Wind Turbine Power Calculations

RWE npower renewables

Mechanical and Electrical Engineering

Power Industry

INTRODUCTION

RWE npower is a leading integrated UK energy company and is part of the RWE Group, one of Europe's leading utilities. We own and operate a diverse portfolio of power plant, including gas-fired combined cycle gas turbine, oil, and coal fired power stations, along with Combined Heat and Power plants on industrial site that supply both electrical power and heat. RWE npower also has a strong in-house operations and engineering capability that supports our existing assets and develops new power plant. Our retail business, npower, is one of the UK's largest suppliers of electricity and gas.

In the UK RWE is also at the forefront of producing energy through renewable resources. npower renewables leads the UK wind power market and is a leader in hydroelectric generation. It developed the UK's first major off-shore wind farm, North Hoyle, off the North Wales coast, which began operation in 2003.

Through the RWE Power International brand, RWE npower sells specialist services that cover every aspect of owning and operating a power plant, from construction, commissioning, operations and maintenance to eventual decommissioning.

Figure 1: Wind Turbine at North Wales coast

SCENARIO

Wind turbines work by converting the kinetic energy in the wind first into rotational kinetic energy in the turbine and then electrical energy that can be supplied, via the national grid, for any purpose around the UK. The energy available for conversion mainly depends on the wind speed and the swept area of the turbine. When planning a wind farm it is important to know the expected power and energy output of each wind turbine to be able to calculate its economic viability.

PROBLEM STATEMENT

With the knowledge that it is of critical economic importance to know the power and therefore energy produced by different types of wind turbine in different conditions, in this exemplar we will calculate the rotational kinetic power produced in a wind turbine at its rated wind speed. This is the minimum wind speed at which a wind turbine produces its rated power.

MATHEMATICAL MODEL

The following table shows the definition of various variables used in this model:

E	=	Kinetic Energy (J)	ρ	=	Density (kg/m^3)
m	=	Mass (kg)	A	=	Swept Area (m^2)
v	=	Wind Speed (m/s)	C_p	=	Power Coefficient
P	=	Power (W)	r	=	Radius (m)
$\frac{dm}{dt}$	=	Mass flow rate (kg/s)	x	=	distance (m)
$\frac{dE}{dt}$	=	Energy Flow Rate (J/s)	t	=	time (s)

Under constant acceleration, the kinetic energy of an object having mass m and velocity v is equal to the work done W in displacing that object from rest to a distance s under a force F, i.e.:

$$E = W = Fs$$

According to Newton's Law, we have:

$$F = ma$$

Hence,

$$E = mas \quad \ldots (1)$$

Using the third equation of motion:

$$v^2 = u^2 + 2as$$

we get:

$$a = \frac{\left(v^2 - u^2\right)}{2s}$$

Since the initial velocity of the object is zero, i.e. $u = 0$, we get:

$$a = \frac{v^2}{2s}$$

Substituting it in equation (1), we get that the kinetic energy of a mass in motions is:

$$E = \frac{1}{2}mv^2 \quad \dots (2)$$

The power in the wind is given by the rate of change of energy:

$$P = \frac{dE}{dt} = \frac{1}{2}v^2\frac{dm}{dt} \quad \dots (3)$$

As mass flow rate is given by:

$$\frac{dm}{dt} = \rho A\frac{dx}{dt}$$

and the rate of change of distance is given by:

$$\frac{dx}{dt} = v$$

we get:

$$\frac{dm}{dt} = \rho Av$$

Hence, from equation (3), the power can be defined as:

$$P = \frac{1}{2}\rho Av^3 \quad \dots (4)$$

A German physicist Albert Betz concluded in 1919 that no wind turbine can convert more than 16/27 (59.3%) of the kinetic energy of the wind into mechanical energy turning a rotor. To this day, this is known as the **Betz Limit** or **Betz' Law.** The theoretical maximum **power efficiency** of *any* design of wind turbine is 0.59 (i.e. no more than 59% of the energy carried by the wind can be extracted by a wind turbine). This is called the “power coefficient” and is defined as:

$$C_{p_{max}} = 0.59$$

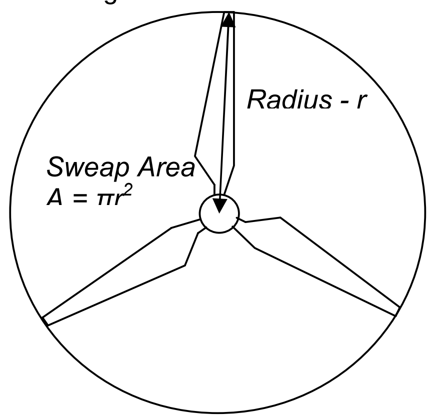
Also, wind turbines cannot operate at this maximum limit. The C_p value is unique to each turbine type and is a function of wind speed that the turbine is operating in. Once we incorporate various engineering requirements of a wind turbine - strength and durability in particular - the real world limit is well below the *Betz Limit* with values of 0.35-0.45 common even in the best designed wind turbines. By the time we take into account the other factors in a complete wind turbine system - e.g. the gearbox, bearings, generator and so on - only 10-30% of the power of the wind is ever actually converted into usable electricity. Hence, the power coefficient needs to be factored in equation (4) and the extractable power from the wind is given by:

$$P_{avail} = \frac{1}{2}\rho Av^3C_p \quad \dots(5)$$

The swept area of the turbine can be calculated from the length of the turbine blades using the equation for the area of a circle:

$$A = \pi r^2 \quad \dots (6)$$

where the radius is equal to the blade length as shown in the figure below:



CALCULATIONS WITH GIVEN DATA

We are given the following data:

Blade length, l	=	52 m
Wind speed, v	=	12 m/sec
Air density, ρ	=	1.23 kg/m^3
Power Coefficient, C_p	=	0.4

Inserting the value for blade length as the radius of the swept area into equation (8) we have:

$$l = r = 52m$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 52^2 \\ &= 8495m^2 \end{aligned}$$

We can then calculate the power converted from the wind into rotational energy in the turbine using equation (7):

$$\begin{aligned} P_{avail} &= \frac{1}{2}\rho Av^3C_p \\ &= \frac{1}{2} \times 123 \times 8495 \times 12^3 \times 0.4 \\ &= 3.6MW \end{aligned}$$

CONCLUSION

This value is normally defined by the turbine designers but it is important to understand the relationship between all of these factors and to use this equation to calculate the power at wind speeds other than the rated wind speed.

Having knowledge of how a turbine behaves in different wind speeds is critical to understand the income lost by any down time of the turbine. It is also useful to understand what power a turbine should be producing so that if there is a problem with the turbine this can be picked up on due to lower than estimated energy values.

Predictions of how much energy will be produced by a turbine are important to the energy market, as energy is sold before it is actually produced. This means that accurate calculations of the energy are very important to balancing the energy in the market and to forecasting a company's income.

EXTENSION ACTIVITY – 1:

Repeat the above calculation using different wind speeds and plot a graph of the power produced at these different values.

What is the relationship between wind speed, v, and power, P?

Please note: This activity assumes that C_p is constant, whereas it is actually a function of wind speed. This will be explored further in Extension Activity 3.

EXTENSION ACTIVITY – 2:

Marine turbines are designed using the same principles as wind turbines. However, they are used in the different conditions and the variables used in the power equation given in equation (5) are also slightly different.

As the marine turbine works in water rather than air, we will use density of water instead of air:

Density of water, ρ_w = 1000 kg/m^3

The average power coefficient, C_p, for marine turbines is also different than that of wind turbines. Currently, the technology for marine turbines is not that much developed to reach the same levels of results as wind turbines. However, the theoretical maximum for marine turbines is still defined by Betz Law with a limit of 0.59 and we will use the following value of this coefficient:

Power Coefficient Marine Turbine, C_{pm} = 0.35

Given this information, rearrange the power equation (5) using marine turbine variables to calculate the length of blade that would be needed to produce the same power by marine turbine as produced by the wind turbine in the example above. Assume v = 2.5m/s, which is the typical rated tidal flow speed.

EXTENSION ACTIVITY – 3:

The power coefficient is not a static value as defined in the main question; it varies with the tip speed ratio of the turbine. Tip speed ratio is defined as:

$$\lambda = \frac{blade\ tip\ speed}{wind\ speed}$$

The blade tip speed can be calculated from the rotational speed of the turbine and the length of the blades used in the turbine using the following equation:

$$blade\ tip\ speed = \frac{rotational\ speed\ (rpm) \times \pi \times D}{60}$$

where D is the diameter of the turbine.

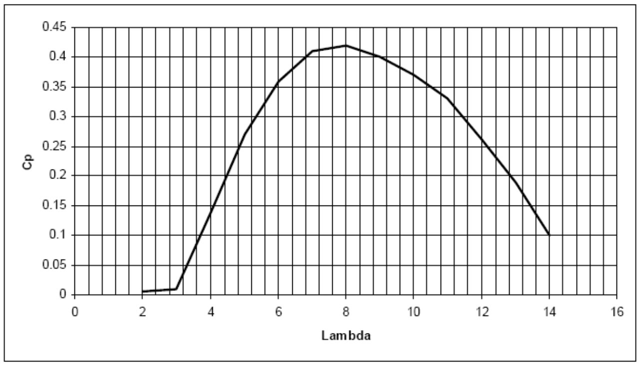
Given that the rotational speed of the turbine is 15rpm, calculate λ using the above two equations and fill it in the following table. Then read the corresponding value of C_p using the graph below. This C_p value can then be used to calculate the power at that wind speed using appropriate form of equation (5). Finally, calculate the energy using the following equation and complete the table:

$$\text{Energy} = \text{Power} \times \text{Time}$$

Please note, there are crosses in the following table where the wind turbine would not operate due to the wind speed being too high or too low.

WIND SPEED (M/S)	TIME (HOURS)	λ VALUE	C_p VALUE	POWER (KW)	ENERGY (KWH)
1	531	X	X	X	X
3	1407	X	X	X	X
5	1831	X	X	X	X
7	1769				
9	1386				
11	913				
13	524				
15	249				
17	105				
19	39				
21	12				
23	3				
25	1	X	X	X	X
27	0	X	X	X	X
TOTAL		X	X		

Table with EA3



C_p vs λ **curve for wind turbine (with EA3)**

WHERE TO FIND MORE

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.

Photo awaited	**Jess, Graduate Engineer, npower renewables** She says: "*Following my degree in Mechanical Engineering with new and renewable energy, I am working for RWE npower renewables as a graduate engineer on our Professional Engineers and Scientists Development Scheme. As a graduate I go on placements around the business including Marine Development, Hydro Commissioning and placements abroad, such as turbine operations and maintenance strategy in Spain. My current team is Turbine Performance where we keep track of the performance of all of our wind turbines. We also help our operations staff with investigations into wind turbine problems.*"

INFORMATION FOR TEACHERS

Keywords:

- Kinetic energy and Power Calculation
- Betz Limit and power coefficient

TOPICS COVERED FROM "MATHEMATICS FOR ENGINEERING"

- Topic 1: Mathematical Models in Engineering
- Topic 2: Models of Growth and Decay
- Topic 5: Geometry
- Topic 6: Differentiation and Integration

LEARNING OUTCOMES

- LO 01: Understand the idea of mathematical modelling
- LO 02: Be familiar with a range of models of change, and growth and decay
- LO 05: Know how 2-D and 3-D coordinate geometry is used to describe lines, planes and conic sections within engineering design and analysis
- LO 06: Know how to use differentiation and integration in the context of engineering analysis and problem solving
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

ASSESSMENT CRITERIA

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 2.3: Set up and solve a differential equation to model a physical situation
- AC 4.1: Identify and describe functions and their graphs
- AC 4.2: Analyse functions represented by polynomial equations
- AC 5.1: Use equations of straight lines, circles, conic sections, and planes
- AC 6.1: Calculate the rate of change of a function
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

LINKS TO OTHER UNITS OF THE ADVANCED DIPLOMA IN ENGINEERING

- Unit-1: Investigating Engineering Business and the Environment
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

ANSWERS TO EXTENSION ACTIVITIES

EA1: Cubic relationship should be shown by graph and seen in the formula

EA2: Blade length = 20.5m

EA3: Total Energy over the year is approximately 10GWh