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A New Approach for Finding an Optimal Solution for Integer Interval Transportation Problems

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Abstract

In this paper a different approach namely separation method based on zero suffix method is applied for finding an optimal solution for integer transportation problems where transportation cost, supply and demand are intervals. The proposed method is a non fuzzy method. The solution procedure is illustrated with a numerical example. The separation method can be served as an important tool for the decision makers when they are handling various types of logistic problems having interval parameters.

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Keywords: *Interval integer transportation problem, Optimal solution, Zero suffix method.*

1 Introduction

In this paper, we propose a new method namely, separation method to find an optimal solution for integer transportation problems where transportation cost, supply and demand are intervals. We develop the separation method without using the midpoint and width of the interval in the objective function of the interval transportation problem which is a non-fuzzy method. The proposed method is based on zero suffix method [8],[9]. Various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter, and the penalty factors.

In real life problems, these conditions may not be satisfied always. To deal with inexact coefficients in transportation problems, many researchers [1]-[3], [5]-[7], [10], [11] have proposed fuzzy and interval programming techniques for solving them. Das et al. [3] proposed a method, called fuzzy technique to solve interval transportation problem by considering the right bound and the midpoint of the interval. Sengupta and Pal [10] proposed a new fuzzy orientated method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function. Here we have proposed new method called as separations based on zero suffix method to find the optimal solution for fully interval integer transportation problems [FIITP].

2 Preliminaries

Let D denote the set of all closed bounded intervals on the real line R. That is, $D = \{[a, b], a \leq b$ where a&b are in R$\}$.

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [6],[4].

Definition 2.1. *Let* $A = [a, b]$ *and* $B = [c, d]$ *be in* D. *Then*
$A \oplus B = [a + c, b + d]$;
$A \otimes B = [a - d, b - c]$;
$kA = [ka, kb]$ *if* k *is a positive real number;*
$kA = [kb, ka]$ *if* k *is negative real number and*
$A \otimes B = [p, q]$ *where* $p = min\{ac, ad, bc, bd\}$ *and* $q = max\{ac, ad, bc, bd\}$

Definition 2.2. *Let* $A = [a, b]$ *and* $B = [c, d]$ *be in* D. *Then*
$A \leq B$ *if* $a \leq c$ *and* $b \leq d$
$A \geq B$ *if* $B \leq A$, *that is,* $a \geq c$ *and* $b \geq d$ *and*
$A = B$ *if* $A \leq B$ *and* $B \leq A$, *that is* $a = c$ *and* $b = d$.

Definition 2.3. *The set* $\{[x_{ij}, y_{ij}],\ rmfor\ all\ i = 1, 2, 3, \cdots, m$ and j $=$ $1, 2, 3, \cdots,$ n$\}$ *is said to be a feasible solution of (FIITP) if they satisfy the equations 1,2 and 3.*

Definition 2.4. *A feasible solutions* $\{[x_{ij}, y_{ij}],$ for all i $= 1, 2, 3, \cdots,$ m and j $=$ $1, 2, 3, \cdots,$ n$\}$ *is said to be a feasible solution of (FIITP) is said to be an optimal solution of (FIITP) if.*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij},d_{ij}]\otimes[x_{ij},y_{ij}] \leq \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij},d_{ij}]\otimes[u_{ij},v_{ij}],$$

for all $i = 1,2,3,\cdots,m$ *and* $j = 1,2,3,\cdots,n$ *and for all feasible* $\{[u_{ij},v_{ij}],\ \text{for i} = 1,2,3,\cdots,\text{m and j} = 1,2,3,\cdots,\text{n}\}$.

Now, we prove the following theorem which finds a relation between optimal solutions of a fully interval integer transportation problem and a pair of induces transportation problems and also, is used in the proposed method.

3 Fully Interval Integer Transportation Problems

Consider the following fully interval integer transportation problem (FIITP):

$$\text{Minimize } [z_1,z_2] = \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij},d_{ij}]\otimes[x_{ij},y_{ij}]$$

$$\text{Subject to } \sum_{j=1}^{n}[x_{ij},y_{ij}] = [a_i,p_i], i = 1,2,\cdots,m \quad (1)$$

$$\sum_{i=1}^{n}[x_{ij},y_{ij}] = [b_j,q_j], j = 1,2,\cdots,n \quad (2)$$

$$x_{ij} \geq 0, y_{ij} \geq 0,\ i = 1,2,3,\cdots,m \text{ and } j = 1,2,3,\cdots,n \text{ and are integers} \quad (3)$$

where c_{ij} and d_{ij} are positive real numbers for all i and j, a_i and p_i are positive real numbers for all i, b_j and q_j are positive real numbers for all j.

Theorem 3.1. *If the set* $\{y^{\circ}_{ij}$ for all i and $j\}$ *is an optimal solution of the upper bound transportation problem (UBITP) of (FIITP) where*

$$\text{(UBITP) Minimize } z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} y_{ij} = p_i, i = 1,2,\cdots,m \quad (4)$$

$$\sum_{i=1}^{m} y_{ij} = q_i, j = 1,2,\cdots,n \quad (5)$$

$$y_{ij} \geq 0, i = 1,2,3,\cdots,m \text{ and } j = 1,2,3,\cdots,n \text{ and are integers} \quad (6)$$

and the set $\{x^\circ_{ij}$ for all i and $j\}$ *is an optimal solution of the lower bound transportation problem (LBITP) of (FIITP) where*

$$\text{(UBITP) Minimize } z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \cdots, m \tag{7}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \cdots, n \tag{8}$$

$$x_{ij} \geq 0, i = 1, 2, 3, \cdots, m \text{ and } j = 1, 2, 3, \cdots, n \text{ and are integers} \tag{9}$$

then the set $\{[x^\circ_{ij}, y^\circ_{ij}]$ for all i and $j\}$ *is an optimal solution of the problem (FIITP) provided* $x^\circ \leq y^\circ_{ij}$, *for all* i *and* j.

Proof. Let $\{[x_{ij}, y_{ij}]$ for all i and $j\}$ be a feasible solution of the problem (FI-ITP). Therefore, $\{x^\circ_{ij}$ for all i and $j\}$ and $\{y^\circ_{ij}$ for all i and $j\}$ are feasible solution of the problems (UBITP) and (LBITP).

Now, since $\{x^\circ_{ij}$ for all i and $j\}$ and $\{y^\circ_{ij}]$ for all i and $j\}$ are feasible solution of the problems (UBITP) and (LBITP), we have

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y^\circ_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij}; \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x^\circ_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

and $x^\circ_{ij} \leq y^\circ_{ij}$, for all $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$.

This implies that $\left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x^\circ_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y^\circ_{ij}\right] \leq \left[\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \leq \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij}\right]$

That is, $\sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x^\circ_{ij}, y^\circ_{ij}] \leq \sum_{i=1}^{m}\sum_{j=1}^{n}[c_{ij}, d_{ij}] \otimes [x_{ij}, y_{ij}]$,

Now, since $\{x^\circ_{ij}$ for all i and $j\}$ and $\{y^\circ_{ij}]$ for all i and $j\}$ satisfy 4 to 9 and $x^\circ_{ij} \leq y^\circ_{ij}$, for all i and j, we can conclude that the set $\{[x^\circ_{ij}, y^\circ_{ij}]$for all i and $j\}$ is an optimal solution of the problem (FIITP).

Hence the theorem. □

4 Zero suffix method

We, now introduce a new method called the zero suffix method for finding an optimal solution to the transportation problem.

The zero suffix method proceeds as follows.

Step 1: Construct the transportation table.

Step 2: Subtract each row entries of the transportation table from the corresponding row minimum after that subtract each column entries of the transportation table from the corresponding column minimum.

Step 3: In the reduced cost matrix there will be atleast one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S,

$$\text{Therefore } S = \left\{ \frac{\text{Add the costs of nearest adjacent sides of zero which are greater than zero}}{\text{No. of costs added}} \right\}$$

Step 4: Choose the maximum of S, if it has one maximum value then first supply to that demand corresponding to the cell. If it has more equal values then select $\{a_i, b_j\}$ and supply to that demand maximum possible.

Step 5: After the above step, the exhausted demands (column) or supplies(row) to be trimmed. The resultant matrix must possess at least one zero is each row and column, else repeat 4.

Step 6: Repeat 4 to 4 until the optimal solution is obtained.

5 Separation method

We, now introduce a new algorithm namely, separation method for finding an optimal solution for a fully interval integer transportation problem.

The separation method proceeds as follows.

Step1: Construct the UBITP of the given FIITP.

Step2: Solve the UBITP by using the zero suffix method to get optimal. Let $\{y^\circ_{ij}]$ for all i and $j\}$ be an optimal solution of the upper bound transportation problem(UBITP).

Step3: Construct the LBITP of the given FIITP.

Step4: Solve the LBITP with the upper bound constraints $d_{ij} \leq y^\circ_{ij}$ for all i and j by using the zero suffix method. Let $\{x^\circ_{ij}]$ for all i and $j\}$ be an optimal solution of LBITP with $x^\circ_{ij} \leq y^\circ_{ij}$, for all i and j.

Step5: The optimal solution of the given FIITP is $\{[x^\circ_{ij}, y^\circ_{ij}]$ for all i and $j\}$ (by the 3.1)

6 Numerical example

Consider the following FIITP

					Supply
	[1,2]	[1,3]	[5,9]	[4,8]	[7,9]
	[1,2]	[7,10]	[2,6]	[3,5]	[17,21]
	[7,9]	[7,11]	[3,5]	[5,7]	[16,18]
Demand	[10,12]	[2,4]	[13,15]	[15,17]	[40,48]

Now, the UBITP of the given problem is given below:

					Supply
	2	3	9	8	9
	2	10	6	5	21
	9	11	5	7	18
Demand	12	4	15	17	48

Now, using the zero suffix method, the optimal solution to the UBITP is $y_{11}^{\circ} = 5, y_{12}^{\circ} = 4, y_{21}^{\circ} = 7, y_{24}^{\circ} = 14, y_{33}^{\circ} = 15$ and $y_{34}^{\circ} = 3$

Now, the LBITP of the given problem with the upper bounded constraints is given below:

					Supply
	1	1	5	4	7
	1	7	2	3	17
	7	7	3	5	16
Demand	10	2	13	15	40

and also, $x_{ij} \leq y_{ij}^{\circ},\ i = 1, 2, 3, \cdots m$ and $j = 1, 2, 3, \cdots n$ and are integers.

Now, using the zero suffix method, the optimal solution to the LBITP with the upper bounded constraints is $x_{11}^{\circ} = 5, x_{12}^{\circ} = 2, x_{21}^{\circ} = 5, x_{24}^{\circ} = 12, x_{33}^{\circ} = 13, x_{34}^{\circ} = 3$.

Thus an optimal solution to the given FIITP is $[x_{11}^{\circ}, y_{11}^{\circ}] = [5, 5], [x_{12}^{\circ}, y_{12}^{\circ}] = [2, 4], [x_{21}^{\circ}, y_{21}^{\circ}] = [5, 7], [x_{24}^{\circ}, y_{24}^{\circ}] = [12, 14], [x_{33}^{\circ}, y_{33}^{\circ}] = [13, 15], [x_{34}^{\circ}, y_{34}^{\circ}] = [3, 3]$ and also the minimum transportation cost is $[102, 202]$.

7 Conclusion

Thus the separation method based on the zero suffix method provides an optimal value of the objective function for the fully interval transportation problem. This method is a systematic procedure, both easy to understand and to apply and also it is a non-fuzzy method. The proposed method provides more options and can be served an important tool for the decision makers when they are handling various types of logistic problems having interval parameters.

8 Open problem

This idea can be extended in the computer science field specifically in net working area and data logistic management.

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