdot 5 = 17.5 \text{ units} \\ A_{t_2} &= \frac{1}{2} \cdot 5 \cdot 5 = 12.5 \text{ units} \\ &= 96 \text{ units}^2 \end{aligned}$$

d) easier to determine necessary lengths

e) right Δ - height = one of legs
equilateral Δ - height can be found using trig, 30° - 60° - 90° , or Pyth. thm.
(depending on what is given)

Check Your Understanding

6. Find the area of the tabletop shown on the coordinate plane.

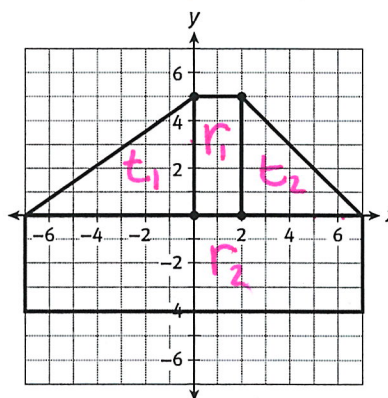
a. Name the shapes that make up the tabletop.

b. **Critique the reasoning of others.** Jalen and Bree each used a different method to determine the area of the tabletop.

Jalen found the sum of the areas of each of the four figures. Bree drew auxiliary lines to create a large rectangle, and then subtracted the area of the two triangular regions not in the tabletop from the large rectangle. Whose method is correct? Explain your reasoning.

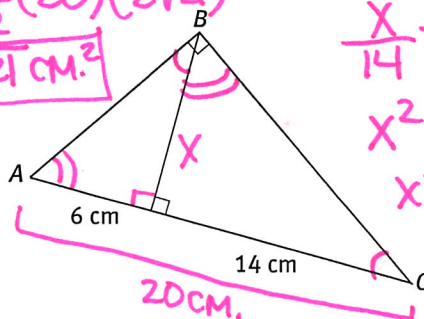
c. Compute the area of the tabletop.

d. What are some benefits of working with composite figures on a coordinate plane?



7. Determine the area of triangle ABC. Leave the answer in radical form.

$$\text{area} = \frac{1}{2} (20) (21\sqrt{2}) \\ = 210\sqrt{2} \text{ cm}^2$$



$$\begin{aligned} \frac{x}{14} &= \frac{6}{x} \\ x^2 &= 84 \\ x &= 2\sqrt{21} \end{aligned}$$

8. Explain how to determine the height of a right triangle versus the height of an equilateral triangle, given the side lengths of the triangles.

Lesson 30-2

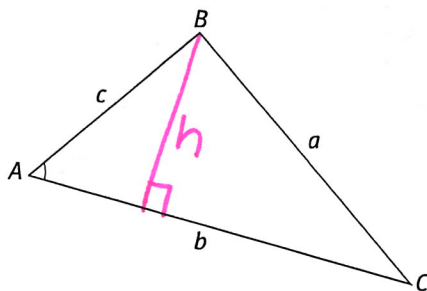
Areas of Triangles

ACTIVITY 30

continued

Next, Rashid shows Lisa how to determine the area of a triangular-shaped tabletop given two side lengths and the measure of the included angle.

9. **Make use of structure.** Complete the proof to discover an area formula that can be used when given a triangle with two side lengths and the measure of the included angle.



- a. Write the formula for the area of a triangle. $\text{area} = \frac{1}{2} \text{ base} \cdot \text{height}$

- b. Draw and label a perpendicular line, h , from point B to \overline{AC} . This represents the height of the triangle.

- c. Complete to find $\sin A$: $\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{c}$

- d. Solve for h : $= h$

$$c \cdot \sin A = \frac{h}{c} \cdot c$$

$$c(\sin A) = h$$

- e. Substitute the equivalent expression you got for h (from part d) into the formula for area of a triangle (from part a).

$$\text{Area} = \frac{1}{2}bh$$

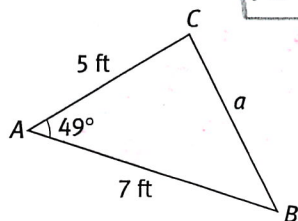
$$= \frac{1}{2}b$$

$$\text{area} = \frac{1}{2}b \cdot c \sin A$$

10. The diagram shown represents tabletop ABC . The tabletop has side lengths 5 ft and 7 ft. The angle between the two sides, $\angle A$, has a measure of 49° . Compute the area of the triangular tabletop, to the nearest tenth.

$$\text{area} = \frac{1}{2} \cdot 7 \cdot 5 \cdot \sin 49^\circ$$

$$= \boxed{13.2 \text{ ft}^2}$$



ACTIVITY 30

continued

Lesson 30-2 Areas of Triangles

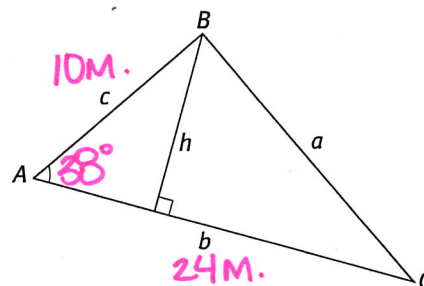
My Notes

Check Your Understanding

11. Find the area of the triangle to the nearest tenth, if $c = 10$ m, $b = 24$ m, and $m\angle A = 38^\circ$.

$$A = \frac{1}{2} \cdot 24 \cdot 10 \cdot \sin 38^\circ$$

$$\approx \boxed{73.9 \text{ M}^2}$$



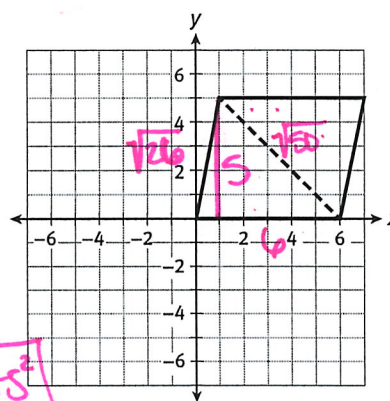
12. A parallelogram is given on a coordinate plane. Use the area formula of a triangle to derive the area formula of a parallelogram. Then compute the area of the figure.

$$\text{area}_{\Delta} = \frac{1}{2}bh$$

$$\text{area}_{2\Delta s} = 2\left(\frac{1}{2}bh\right)$$

$$= bh$$

$$= 5 \cdot 6 = \boxed{30 \text{ units}^2}$$

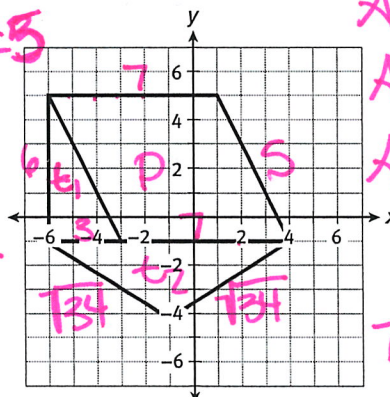


LESSON 30-2 PRACTICE

For Items 13–16, compute the area of each triangular-shaped tabletop, to the nearest tenth.

- Equilateral triangle with 8.5 ft sides
- Triangle ABC with vertices at (2, 1), (8, 1), and (6, 4)
- Base measure of 15 ft, another side length of 12 ft, and an included angle of 30°
- Isosceles triangle with sides of 4 ft, 7 ft, and 7 ft

17. **Make sense of problems.** Determine the perimeter and area of the tabletop shown on the coordinate plane below.



$$A_{t_1} = \frac{1}{2} \cdot 6 \cdot 3 = 9$$

$$A_{t_2} = \frac{1}{2} \cdot 10 \cdot 5 = 25$$

$$A_p = 6 \cdot 7 = 42$$

$$\boxed{66 \text{ ft}^2}$$

$$7 + 5 + \sqrt{34} + \sqrt{34} + 6$$

$$\approx \boxed{29.66 \text{ ft}}$$