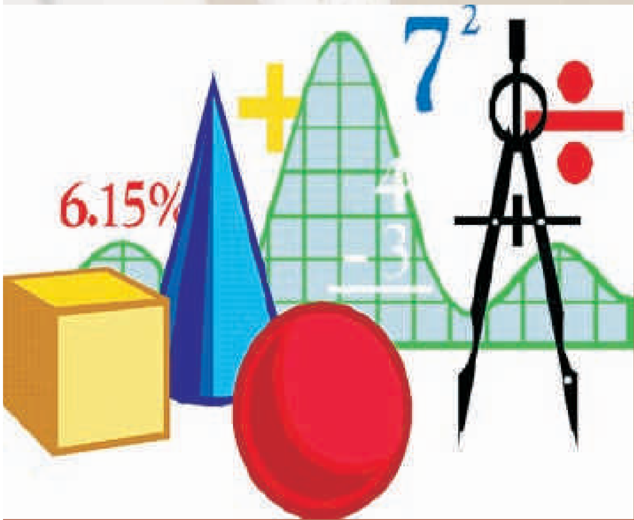
MATHEMATICS



LOGARITHM

Achiever's Comprehensive Course (ACC)

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BASIC MATHEMATICS :

Remainder Theorem :

Let p(x) be any polynomial of degree geater than or equal to one and 'a' be any real number. If p(x) is divided by (x – a), then the remainder is equal to p(a).

Factor Theorem :

Let p(x) be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that p(a) = 0, then (x – a) is a factor of p(x). Conversely, if (x – a) is a factor of p(x), then p(a) = 0.

Note : Let p(x) be any polynomial of degree greater than or equal to one. If leading coefficient of p(x) is 1 then p(x) is called monic. (Leading coefficient means coefficient of highest power.)

SOME IMPORTANT IDENTITIES :

(1) $(a + b)^2 = a^2 + 2ab + b^2 = (a - b)^2 + 4ab$

(2) $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$

(3) $a^2 - b^2 = (a + b)(a - b)$

(4) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(5) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(6) $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 + b^2 - ab)$

(7) $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + b^2 + ab)$

(8) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = a^2 + b^2 + c^2 + 2abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$.

(9) $a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$

(10) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$= \frac{1}{2}(a + b + c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

If $(a + b + c) = 0$, then $a^3 + b^3 + c^2 = 3abc$.

(11) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2) = (a^2 + b^2)(a - b)(a + b)$

(12) If $a, b \ge 0$ then $(a - b) = \left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right)$

(13) $a^4 + a^2 + 1 = (a^4 + 2a^2 + 1) - a^2 = (a^2 + 1)^2 - a^2 = (a^2 + a + 1)(a^2 - a + 1)$

Definition of Indices :

The product of m factors each equal to a is represented by a^m. So, $a^m = a \cdot a \cdot a \, ... \, a$ (m times). Here a is called the base and m is the index (or power or exponent).

Law of Indices :

(1) $a^{m+n} = a^m \cdot a^n$, where m and n are rational numbers.

(2) $a^{-m} = \dfrac{1}{a^m}$, provided $a \neq 0$.

(3) $a^0 = 1$, provided $a \neq 0$.

(4) $a^{m-n} = \dfrac{a^m}{a^n}$, where m and n are rational numbers, $a \neq 0$.

(5) $(a^m)^n = a^{mn}$.

(6) $a^{\frac{p}{q}} = \sqrt[q]{a^p}$

(7) $(ab)^n = a^n b^n$.

Intervals :

Intervals are basically subsets of R (the set of all real numbers) and are commonly used in solving inequaltities. If $a, b \in R$ such that $a < b$, then we can defined four types of intervals as follows :

Name	**Representation**	**Discription.**
Open interval	(a, b)	$\{x : a < x < b\}$ i.e., end points are not included.
Close interval	[a, b]	$\{x : a \le x \le b\}$ i.e., end points are also included. This is possible only when both a and b are finite.
Open-closed interval	(a, b]	$\{x : a < x \le b\}$ i.e., a is excluded and b is included.
Closed-open interval	[a, b)	$\{x : a \le x < b\}$ i.e., a is included and b is excluded.

Note :

(1) The infinite intervals are defined as follows :

(i) $(a, \infty) = \{x : x > a\}$ (ii) $[a, \infty) = \{x \mid x \ge a\}$

(iii) $(-\infty, b) = \{x : x < b\}$ (iv) $(-\infty, b] = \{x : x \le b\}$

(v) $(-\infty, \infty) = \{x : x \in R\}$

(2) $x \in \{1, 2\}$ denotes some particular values of x, i.e., $x = 1, 2$.

(3) If their is no value of x, then we say $x \in \phi$ (i.e., null set or void set or empty set).

Proportion :

When two ratios are equal, then the four quantities compositing then are said to be proportional.

If $\frac{a}{b} = \frac{c}{d}$, then it is written as a : b = c : d or a : b : : c : d.

Note :

(1) a and d are known as extremes while b and c are known as means.

(2) Product of extremes = product of means.

(3) If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$ (Invertando)

(4) If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$ (Alternando)

(5) If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} + 1 = \frac{c}{d} + 1 \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

(6) If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b} - 1 = \frac{c}{d} - 1 \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo).

(7) If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo and dividendo)

(8) If $\frac{a}{b} = \frac{b}{c}$ then $b^2 = ac$. Here b is called mean proportional of a and c.

Histrorical Development of Number System :

I. Natural Number's

Number's used for counting are called as Natural number's.

{1, 2, 3, 4, ...}

II. Whole number's

Including zero (0) | cypher | शून्य | duck |love| knot along with natural numbers called as whole numbers.

w = {0,1, 2, 3 ....}

i.e. $N \subset W$

0 is neither positive nor negative

III Integer's

Integer's given by

I = {..........–2, –1, 0, 1, 2, 3.......}

i.e. $N \subset W \subset I$

Type of Integer's

(a) None negative integers	{ 0, 1, 2, 3, ...}
(b) Negative integers (I^-)	{...........–3, –2, –1}
(c) Non positive integers	{...........–3, –2 –1, 0}
(d) Positve integers (I^+)	{1, 2, 3 ....}

IV. Rational Number's

Number's which are of the form p/q where p, q, $\in$ I & q $\neq$ 0 called as rational number's.

Rational numbers are also represented by recurring & terminating or repeating decimal's

e.g. $1.\overline{3}$ = 1.333 ....

$$x = 1.3333 \ldots$$
$$10x = 13.33\ldots$$
$$9x = 12$$
$$x = \frac{4}{3}$$

Every rational is either a terminating or a recurring decimal

V Irrational number's

The number's which cannot be expressed in the form p/q (p,q $\in$ I) are called as irrational numbers. The decimal representation of these number is non-terminating and non repeating.

$$\sqrt{2} = 1.414 \ \ldots\ldots\ldots\ldots$$

π is an irrational number

VI Real Number's

Set of real number's is union of the set of rational number's and the set of irrational numbers.

$$\text{Real} \rightarrow \text{Rational} + \text{Irrational}$$
$$N \subset W \subset I \subset Q \subset R \subset Z$$

VII. Prime Number's

Number's which are devisible by 1 or itself

e.g. {2, 3, 5, 7, 11, 13 ....}

VIII Composite Number's

Number's which are multiples of prime are called composite number's

{4, 6, 8, 9 ....}

IX Coprime or relatively prime number's

The number's having heighest common factor 1 are called relatively prime.

e.g. (2, 9), (16, 25 ......)

X Twin primes :

The prime number's which having the diffrence of 2

e.g. (5, 3), (7, 5), (13,11) ....

1 is niether a prime nor a composite number.

When studying logarithms it is important to note that all the propertise of logarithms are consequences of the corresponding properties of power, which means that sudent should have a good working knowledge of powers are a foundation for tacking logarithms

LOGARITHM :

Definition : Every positive real number N can be expressed in exponential form as

$N = a^x$ ....(1) ***e.g.*** $49 = 7^2$

where 'a' is also a positive real different than unity and is called the base and 'x' is called the exponent.

We can write the relation (1) in logarithmic form as

$\log_a N = x$ ....(2)

Hence the two relations

$$\left.\begin{array}{r} a^x = N \\ \text{and} \quad \log_a N = x \end{array}\right]$$

are identical where $N > 0$, $a > 0$, $a \neq 1$

Hence logarithm of a number to some base is the exponent by which the base must be raised in order to get that number. Logarithm of zero does not exist and logarithm of (–) ve reals are not defined in the system of real numbers.

i.e a is raised what power to get N

Illustration :

Find value of

(i) $\log_{81} 27$ *(ii)* $\log_{10} 100$ *(iii)* $\log_{1/3} 9\sqrt{3}$

Sol. *(i)* Let $\log_{81} 27 = x$

$\Rightarrow$ $27 = 81^x$

$\Rightarrow$ $3^3 = 3^{4x}$ *gives* $x = 3/4$

(ii) Let $\log_{10} 100 = x$

$\Rightarrow$ $100 = 10^x$

$\Rightarrow$ $10^2 = 10^x$ *gives* $x = 2$

(iii) Let $\log_{1/3} 9\sqrt{3} = x$

$\Rightarrow$ $9\sqrt{3} = \left(\frac{1}{3}\right)^x$

$\Rightarrow$ $3^{5/2} = 3^{-x}$ *gives* $x = -5/2$

Note that :

(a) Unity has been excluded from the base of the logarithm as in this case

$\log_1 N$ will not be possible and if $N = 1$

then $\log_1 1$ will have infinitely many solutions and will not be unique which is necessary in the functional notation.

(b) $a^{\log_a N} = N$ is an identify for all $N > 0$ and $a > 0$, $a \neq 1$ e.g. $2^{\log_2 5} = 5$

(c) The number N in (2) is called the antilog of 'x' to the base 'a'. Hence

If $\log_2 512$ is 9 then $\text{antilog}_2 9$ is equal to $2^9 = 512$

(d) Using the basic definition of log we have 3 important deductions :

(i) $\log_N N = 1$ i.e. logarithm of a number to the same base is 1.

(ii) $\log_{\frac{1}{N}} N = -1$ i.e. logarithm of a number to its reciprocal is – 1.

(iii) $\log_a 1 = 0$ i.e. logarithm of unity to any base is zero.

(basic constraints on number and base must be observed.)

(iv) $a^{\log_a n} = n$ is an identify for all N > 0 and a> 0 ; a ≠ 1 e.g. $2^{\log_2 5} = 5$

(e) Whenever the number and base are on the same side of unity then logarithm of that number to the base is (+ve), however if the number and base are located an diffrent side of unity then logarithm of that number to the base is (–ve)

e.g. (i) $\log_{10}100 = 2$

(ii) $\log_{1/10}100 = -2$

(f) For a non negative number 'a' & n ≥ 2, n ∈ N $\sqrt[n]{a} = a^{1/n}$

Illustration :

(i) $\log_{\sin 30°} \cos 60^o = 1$ *(ii)* $\log_{3/4} 1.\overline{3} = -1$ *(iii)* $\log_{2-\sqrt{3}} 2+\sqrt{3} = -1$

(iv) $\log_5\sqrt{5\sqrt{5\sqrt{5.......\infty}}} = 1$

Sol. *Let* $\sqrt{5\sqrt{5\sqrt{5.....\infty}}} = x$

$\Rightarrow \sqrt{5x} = x \quad \Rightarrow \quad x^2 = 5x \quad \Rightarrow \quad x = 5 \quad \Rightarrow \quad \log_5 5 = 1$

(v) *(log tan 1º) (log tan 2º) (log tan 3º) ....(log tan 89º) = 0*

Sol. *Since tan 45° = 1 thus log tan 45° = 0*

(vi) $7^{\log_7 x} + 2x + 9 = 0$

Sol. $3x + 9 = 0 \Rightarrow (x = -3)$ *as it makes initial problem undefined*

$x = \phi$

(vii) $2^{\log_2(x-3)} + 2(x-3) - 12 = 0$

Sol. $x - 3 + 2x - 6 - 12 = 0$

$3x = 21 \Rightarrow x = 7$

(viii) $\log_2(x-3) = 4$

Sol. $x-3 = 2^4$

$x = 19$

Practice Problem

Q.1 Find the logarithms of the following numbers to the base 2:

(i) $\sqrt[3]{8}$ (ii) $2\sqrt{2}$ (iii) $\frac{1}{\sqrt[5]{2}}$ (iv) $\frac{1}{\sqrt[7]{8}}$

Q.2 Find the logarithms of the following numbers to the base $\frac{1}{3}$

(i) 81 (ii) $\sqrt[3]{3}$ (iii) $\frac{1}{\sqrt[7]{3}}$ (iv) $9\sqrt{3}$ (v) $\frac{1}{9\sqrt[4]{3}}$

Q.3 Find all number a for which each of the following equalities hold true?

(i) $\log_2 a = 2$ (ii) $\log_{10}\left(a(a+3)\right) = 1$

(iii) $\log_{1/3}(a^2 - 1) = -1$ (iv) $\log_2(a^2 - 5) = 2$

Q.4 Find all values of x for which the following equalities hold true?

(i) $\log_2 x^2 = 1$ (ii) $\log_3 x = \log_3(2-x)$ (iii) $\log_4 x^2 = \log_4 x$

(iv) $\log_{1/2}(2x+1) = \log_{1/2}(x+1)$ (v) $\log_{1/3}(x^2+8) = -2$

Q.5 If $2\left(\sqrt{3+\sqrt{5-\sqrt{13+\sqrt{48}}}}\right) = \sqrt{a} + \sqrt{b}$ where a and b are natural number find (a + b).

Answer key

Q.1 (i) 1, (ii) 3/2, (iii) – 1/5, (iv) – 3/7

Q.2 (i) – 4, (ii) – 1/3, (iii) 1/7, (iv) – 5/2, (v) 9/4

Q.3 (i) 4, (ii) –5, 2, (iii) –2, 2, (iv) –3, 3

Q.4 (i) $\sqrt{2}, -\sqrt{2}$, (ii) 1, (iii) 1, (iv) 0, (v) 1, – 1

Q.5 8

PRINCIPAL PROPERTISE OF LOGARITHM :

If m, n are arbitrary positive real numbers where

$a > 0\ ;\ a \neq 1$

(1) $\log_a m + \log_a n = \log_a mn$ $(m > 0, n > 0)$

Proof: Let $x_1 = \log_a m$; $m = a^x$

$x_2 = \log_a n$; $n = a^{x_2}$

Now $mn = a^x\ ;\ a^{x_2}$

$mn = a^{x_1+x_2}$

$x_1 + x_2 = \log_a mn$

$\log_a m + \log_a n = \log_a mn$

(2) $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\frac{m}{n} = a^{x_1 - x_2}$

$x_1 - x_2 = \log_a \frac{m}{n}$

$\log_a m - \log_a n = \log_a \frac{m}{n}$

Solved Examples

Q.1 Find the value of x satisfying $\log_{10}(2^x + x - 41) = x(1 - \log_{10}5)$.

Sol. We have,

$\log_{10}(2^x + x - 41) = x(1 - \log_{10}5)$

$\Rightarrow$ $\log_{10}(2^x + x - 41) = x\log_{10}2 = \log_{10}(2^x)$

$\Rightarrow$ $2^x + x - 41 = 2^x \Rightarrow x = 41$. Ans.

Q.2 If the product of the roots of the equation, $x^{\left(\frac{3}{4}\right)(\log_2 x)^2 + \log_2 x - \left(\frac{5}{4}\right)} = \sqrt{2}$ is $\frac{1}{\sqrt[b]{a}}$ (where a, b $\in$ N) then the value of (a + b).

Sol. Take log on both the sides with base 2

$$\left(\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right)\log_2 x = \frac{1}{2}$$

$\log_2 x = y$

$3y^3 + 4y^2 - 5y - 2 = 0 \Rightarrow$ $3y^2(y-1) + 7y(y-1) + 2(y-1) = 0$

$\Rightarrow$ $(y-1)(3y^2 + 7y + 2) = 0$ $\Rightarrow$ $(y-1)(3y+1)(y+2) = 0$

$\Rightarrow$ $y = 1$ or $y = -2$ or $y = \frac{-1}{3}$

$\therefore$ $x = 2;\ \frac{1}{4};\ \frac{1}{2^{1/3}} \Rightarrow x_1x_2x_3 = \frac{1}{\sqrt[3]{16}} \Rightarrow a + b = 19$

Q.3 For $0 < a \neq 1$, find the number of ordered pair (x, y) satisfying the equation $\log_{a^2}|x+y| = \frac{1}{2}$ and $\log_a y - \log_a|x| = \log_{a^2} 4$.

Sol. We have $\log_{a^2}|x+y| = \frac{1}{2}$ $\Rightarrow$ $|x+y| = a$ $\Rightarrow$ $x + y = \pm a$ .....(1)

Also, $\log_a\left(\frac{y}{|x|}\right) = \log_{a^2} 4$ $\Rightarrow$ $y = 2|x|$ .....(2)

If $x > 0$, then $x = \frac{a}{3}$, $y = \frac{2a}{3}$

If $x < 0$, then $y = 2a$, $x = -a$

$\therefore$ possible ordered pairs $= \left(\frac{a}{3}, \frac{2a}{3}\right)$ and $(-a, 2a)$

Q.4 The system of equations

$\log_{10}(2000xy) - \log_{10}x \cdot \log_{10}y = 4$

$\log_{10}(2yz) - \log_{10}y \cdot \log_{10}z = 1$

and $\log_{10}(zx) - \log_{10}z \cdot \log_{10}x = 0$

has two solutions (x_1, y_1, z_1) and (x_2, y_2, z_2). Find $(y_1 + y_2)$.

Sol. From (1),

$3 + \log_{10}(2xy) - \log_{10}x \cdot \log_{10}y = 4$ ....(i)

or $\log_{10}(xy) - \log_{10}x \cdot \log_{10}y = 1 - \log_{10}(2)$

EXERCISE-1 (Exercise for JEE Main)

[SINGLE CORRECT CHOICE TYPE]

Q.1 The sum $\sqrt{\frac{5}{4}+\sqrt{\frac{3}{2}}} + \sqrt{\frac{5}{4}-\sqrt{\frac{3}{2}}}$ is equal to

(A) $\tan\frac{\pi}{3}$ (B) $\cot\frac{\pi}{3}$ (C) $\sec\frac{\pi}{3}$ (D) $\sin\frac{\pi}{3}$ **[3010110650]**

Q.2 For $N > 1$, the product $\frac{1}{\log_2 N}\cdot\frac{1}{\log_N 8}\cdot\frac{1}{\log_{32} N}\cdot\frac{1}{\log_N 128}$ simplifies to

(A) $\frac{3}{7}$ (B) $\frac{3}{7\ln 2}$ (C) $\frac{3}{5\ln 2}$ (D) $\frac{5}{21}$ **[3010110244]**

Q.3 If p is the smallest value of x satisfying the equation $2^x + \frac{15}{2^x} = 8$ then the value of 4^p is equal to

(A) 9 (B) 16 (C) 25 (D) 1 **[3010110950]**

Q.4 The sum of two numbers a and b is $\sqrt{18}$ and their difference is $\sqrt{14}$. The value of $\log_b a$ is equal to

(A) – 1 (B) 2 (C) 1 (D) $\frac{1}{2}$ **[3010112439]**

Q.5 The value of the expression $(\log_{10}2)^3 + \log_{10}8\cdot\log_{10}5 + (\log_{10}5)^3$ is

(A) rational which is less than 1 (B) rational which is greater than 1
(C) equal to 1 (D) an irrational number **[3010111646]**

Q.6 Let $N = 10^{3\log 2 - 2\log\left(\log 10^3\right) + \log\left(\left(\log 10^6\right)^2\right)}$ where base of the logarithm is 10. The characteristic of the logarithm of N to the base 3, is equal to

(A) 2 (B) 3 (C) 4 (D) 5 **[3010112388]**

Q.7 If $x = \frac{\sqrt{10}+\sqrt{2}}{2}$ and $y = \frac{\sqrt{10}-\sqrt{2}}{2}$, then the value of $\log_2(x^2+xy+y^2)$, is equal to

(A) 0 (B) 2 (C) 3 (D) 4 **[3010112337]**

Q.8 Suppose that x < 0. Which of the following is equal to $\left|2x-\sqrt{(x-2)^2}\right|$

(A) x – 2 (B) 3x – 2 (C) 3x + 2 (D) – 3x + 2
[3010112438]

EXERCISE-2 (Exercise for JEE Advanced)

[PARAGRAPH TYPE]

Paragraph for Question no. 1 to 3

A denotes the product xyz where x, y and z satisfy

$$\log_3 x = \log 5 - \log 7$$
$$\log_5 y = \log 7 - \log 3$$
$$\log_7 z = \log 3 - \log 5$$

B denotes the sum of square of solution of the equation

$$\log_2(\log_2 x^6 - 3) - \log_2(\log_2 x^4 - 5) = \log_2 3$$

C denotes characterstic of logarithm

$$\log_2(\log_2 3) - \log_2(\log_4 3) + \log_2(\log_4 5) - \log_2(\log_6 5) + \log_2(\log_6 7) - \log_2(\log_8 7)$$

Q.1 Find value of A + B + C
(A) 18 (B) 34 (C) 32 (D) 24

Q.2 Find $\log_2 A + \log_2 B + \log_2 C$
(A) 5 (B) 6 (C) 7 (D) 4

Q.3 Find | A – B + C |
(A) – 30 (B) 32 (C) 28 (D) 30

[3010112328]

[MULTIPLE CORRECT CHOICE TYPE]

Q.4 Let $N = \dfrac{\log_3 135}{\log_{15} 3} - \dfrac{\log_3 5}{\log_{405} 3}$. Then N is
(A) a natural number (B) coprime with 3
(C) a rational number (D) a composite number **[3010112387]**

Q.5 If $\left(a^{\log_b x}\right)^2 - 5\, x^{\log_b a} + 6 = 0$, where $a > 0$, $b > 0$ & $ab \neq 1$, then the value of x can be equal to
(A) $2^{\log_b a}$ (B) $3^{\log_a b}$ (C) $b^{\log_a 2}$ (D) $a^{\log_b 3}$ **[3010112336]**

Q.6 Which of the following statement(s) is/are true ?
(A) $\log_{10} 2$ lies between $\frac{1}{4}$ and $\frac{1}{3}$ (B) $\log_{\text{cosec}\left(\frac{\pi}{6}\right)}\left(\cos\frac{\pi}{3}\right) = -1$
(C) $e^{\ln(\ln 3)}$ is smaller than 1
(D) $\log_{10} 1 + \frac{1}{2}\log_{10} 3 + \log_{10}\left(2+\sqrt{3}\right) = \log_{10}\left(1+\sqrt{3}+\left(2+\sqrt{3}\right)\right)$ **[3010112432]**

EXERCISE-3 (Miscellaneous Exercise)

Q.1 Let **A** denotes the value of $\log_{10}\left(\frac{ab+\sqrt{(ab)^2-4(a+b)}}{2}\right)$ + $\log_{10}\left(\frac{ab-\sqrt{(ab)^2-4(a+b)}}{2}\right)$

when a = 43 and b = 57

and **B** denotes the value of the expression $\left(2^{\log_6 18}\right)\cdot\left(3^{\log_6 3}\right)$.

Find the value of $(A \cdot B)$. **[3010111267]**

Q.2 (a) If $x = \log_3 4$ and $y = \log_5 3$, find the value of $\log_3 10$ and $\log_3(1.2)$ in terms of x and y.

(b) If $k^{\log_2 5} = 16$, find the value of $k^{(\log_2 5)^2}$. **[3010110921]**

Q.3 If mantissa of a number N to the base 32 is varying from 0.2 to 0.8 both inclusive, and whose characteristic is 1, then find the number of integral values of N. **[3010110177]**

Q.4 For x, y ∈ N, if $3^{2x-y+1} = 3^{y-2x+1} - 8$ and $\log_6\left|2x^2y - xy^2\right| = 1 + \log_{36}(xy)$, then find the absolute value of $(x - y)$. **[3010110550]**

Q.5 Let $\log_2 x + \log_4 y + \log_4 z = 2$

$\log_9 x + \log_3 y + \log_9 z = 2$

and $\log_{16} x + \log_{16} y + \log_4 z = 2$.

Find the value of $\frac{yz}{x}$. **[3010110900]**

Q.6 Find the value of x satisfying $\log_{10}(2^x + x - 41) = x(1 - \log_{10} 5)$. **[3010110220]**

Q.7 Positive numbers x, y and z satisfy $xyz = 10^{81}$ and $(\log_{10}x)(\log_{10}yz) + (\log_{10}y)(\log_{10}z) = 468$.

Find the value of $(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2$

[3010111000]

Q.8 Find the number of integral solution of the equation $\log_{\sqrt{x}}(x + |x-2|) = \log_x(5x - 6 + 5|x-2|)$.

[3010110092]

Q.9 Suppose p, q, r and s ∈ N satisfying the relation $p + \cfrac{1}{q + \cfrac{1}{r + \cfrac{1}{s}}} = \frac{89}{68}$, then find the value of $(pq + rs)$.

[3010110887]

Q.10 If 'x' and 'y' are real numbers such that, $2\log(2y - 3x) = \log x + \log y$, find $\frac{x}{y}$.

[3010110291]

EXERCISE-4

(IIT JEE Previous Year's Questions)

Q.1 The least value of the expression $2\log_{10}x - \log_x(0.01)$, for $x > 1$ is : [IIT 1980]

(A) 10 (B) 2 (C) –0.01 (D) None of these

[3010110151]

Q.2 Solve for x the following equation : [IIT 1987, 3M]

$$\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$$

[3010110279]

Q.3 The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has : [IIT 1989, 2M]

(A) at least one real solution (B) exactly three real solution

(C) exactly one irrational (D) Complex roots

[3010110651]

Q.4 The nuber of solution of $\log_4(x-1) = \log_2(x-3)$ is : [IIT 2001]

(A) 3 (B) 1 (C) 2 (D) 0

[3010110575]

Q.5 Let (x_0, y_0) be the solution of the following equations

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$
$$3^{\ln x} = 2^{\ln y}.$$

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6 [JEE 2011, 3]

[3010111020]

Q.6 The value of $6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}......}}}\right)$ is [JEE 2012, 4]

[3010112474]

ANSWER KEY

EXERCISE-1

Q.1	A	Q.2	D	Q.3	A	Q.4	A	Q.5	C
Q.6	B	Q.7	C	Q.8	D	Q.9	D	Q.10	C
Q.11	C	Q.12	A						

EXERCISE-2

Q.1	B	Q.2	A	Q.3	D	Q.4	A, C	Q.5	B, C
Q.6	A, B, D	Q.7	A, B, C	Q.8	A, C, D				

Q.9 (A) P, (B) P, R, S, (C) P, R, (D) P, Q, R

Q.10 (A) Q, R, S, T; (B) P; (C) Q, R, S, T; (D) P, R, S

EXERCISE-3

Q.1 12 Q.2 (a) $\frac{xy+2}{2y}$, $\frac{xy+2y-2}{2y}$; (b) 625 Q.3 449 Q.4 5

Q.5 54 Q.6 41 Q.7 5625 Q.8 1 Q.9 23

Q.10 4/9 Q.11 (a) 0.5386; $\overline{1}.5386$; $\overline{3}.5386$ (b) 2058 (c) 0.3522 (d) 3

Q.12 (a) 140 (b) 12 (c) 47 Q.13 54 Q.14 2 Q.15 12

Q.17 $x \in [1/3, 3] - \{1\}$ Q.18 $2s + 10s^2 - 3(s^3 + 1)$ Q.19 y = 6

EXERCISE-4

Q.1 D Q.2 x = –1/4 is the only solution Q.3 B Q.4 B

Q.5 C Q.6 4

HINTS & SOLUTIONS

EXERCISE-1 (Exercise for JEE Main)

[SINGLE CORRECT CHOICE TYPE]

1. Let $x = \sqrt{\frac{5}{4}+\sqrt{\frac{3}{2}}} + \sqrt{\frac{5}{4}-\sqrt{\frac{3}{2}}} \Rightarrow x^2 = \frac{5}{2} + 2\sqrt{\frac{25}{16}-\frac{3}{2}} = \frac{5}{2} + 2\cdot\frac{1}{4} = 3$

$\Rightarrow \quad x = \sqrt{3} = \tan\frac{\pi}{3}$.

Alternative :

Let $S = \sqrt{\frac{5}{4}+\frac{\sqrt{24}}{4}} + \sqrt{\frac{5}{4}-\frac{\sqrt{24}}{4}} = \frac{\sqrt{5+2\sqrt{6}}+\sqrt{5-2\sqrt{6}}}{2} = \frac{\left(\sqrt{3}+\sqrt{2}\right)+\left(\sqrt{3}-\sqrt{2}\right)}{2} = \sqrt{3}$.**Ans.**

2. $\frac{1}{\log_2 N}\cdot\frac{1}{\log_N 8}\cdot\frac{1}{\log_{32} N}\cdot\frac{1}{\log_N 128} = \frac{ln 2}{ln N}\cdot\frac{ln N}{3 ln 2}\cdot\frac{5 ln 2}{ln N}\cdot\frac{ln N}{7 ln 2} = \frac{5}{21}$ **Ans.**

3. We have,

$2^{2x} - 8\cdot 2^x + 15 = 0 \Rightarrow (2^x - 3)(2^x - 5) = 0 \Rightarrow 2^x = 3$ or $2^x = 5$

Hence smallest x is obtained by equating $2^x = 3 \Rightarrow x = \log_2 3$

So, $p = \log_2 3$

Hence, $4^p = 2^{2\log_2 3} = 2^{\log_2 9} = 9$. **Ans.**

4. We have, $a + b = \sqrt{18}$

$a - b = \sqrt{14}$

squaring & subtract, we get $4ab = 4 \Rightarrow ab = 1$

Hence number are reciprocal of each other $\Rightarrow \log_b a = -1$. **Ans.**

5. $\log_{10}2 = a$ and $\log_{10}5 = b \Rightarrow a + b = 1; \; a^3 + 3ab + b^3 = ?$

Now $(a + b)^3 = 1 \Rightarrow a^3 + b^3 + 3ab = 1 \Rightarrow$ (C)

6. $N = 10^p$; $p = \log_{10}8 - \log_{10}9 + 2\log_{10}6$

$p = \log\left(\frac{8\cdot 36}{9}\right) = \log_{10}32$

$\therefore N = 10^{\log_{10}32} = 32$

Hence characteristic of $\log_3 32$ is 3. **Ans.**

7. $\log_2\left((x+y)^2 - xy\right)$

but $x + y = \sqrt{10}$; $x - y = \sqrt{2}$; $xy = \frac{10-2}{4} = 2$

$\log_2(10-2) = \log_2 8 = 3$ Ans.

8. $y = \left|2x - |x-2|\right| = \left|2x-(2-x)\right| = |3x-2|$ as $x < 0$ hence $y = 2 - 3x$ **Ans.**

9. $N = \left(2^{\log_{70}\left((70)^2\times 2\right)}\right)\left(5^{\log_{70}(70\times 2)}\right)\left(7^{\log_{70}2}\right) = \left(2^{2+\log_{70}2}\right)\left(5^{1+\log_{70}2}\right)\left(7^{\log_{70}2}\right)$

$= 20\,(2\times 5\times 7)^{\log_{70}2} = 20\left(70^{\log_{70}2}\right) = 20\times 2 = 40.$ **Ans.**

10. Clearly, $p^{\frac{\log_q(\log_q r)}{(\log_q p)}} = p^{\log_p(\log_q r)} = \log_q r$

and let $y = \sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}$, $y > 0 \Rightarrow y = \sqrt{6+y} \Rightarrow y^2 = 6 + y$

$\Rightarrow$ $y^2 - y - 6 = 0 \Rightarrow (y-3)(y+2) = 0$

But $y > 0$, so $y = 3$.

$\therefore$ Given expression $\log_3(\log_q r)$

$= q^{3^{\log_3(\log_q r)}} = q^{(\log_q r)} = r$. **Ans.**

11. As, $\dfrac{1}{\log_a\left(2-\sqrt{3}\right)} + \dfrac{1}{\log_b\left(\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right)} = \log_{2-\sqrt{3}} a + \log_{\frac{\sqrt{3}-1}{\sqrt{3}+1}} b$

$= \log_{2-\sqrt{3}} a + \log_{2-\sqrt{3}} b = \log_{2-\sqrt{3}}(ab)$

Now, $\left(2+\sqrt{3}\right)^{\log_{2-\sqrt{3}}(ab)} = \dfrac{1}{12} \Rightarrow \left(2-\sqrt{3}\right)^{\log_{2-\sqrt{3}}\left(\frac{1}{ab}\right)} = \dfrac{1}{12}$

$\Rightarrow \dfrac{1}{ab} = \dfrac{1}{12} \Rightarrow ab = 12$

As a, b are co-prime numbers, so either $a = 4, b = 3$ or $a = 3, b = 4$.

Hence , $(a + b) = 7$. Ans.

12. $2^{(\log_2 3)^x} = 3^{(\log_3 2)^x}$

Taking log to the base 2 on both the sides, we get

$(\log_2 3)^x \cdot \log_2 2 = (\log_3 2)^x \log_2 3$

$(\log_2 3)^{x-1} = (\log_3 2)^x \Rightarrow \dfrac{(\log_2 3)^{x-1}}{(\log_3 2)^x} = 1$

$(\log_2 3)^{2x-1} = 1 = (\log_2 3)^0$

$\Rightarrow$ $2x - 1 = 0$ $\Rightarrow$ $x = \dfrac{1}{2}$ **Ans.**

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Email: admin@bansal.ac.in