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SYSTEM IDENTIFICATION

A Frequency Domain Approach

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To all people of goodwill

Rik

To Annick, Ine, Maarten, and Sanne

Johan

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Preface

Identification is a powerful technique for building accurate models of complex systems from noisy data. It consists of three basic steps, which are interrelated: (1) the design of an experiment; (2) the construction of a model, black box or from physical laws; and (3) the estimation of the model parameters from the measurements. The art of modeling lies in proper use of the skills and specialized knowledge of experts in the field of study, who decide what approximations can be made, suggest how to manipulate the system, reveal the important aspects, and so on. Consequently, modeling should preferably be executed by these experts themselves. Naturally, they require relevant tools for extracting information of interest. However, most experts will not be familiar with identification theory and will struggle in each new situation with the same difficulties while developing their own identification techniques, losing time over problems already solved in the literature of identification.

This book presents a thorough description of methods to model linear dynamic time-invariant systems by their transfer function. The relations between the transfer function and the physical parameters of the system are very dependent upon the specific problem. Because transfer function models are generally valid, we have restricted the scope of the book to these alone, so as to develop and study general purpose identification techniques. This should not be unnecessarily restricting for readers who are more interested in the physical parameters of a system: the transfer function still contains all the information that is available in the measurements, and it can be considered to be an intermediate model between the measurements and the physical parameters. Also, the transfer function model is very suitable for those readers looking for a black box description of the input-output relations of a system. And, of course, the model is directly applicable to predict the output of the system.

In this book, we use mainly frequency domain representations of the data. In combination with periodic excitations, this opens many possibilities to identify continuous-time (Laplace-domain) or discrete-time (z-domain) models, if necessary extended with an arbitrary and unknown delay. Although we strongly advocate using periodic excitations, we also extend the methods and models to deal with arbitrary excitations. The "classical" time-domain identification methods that are specifically directed toward these signals are briefly covered and encapsulated in the identification framework that we offer to the reader.

This book provides answers to questions at different levels, such as: What is identification and why do I need it? How to measure the frequency response function of a linear dynamic system? How to identify a dynamic system? All these are very basic questions, directly focused on the interests of the practitioner. Especially for these readers, we have added guidelines to many chapters for the user, giving explicit and clear advice on what are good choices in order to attain a sound solution. Another important part of the material is intended for readers who want to study identification techniques at a more profound level. Questions on how to analyze and prove the properties of an identification scheme are addressed in this part. This study is not restricted to the identification of linear dynamic systems; it is valid for a very wide class of weighted, nonlinear least squares estimators. As such, this book provides a great deal of information for readers who want to set up their own identification scheme to solve their specific problem.

The structure of the book can be split into four parts: (1) collection of raw data or nonparametric identification; (2) parametric identification; (3) comparison with existing frameworks, guidelines, and illustrations; (4) profound development of theoretical tools.

In the first part, after the introductory chapter on identification, we discuss the collection of the raw data: How to measure a frequency response function of a system. What is the impact of nonlinear distortions? How to recognize, qualify, and quantify nonlinear distortions. How to select the excitation signals in order to get the best measurements. This nonparametric approach to identification is discussed in detail in Chapters 2, 3, and 4.

In the second part, we focus on the identification of parametric models. Signal and system models are presented, using a frequency and a time domain representation. The equivalence and impact of leakage effects and initial conditions are shown. Nonparametric and parametric noise models are introduced. The estimation of the parameters in these models is studied in detail. Weighted (nonlinear) least squares methods, maximum likelihood, and subspace methods are discussed and analyzed. First, we assume that the disturbing noise model is known; next, the methods are extended to the more realistic situation of unknown noise models that have to be extracted from the data, together with the system model. Special attention is paid to the numerical conditioning of the sets of equations to be solved. Taking some precautions, very high order systems, with 100 poles and zeros or even more, can be identified. Finally, validation tools to verify the quality of the models are explained. The presence of unmodeled dynamics or nonlinear distortions is detected, and simple rules to guide even the inexperienced user to a good solution are given. This material is presented in Chapters 5 to 9.

The third part begins with an extensive comparison of what is classically called *time and frequency domain identification*. It is shown that, basically, both approaches are equivalent, but some questions are more naturally answered in one domain instead of the other. The most important question is periodic excitations versus nonperiodic or arbitrary excitations. Next, we provide the practitioner with detailed guidelines to help avoid pitfalls from the very beginning of the process (collecting the raw data), over the selection of appropriate identification methods until the model validation. Finally, we illustrate many of the developed ideas in a wide variety of examples from different fields. This part covers Chapters 10, 11, and 12.

The last part of the book is intended for readers who want to acquire a thorough understanding of the material or those who want to develop their own identification scheme. Not only do we give an introduction to the stochastic concepts we use, but we also show, in a structured approach, how to prove the properties of an estimator. This avoids the need for each freshman in this field to find out, time and again, the basic steps to solve such a problem. Starting from this background, a general but detailed framework is set up to analyze the properties of nonlinear least squares estimators with deterministic and stochastic weighting.

For the special and quite important class of semilinear models, it is possible to make this analysis in much more detail. This material is covered in Chapters 13 to 18.

It is possible to extract a number of undergraduate courses from this book. In most of the chapters that can be used in these courses, we added exercises that introduce the students to the typical problems that appear when applying the methods to solve practical problems.

A first, quite general undergraduate course subject is the measurement of frequency response functions of dynamic systems, as discussed in Chapters 2 to 4.

A second possibility is a first introduction to the identification of linear dynamic systems. Such an undergraduate course should include Chapter 1 and some selected parts of Chapters 5, 6, 7, 8, and 9.

A last course, at the graduate level, is an advanced course on identification based on the methods that are explained in Chapters 15, 16, 17, and 18. This gives an excellent introduction for students who want to develop their own algorithms.

A complete MATLAB® toolbox, which includes the techniques developed in this book, is available. It can be used with a graphical user interface, avoiding most problems and nasty questions for the inexperienced user. At the basic level, this toolbox produces almost autonomously a good model. At the intermediate or advanced level, the user obtains access to some of the parameters in order to optimize the operation of the toolbox to solve dedicated modeling problem. Finally, for those who want to use it as a research tool, there is also a command level that gives full access to all the parameters that can be set to optimize and influence the behavior of the algorithms. More information on this package can be obtained by sending an E-mail to one of the authors: rik.pintelon@vub.ac.be or johan.schoukens@vub.ac.be

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Rik Pintelon

Johan Schoukens

List of Operators and Notational Conventions

$\mathbb{A}$	outline uppercase font denotes a set, for example, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ are, respectively, the natural, the integer, the rational, the real, and the complex numbers
$\otimes$	the Kronecker matrix product
$*$	convolution operator
Re( )	real part of
Im( )	imaginary part of
$\arg\min_x f(x)$	the minimizing argument of $f(x)$
$O(x)$	an arbitrary function with the property $\lim_{x \to 0} \lvert O(x)/x \rvert < \infty$
$o(x)$	an arbitrary function with the property $\lim_{x \to 0} \lvert o(x)/x \rvert = 0$
$\hat{\theta}$	estimated value of θ
$\bar{x}$	complex conjugate of x
subscript 0	true value
subscript Re	$A_{\mathrm{Re}} = \begin{bmatrix} \mathrm{Re}(A) & -\mathrm{Im}(A) \\ \mathrm{Im}(A) & \mathrm{Re}(A) \end{bmatrix}$
subscript re	$A_{\mathrm{re}} = \begin{bmatrix} \mathrm{Re}(A) \\ \mathrm{Im}(A) \end{bmatrix}$
subscript u	with respect to the input of the system
subscript y	with respect to the output of the system
subscript *	limiting estimate
superscript T	matrix transpose

superscript $-T$	transpose of the inverse matrix
superscript H	Hermitian transpose: complex conjugate transpose of a matrix
superscript $-H$	Hermitian transpose of the inverse matrix
superscript +	Moore-Penrose pseudoinverse
superscript $\perp$	orthogonal complement of a subspace or a matrix
$\angle x$	phase (argument) of the complex number x
$X_{[i]}(s)$	ith entry of the vector function $X(s)$
$A_{[i,j]}(s)$	i, jth entry of the matrix function $A(s)$
$A_{[:,j]}$	jth column of A
$A_{[i,:]}$	ith row of A
$X^{[k]}(s)$	kth realization of a random process $X(s)$
$\lambda(A)$	eigenvalue of a square matrix A
$\sigma(A)$	singular value of an $n \times m$ matrix A
$\kappa(A) = (\max_i \sigma_i(A))/(\min_i \sigma_i(A))$	condition number of an $n \times m$ matrix A
$\|x\| = \sqrt{(\mathrm{Re}(x))^2 + (\mathrm{Im}(x))^2}$	magnitude of a complex number x
$\\|A\\|_1 = \max_{1 \le j \le m} \sum_{i=1}^{n} \|A_{[i,j]}\|$	1-norm of an $n \times m$ matrix A
$\\|A\\|_2 = \max_{1 \le i \le m} \sigma_i(A)$	2-norm of an $n \times m$ $(n \ge m)$ matrix A
$\\|X\\|_2 = \sqrt{X^H X}$	2-norm of the column vector X
$\\|A\\|_\infty = \max_{1 \le i \le n} \sum_{j=1}^{m} \|A_{[i,j]}\|$	∞-norm of an $n \times m$ matrix A
$\\|A\\|_F = \sqrt{\mathrm{tr}(A^H A)}$	Frobenius norm of an $n \times m$ matrix A
$\mathrm{diag}(A_1, A_2, \ldots, A_K)$	block diagonal matrix with blocks A_k, $k = 1, 2, \ldots, K$
$\mathrm{herm}(A) = (A + A^H)/2$	Hermitian symmetric part of an $n \times m$ matrix A
$\mathrm{null}(A)$	null space of the $n \times m$ matrix A, linear subspace of $\mathbb{C}^m$ defined by $Ax = 0$
$\mathrm{range}(A)$	range of the $n \times m$ matrix A, linear subspace of $\mathbb{C}^n$ that is reachable by making linear combinations of the columns of A $(\mathrm{range}(A) = (\mathrm{null}(A^T))^{\perp})$
$\mathrm{rank}(A)$	rank of the $n \times m$ matrix A, maximum number of linear independent rows (columns) of A

$\text{span}\{a_1, a_2, \ldots, a_m\}$	the span of the vectors $a_1, a_2, \ldots, a_m$ is the linear subspace obtained by making all possible linear combinations of $a_1, a_2, \ldots, a_m$
$\text{tr}(A) = \sum_{i=1}^{n} A_{[i,i]}$	trace of an $n \times n$ matrix A
$\text{vec}(A)$	a column vector formed by stacking the columns of the matrix A on top of each other
a.s.lim	almost sure limit, limit with probability one
l.i.m.	limit in mean square
plim	limit in probability
Lim	limit in distribution
$\mathcal{E}\{\ \}$	mathematical expectation
Prob( )	probability
$b_X = X - \mathcal{E}\{X\}$	bias of the estimate X
$\text{Cov}(X, Y) = \mathcal{E}\{(X - \mathcal{E}\{X\})(Y - \mathcal{E}\{Y\})^H\}$	cross-covariance matrix of X and Y
$\text{covar}(x, y) = \mathcal{E}\{(x - \mathcal{E}\{x\})\overline{(y - \mathcal{E}\{y\})}\}$	covariance of x and y
cum( )	cumulant
$\text{var}(x) = \mathcal{E}\{\lvert x - \mathcal{E}\{x\}\rvert^2\}$	variance of x
$C_X = \text{Cov}(X) = \text{Cov}(X, X)$	covariance matrix of X
$\hat{C}_X = \frac{1}{M-1}\sum_{m=1}^{M}(X^{[m]} - \hat{X})(X^{[m]} - \hat{X})^H$	sample covariance matrix of M realizations of X
$C_{XY} = \text{Cov}(X, Y)$	cross-covariance matrix of X and Y
$\hat{C}_{XY} = \frac{1}{M-1}\sum_{m=1}^{M}(X^{[m]} - \hat{X})(Y^{[m]} - \hat{Y})^H$	sample cross-covariance matrix of M realizations of X and Y
$CR(X)$	Cramér-Rao lower bound on X
$\text{DFT}(x(t))$	discrete Fourier transform of the samples $x(t)$, $t = 0, 1, \ldots, N-1$
$Fi(X)$	Fisher information matrix with respect to the parameters X
I_m	$m \times m$ identity matrix
q	backward shift operator: $qu(kT_s) = u((k-1)T_s)$
$\text{MSE}(X) = \mathcal{E}\{(X - X_0)(X - X_0)^H\}$	mean square error of the estimate X
$R_{xx}(\tau) = \mathcal{E}\{x(t)x^H(t-\tau)\}$	autocorrelation of $x(t)$
$R_{xy}(\tau) = \mathcal{E}\{x(t)y^H(t-\tau)\}$	cross-correlation of $x(t)$ and $y(t)$
$S_{XX}(j\omega)$	Fourier transform of $R_{xx}(\tau)$ (autopower spectrum of $x(t)$)
$S_{XY}(j\omega)$	Fourier transform of $R_{xy}(\tau)$ (cross-power spectrum of $x(t)$ and $y(t)$)

$\hat{X} = \frac{1}{M}\sum_{m=1}^{M} X^{[m]}$	sample mean of M realizations (experiments) of X
$\mu_x = \mathcal{E}\{x\}$	mean value of x
$\sigma_x^2 = \text{var}(x)$	variance of the x
$\hat{\sigma}_x^2 = \frac{1}{M-1}\sum_{m=1}^{M} \lvert x^{[m]} - \hat{x}\rvert^2$	sample variance of M realizations of x
$\sigma_{xy}^2 = \text{covar}(x, y)$	covariance of x and y
$\hat{\sigma}_{xy}^2 = \frac{1}{M-1}\sum_{m=1}^{M} (x^{[m]} - \hat{x})(\overline{y^{[m]} - \hat{y}})$	sample covariance of M realizations of x and y

List of Symbols

$A(\Omega, \theta) = \sum_{r=0}^{n_p} a_r p_r(\Omega)$	denominator polynomial plant model expanded in the polynomial basis $p_r(\Omega)$
$A(\Omega, \theta) = \sum_{r=0}^{n_a} a_r \Omega^r$	denominator polynomial plant model
$B(\Omega, \theta) = \sum_{r=0}^{n_q} b_r q_r(\Omega)$	numerator plant model expanded in the polynomial basis $q_r(\Omega)$
$B(\Omega, \theta) = \sum_{r=0}^{n_b} b_r \Omega^r$	numerator polynomial plant model
$C(z^{-1}, \theta) = \sum_{r=0}^{n_c} c_r z^{-r}$	numerator polynomial noise model
$D(z^{-1}, \theta) = \sum_{k=0}^{n_d} d_k z^{-k}$	denominator polynomial noise model
$e(t)$	white noise at time t
$E(k)$	discrete Fourier transform of the samples $e(tT_s)$, $t = 0, 1, \ldots, N-1$
f	frequency
F	number of frequency domain data samples
f_s	sampling frequency
$G(j\omega)$	frequency response function
$G_R(j\omega)$	best linear approximation of a nonlinear plant
$G(\Omega, \theta) = B(\Omega, \theta)/A(\Omega, \theta)$	parametric plant model
$H(z^{-1}, \theta) = C(z^{-1}, \theta)/D(z^{-1}, \theta)$	parametric noise model
$I(\Omega, \theta) = \sum_{r=0}^{n_i} i_r \Omega^r$	polynomial of the initial and the final conditions of the plant model $B(\Omega, \theta)/A(\Omega, \theta)$
j	$j^2 = -1$

$J(z^{-1}, \theta) = \sum_{r=0}^{n_d} j_r z^{-r}$	polynomial of the initial and the final conditions of the noise model $C(z^{-1}, \theta)/D(z^{-1}, \theta)$
M	number of (repeated) experiments
N	number of time domain data samples
n_a, n_b, n_c, n_d, n_i and n_j	order of the polynomials $A(\Omega, \theta)$, $B(\Omega, \theta)$, $C(z^{-1}, \theta)$, $D(z^{-1}, \theta)$, $I(\Omega, \theta)$, and $J(z^{-1}, \theta)$
n_θ	dimension of the parameter vector θ
$n_u(t)$, $n_y(t)$	disturbing time domain noise on the input $u(t)$ and output $y(t)$ signals, respectively
$N_U(k)$, $N_Y(k)$	discrete Fourier transform of the samples $n_u(tT_s)$ and $n_y(tT_s)$, $k = 0, 1, \ldots, N-1$, respectively
s	Laplace transform variable
s_k	Laplace transform variable evaluated along the imaginary axis at DFT frequency k: $s_k = j\omega_k$
$T(\Omega, \theta) = I(\Omega, \theta)/A(\Omega, \theta)$	parametric transient model of the plant $B(\Omega, \theta)/A(\Omega, \theta)$
t	continuous or discrete time variable
T_s	sampling period
$U(e^{j\omega T_s})$, $Y(e^{j\omega T_s})$	Fourier transform of $u(tT_s)$ and $y(tT_s)$
$U(k)$, $Y(k)$	discrete Fourier transform of the samples $u(tT_s)$ and $y(tT_s)$, $t = 0, 1, \ldots, N-1$
U_k, Y_k	Fourier coefficients of the periodic signals $u(t)$, $y(t)$
$U(j\omega)$, $Y(j\omega)$	Fourier transform of $u(t)$ and $y(t)$
$U(s)$, $Y(s)$	one-sided Laplace transform of $u(t)$ and $y(t)$
$u(t)$, $y(t)$	input and output time signals
$U(z)$, $Y(z)$	one-sided Z-transform of $u(tT_s)$, $y(tT_s)$
$V_*(\theta)$	asymptotic $(F \to \infty)$ cost function
$V_F(\theta, z)$	cost function based on F measurements
$V_F'(\theta, z)$	derivative cost function w.r.t. θ (dimension $1 \times n_\theta$)
$V_F''(\theta, z)$	second-order derivative (Hessian) cost function w.r.t. θ (dimension $n_\theta \times n_\theta$)
$Z(k) = [Y(k) U(k)]^T$	data vector containing the measured input and output (DFT) spectra at (DFT) frequency k
$Z = [Z^T(1) Z^T(2) \ldots Z^T(F)]^T$	data vector containing the measured input and output DFT spectra (dimension $2F$)

z	Z-transform variable
z_k	Z-transform variable evaluated along the unit circle at DFT frequency k: $z_k = e^{j\omega_k T_s} = e^{j2\pi k/N}$
$\varepsilon(\theta, Z)$	column vector of the (weighted) model residuals (dimension F)
θ	column vector of the model parameters
$\tilde{\theta}(Z_0)$	minimizing argument of the cost function $V_F(\theta)$
$\hat{\theta}(Z)$	estimated model parameters, minimizing argument of the cost function $V_F(\theta, Z)$
$\hat{\underline{\theta}}(Z)$	truncated estimator
$\sigma_U^2(k) = \text{var}(U(k))$	variance of the measured input DFT spectrum
$\sigma_Y^2(k) = \text{var}(Y(k))$	variance of the measured output DFT spectrum
$\sigma_{YU}^2(k) = \text{covar}(Y(k), U(k))$	covariance of the measured output and input DFT spectra
τ	time delay (normalized with the sampling period for discrete time systems)
$J(\theta, Z) = \partial\varepsilon(\theta, Z)/\partial\theta$	gradient of residuals $\varepsilon(\theta, Z)$ w.r.t. the parameters θ (dimension $F \times n_\theta$)
$\omega = 2\pi f$	angular frequency
Ω	generalized transform variable: Laplace domain $\Omega = s$, Z-domain $\Omega = z^{-1}$, Richardson domain $\Omega = \tanh(\tau_R s)$, and diffusion phenomena $\Omega = \sqrt{s}$
Ω_k	generalized transform variable evaluated at DFT frequency k: Laplace domain $\Omega_k = j\omega_k$, Z-domain $\Omega_k = e^{-j\omega_k T_s}$, Richardson domain $\Omega_k = \tanh(\tau_R j\omega_k)$, and diffusion phenomena $\Omega_k = \sqrt{j\omega_k}$, with $\omega_k = 2\pi k/N$

List of Abbreviations

ARMA	AutoRegressive Moving Average
ARMAX	AutoRegressive Moving Average with eXternal input
ARX	AutoRegressive with eXternal input
BJ	Box-Jenkins (model structure)
BTLS	Bootstrapped Total Least Squares
CRB	Cramér-Rao bound for biased estimators
DFT	Discrete Fourier Transform
DUT	Device Under Test
EV	Errors-in-Variables
FFT	Fast Fourier Transform
FRF	Frequency Response Function
GSVD	Generalized Singular Value Decomposition
GTLS	Generalized Total Least Squares
iid	independent identically distributed
IV	Instrumental Variables
IWLS	Iterative weighted linear least squares
IQML	Iterative Quadratic Maximum Likelihood
LS	Least Squares
ML	Maximum Likelihood
NLS	Nonlinear Least Squares
NLS-FRF	Nonlinear Least Squares based on FRF measurements
NLS-IO	Nonlinear Least Squares based on Input-Output measurements

OE	Output Error (model structure)
pdf	probability density function
PE	Prediction Error
rms	root mean square value
SBTLS	sample BTLS
SGTLS	sample GTLS
SISO	Single Input, Single Output
SML	sample ML
SNR	Signal-to-Noise Ratio
SSUB	sample SUB
SUB	subspace
SVD	Singular Value Decomposition
TLS	Total Least Squares
UCRB	Cramér-Rao Bound for Unbiased estimators
w.p.1	with probability one
WGTLS	Weighted Generalized Total Least Squares
WLS	Weighted Least Squares