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# New Dissimilarity Measures in Evidence Theory

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***Abstract*—The dissimilarity of evidence, which represent the degree of dissimilarity between bodies of evidence (BOE's), has attracted more and more research interests and has been used in many applications based on evidence theory. In this paper, some novel dissimilarities of evidence are proposed by using fuzzy sets theory (FST). The basic belief assignments (bba's) are first transformed to the measures in FST and then by using the dissimilarity (or similarity measure) in FST, the dissimilarities between bba's are defined. Some numerical examples are provided to verify the rationality of the proposed dissimilarities of evidence.**

**Keywords: Evidence theory, dissimilarity, distance of evidence, fuzzy sets theory.**

## I. Introduction

Dempster-Shafer evidence theory [1], which can effectively represent the uncertain information and implement the evidence combination, has been widely used in many applications in the fields of information fusion. With the development of evidence theory, several refined or extended evidence theories have emerged, e.g. the transferable belief model (TBM) [2] and DSmT [3], which can counteract some drawbacks of traditional evidence theory.

Recently, the research on dissimilarity measures in evidence theory has attracted more and more interest [4]. The dissimilarity measure of evidence can describe the degree of dissimilarity or similarity between bodies of evidence (BOE's). Several types of dissimilarities of evidence have been proposed and used in many applications such as performance evaluation [5] [6], sensor reliability evaluation [7], conflict evidence combination [8], conflict modeling in evidence combination [9], target association [10], clustering analysis [11], etc. In 1993, Tessem et al. [12] proposed the betting commitment distance to evaluate the approximations for efficient computation in evidence theory. Two basic belief assignments (bba's) are first transformed into two pignistic probability, then the dissimilarity between the two pignistic probability are calculated by using Minkowski family distance to represent the dissimilarity between their corresponding original bba's. Bauer [13] did the similar work in 1997. In Liu's work [9], she jointly used the betting commitment distance and the conflicting coefficient to represent the conflict between BOE's. Fixsen and Mahler [5] proposed a "classification miss-distance metric" based on the bba's and the Bayesian a priori distribution matrix. Jousselme [6] proposed a distance of evidence based on the geometric interpretation of evidence and Euclidean metrics. In Jousselme's distance, Jaccard coefficient representing the similarity between focal elements, is used. In Deng's work [8], Jousselme's distance is used for modifying the BOE's to suppress the counter-intuitive results in highly conflicting evidence combination according to traditional Dempster's rule. Zouhal and Denoeux [14] defined a mean square distance based on pignistic probabilities transformed from bba's. It can effectively improve the performance of the classifier based on $k$-NN rule and Dempster's rule of combination. In Wen *et al.*'s work [15], a cosine measure is defined to describe the similarity between two bba's. In the work of Liu and Dezert [16], a novel dissimilarity of evidence is defined based on DSmP and Minkowski's distance. There are still other types of dissimilarities of evidence, details can be found in one latest paper of Jousselme [4], which is a good overview of all the available research works of dissimilarities of evidence.

Based on the available researches, we find that there are two types of definition of dissimilarities of evidence. For the first type, the dissimilarity of evidence is calculated directly based on the bba's by mechanical using the distance measures in Euclidean geometry. For example, Jousselme's distance [6], Fixsen and Mahler's similarity [5] and Wen's cosine similarity [15], etc. For the second type, the bba's are first transformed to the probabilities by some certain probabilistic transformation approaches and then the dissimilarity of evidence is calculated indirectly based on the distances between probabilities. For example, the betting commitment distance [12], the DSmP dissimilarity [16] and the mean square distance [14] based on pignistic probability. To define the dissimilarity of evidence directly based on bba's can avoid the loss of information caused by the probability transformation, but it is lack of solid foundation and relatively difficult. This is because to design strict distance metrics in the brand new evidential geometric space is relatively hard and needs lots of original and groundbreaking works. The available dissimilarities of evidence are used for reference from the Euclidean space. Since the geometric interpretation of evidence theory is still lacking of solid justifications, the mechanical use of the distance measures in Euclidean geometry might not be proper. For the second type definitions, although the probability transformation generally yields to a loss of information which has an impact on the measurement precision of dissimilarity between bba's, several simple dissimilarities based on subjective probability have been proposed and are still commonly used. It should be noted that although several works named their definitions of dissimilarity between bba's as "distance", unless they can be proved to satisfy the requirements [6] of nonnegativity, nondegeneracy symmetry and triangle inequality, they can only be called the "dissimilarity" but not the "distance".

In this paper we propose new indirect ways to define dissimilarities between evidences. Our major idea is described as follows. Fuzzy sets theory [17] is also an effective tool to handle uncertainty. Besides the probability, the basic belief assignments (random sets) can also be transformed into fuzzy sets through fuzzy membership functions (FMF's) and the intuitionist fuzzy sets [18]. Furthermore, for fuzzy sets and the intuitionist fuzzy sets, there already exist some dissimilarity (or similarity) measures [20] [21] [22] [23]. Thus we present some new dissimilarity measures between evidences which are defined from the dissimilarity (or similarity) measures in fuzzy sets and intuitionist fuzzy sets. Some numerical examples and related analysis are provided, which show the rationality of our new proposed dissimilarity measures between evidences.

## II. Basics of evidence theory

In Dempster-Shafer evidence theory (DST) [1], all the elements in the frame of discernment (FOD) $\Theta$ are mutually exclusive. A basic belief assignment (bba, also called mass function) is a mapping $m : 2^{\Theta} \to [0,1]$ satisfying:

$$\sum\nolimits_{A\subseteq\Theta} m(A) = 1,\ m(\emptyset) = 0 \tag{1}$$

Belief function and plausibility function are defined respectively in (2) and (3):

$$Bel(A) = \sum\nolimits_{B\subseteq A} m(B) \tag{2}$$

$$Pl(A) = \sum\nolimits_{A\cap B\neq\emptyset} m(B) \tag{3}$$

Dempster's rule of combination for combining two distinct BOE's characterized by bba's $m_1(.)$ and $m_2(.)$ is defined by:

$$m(A) = \begin{cases} 0,\ A=\emptyset \\ \dfrac{\sum\limits_{A_i\cap B_j=A} m_1(A_i)m_2(B_j)}{1-K},\ A\neq\emptyset \end{cases} \tag{4}$$

where coefficient $K = \sum_{A_i\cap B_j=\emptyset} m_1(A_i)m_2(B_j)$ represent the conflict between the BOE's.

Dempster's rule of combination is recommended to do the fusion of BOE's in DST [1]. This rule is both associative and commutative and can be extended for combining $n > 2$ distinct sources of evidence as well. Its behavior is however very questionable as soon as the sources become highly conflicting and that's why other rules have been proposed in the literature [3] for dealing with this unexpected behavior.

## III. Classical dissimilarities between bba's

Dissimilarity measure between bba's is used to represent the degree of dissimilarity between different BOE's. As aforementioned in Section I there have emerged several definitions for dissimilarities between bba's. There are two major types of definitions. The first type is established directly based on bba's. It is based on the geometric interpretation of evidence theory. The second type is indirectly based on probability measures. The bba's are first transformed to subjective probabilities based on some probability transformations. Then a distance between probabilities is used to describe the dissimilarity between BOE's. In this section, we recall two kinds of widely used dissimilarities of evidence including Jousselme's distance (representative of the first type) and betting commitment distance (representative of the second type).

### A. Jousselme's distance

Jousselme [6] proposed a distance of evidence denoted by $d_J$ according to (5) using the form of Euclidean metric.

$$d_J(m_1, m_2) = \sqrt{(m_1 - m_2)^T \mathbf{Jac}\ (m_1 - m_2)} \tag{5}$$

For the element in Jaccard's weighting matrix $\mathbf{Jac}$,

$$\mathbf{Jac}(A, B) = \frac{|A\cap B|}{|A\cup B|} \tag{6}$$

Jousselme's distance[1] has been used in weighted average evidence combination and in characterizing the reliability of information sources [7] [8]. In some work, Jousselme's distance was used to construct another similarity measure [24].

### B. Tessem's distance

In Tessem's work [12], a distance of evidence is based on the pignistic probability transformation used in TBM [2]. Each bba $m(.)$ is transformed to the corresponding pignistic probability according to (7):

$$\mathrm{BetP}_m(A) = \sum\nolimits_{B\subseteq\Theta} \frac{|A\cap B|}{|B|} m(B) \tag{7}$$

where $2^{\Theta}$ is the power set of the FOD. The betting commitment distance (or Tessem's distance) $d_T$ is computed by:

$$d_T(m_1, m_2) = \max_{A\subseteq\Theta}\{|\mathrm{BetP}_1(A) - \mathrm{BetP}_2(A)|\} \tag{8}$$

Tessem's distance has been used in conflicting evidence combination and in Liu's work, it has been jointly used with conflict coefficient $K$ in establishing a two-dimensional measure to better describe the conflict between BOE's. In fact, Tessem's distance belongs to Minkowski's family of distances.

Besides Jousselme's and Tessem's distances, there exist other dissimilarities in evidence theory which all have their own advantages and drawbacks (see details in [6]). Until now, there is no well-admitted dissimilarity measure between bba's. The search for new dissimilarities between bba's is still a main problem for the community working with belief functions.

[1] Actually the measure $d_J(m_1, m_2)$ has not been proved to be a true distance measure in [6] because no proof of the strict positiveness of $\mathbf{Jac}$ matrix has been published so far. We conjecture it is.

## IV. New dissimilarities based on fuzzy sets theory

As shown in the previous section, Tessem's distance is not established directly based on bba's, but through a lossy BetP transformation of bba's into subjective probabilities. The reason for such an operation is that for probability, there exist several well-established definitions of distances or dissimilarities between probabilities. The difficulty for designing the distance or dissimilarity of evidence is thus reduced in comparison to a direct construction of a distance or dissimilarity from bba's (a lossless approach).

Besides the probability, there exist other types of measures of uncertainty, e.g. the fuzzy membership function in fuzzy set theory (FST), etc. If we are able to transform the bba's into a measure of uncertainty used in FST and use the similarity (or dissimilarity) definitions in FST, then dissimilarity of evidence can be also derived. Our contribution in this paper is to construct such dissimilarity between bba's indirectly from FST. Some basics of FST are recalled in next subsection. Then new definitions of dissmilarity will be presented.

### A. Fuzzy sets and fuzzy membership function

The fuzzy sets theory was proposed by Zadeh [17]. Fuzzy mathematics shows its simplicity and power when it is used to process the large-scale and complicated systems which can not be processed precisely. It may complement some defects of classical mathematics. A fuzzy set is defined as follows: Let $\mu_A : \Theta \mapsto [0,1], \theta \mapsto \mu_A(\theta)$ be a given mapping over $\Theta$, then $A$ is called a fuzzy set over $\Theta$ and $\mu_A(\theta)$ is called the fuzzy membership function (FMF) of $A$. When there is no confusion, $\mu_A(\theta)$ is briefly denoted by $\mu(\theta)$ in the sequel.

### B. Intuitionistic fuzzy sets

Intuitionistic fuzzy set has been proposed by Atanassov in [18] and it is one of the possible generalizations of fuzzy sets theory. It appears to be relevant and useful in some applications. An intuitionist fuzzy set is defined as follows: Let $\Theta = \theta_1, \theta_2, ..., \theta_n$ be a finite universal set. An intuitionist fuzzy set $A$ in $\Theta$ is an object of the following form:

$$A = \{< \theta_j, \mu_A(\theta_j), \nu_A(\theta_j) > | \theta_j \in \Theta\} \tag{9}$$

where the functions defined in (10) and (11)

$$\mu_A : \Theta \to [0,1], \theta_j \in \Theta \to \mu_A(x_j) \in [0,1] \tag{10}$$

$$\nu_A : \Theta \to [0,1], \theta_j \in \Theta \to \nu_A(\theta_j) \in [0,1] \tag{11}$$

define the degree of membership and degree of non-membership of the element $\theta_j \in \Theta$ to the set $A \subseteq \Theta$, respectively, and for every $\theta_j \in \Theta$, $0 \leq \mu_A(\theta_j) + \nu_A(\theta_j) \leq 1$. The $\pi_A(\theta_i)$ defined in (12)

$$\pi_A(\theta_i) = 1 - \mu_A(\theta_i) - \nu_A(\theta_i) \tag{12}$$

is called the the intuitionist fuzzy index (or the hesitation degree) of the element $\theta_j$ in the set $A$. When there is no confusion, $\mu_A(\theta)$ and $\nu_A(\theta)$ is briefly denoted by $\mu(\theta)$ and $\nu(\theta)$ in the sequel.

There are several available similarity (or dissimilarity) definitions in fuzzy sets and intuitionist fuzzy sets. There exist relationships between FST and evidence theory (DST) and several works have pointed out these relationships already. If the bba's can be properly transformed into FMF or intuitionist fuzzy sets, the dissimilarity of evidence can be constructed directly by using similarity (or dissimilarity) measures in FST.

### C. New dissimilarities between bba's based on FST

Some methods for converting a given bba $m(.)$ into a FMF $\mu(.)$ and an intuitionist fuzzy set are presented here.

*1) Converting a bba into a FMF*

Let's consider a given FOD $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$. Any bba $m(.)$ defined over $\Theta$ can be transformed into FMF from the plausibility on singletons $\theta_1$, $\theta_2$,…, $\theta_n$ by taking

$$\mu = \begin{bmatrix} \mu(\theta_1) \\ \mu(\theta_2) \\ \vdots \\ \mu(\theta_n) \end{bmatrix} = \begin{bmatrix} Pl(\theta_1) \\ Pl(\theta_2) \\ \vdots \\ Pl(\theta_n) \end{bmatrix} \tag{13}$$

or from the credibility on singletons by taking

$$\mu = \begin{bmatrix} \mu(\theta_1) \\ \mu(\theta_2) \\ \vdots \\ \mu(\theta_n) \end{bmatrix} = \begin{bmatrix} Bel(\theta_1) \\ Bel(\theta_2) \\ \vdots \\ Bel(\theta_n) \end{bmatrix} \tag{14}$$

*2) Converting a bba into an intuitionist fuzzy set*

Recently, there have emerged some research works on the relationship between the intuitionist fuzzy sets and the framework of DST. For a given FOD $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, $m(.)$ is the bba defined over $\Theta$. The bba $m(.)$ can be transformed into a measure of intuitionist fuzzy sets as follows: at first, one calculates the plausibility and belief functions of the singletons $\theta_1$, $\theta_2$, …, $\theta_n$, and then for $\theta_i \in \Theta$, $i = 1, ..., n$, the membership function and non-membership function for an intuitionist fuzzy set are defined as follows [19]:

$$\begin{cases} \mu(\theta_i) = Bel(\theta_i) \\ \nu(\theta_i) = 1 - Pl(\theta_i) \end{cases} \tag{15}$$

For two bba's $m_1(.)$ and $m_2(.)$ defined on the FOD $\Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, by using the transformation approaches and the similarity (or dissimilarity) measures commonly used in fuzzy sets theory, we propose the two following new dissimilarity measures between bba's:

• **FMF-based dissimilarities** $d_F$: One transforms bba's $m_1(.)$ and $m_2(.)$ into their corresponding FMFs: $\mu^{(1)}$ and $\mu^{(2)}$. To avoid zero in denominator in the very special case when all $\theta_i$ are not focal elements, we use FMF based on (13) in the simulations in next section. As soon as at least one singleton $\theta_i$ is a focal element of the bba $m(.)$, then (13) or (14) can be used for generating FMF from $m(.)$. From the similarity definition between FMFs defined in [20], $d_F$ is defined by:

$$d_F(m_1, m_2) = 1 - \frac{\sum_{i=1}^{n} (\mu^{(1)}(\theta_i) \wedge \mu^{(2)}(\theta_i))}{\sum_{i=1}^{n} (\mu^{(1)}(\theta_i) \vee \mu^{(2)}(\theta_i))} \tag{16}$$

In (16), the operator $\wedge$ represent conjunction(min) and $\vee$ represent the disjunction(max). For two FMFs defined on $\{\theta_1, \theta_2, \theta_3\}$, $\mu_1 = [0.5, 0.3, 0.7]$ and $\mu_2 = [0.3, 0.6, 0.2]$, $\mu_1 \wedge \mu_2 = [0.3, 0.3, 0.2]$ and $\mu_1 \vee \mu_2 = [0.5, 0.6, 0.7]$.

• **Intuitionistic fuzzy sets-based dissimilarities** $d_{IF1}$, $d_{IF2}$, or $d_{IF3}$: one transforms $m_1(.)$ and $m_2(.)$ into their corresponding intuitionist fuzzy sets: $\{< \theta_j, \mu^{(1)}, \nu^{(1)} > |\theta_j \in \Theta\}$ and $\{< \theta_j, \mu^{(2)}, \nu^{(2)} > |\theta_j \in \Theta\}$. From the dissimilarity measures between intuitionist fuzzy sets defined in [21] and the similarity measures defined in [22], [23], one defines the new dissimilarity $d_{IF1}$, $d_{IF2}$ and $d_{IF3}$ respectively as follow:

$$d_{IF1}(m_1, m_2) = \frac{1}{2n} \sum_{i=1}^{n} \left( \left|\mu^{(1)}(\theta_i) - \mu^{(2)}(\theta_i)\right| + \left|\nu^{(1)}(\theta_i) - \nu^{(2)}(\theta_i)\right| + \left|\pi^{(1)}(\theta_i) - \pi^{(2)}(\theta_i)\right| \right) \quad (17)$$

$$d_{IF2}(m_1, m_2) = \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} (\varphi^{(1)}(\theta_i) - \varphi^{(2)}(\theta_i))^p} \quad (18)$$

where $\varphi^{(1)}$ and $\varphi^{(2)}$ are given by

$$\varphi^{(1)}(\theta_i) = \frac{\mu^{(1)}(\theta_i) + 1 - \nu^{(1)}(\theta_i)}{2} \quad (19)$$

$$\varphi^{(2)}(\theta_i) = \frac{\mu^{(2)}(\theta_i) + 1 - \nu^{(2)}(\theta_i)}{2} \quad (20)$$

$$d_{IF3}(m_1, m_2) = \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} (\varphi_{\mu}^{(1,2)}(\theta_i) - \varphi_{\nu}^{(1,2)}(\theta_i))^p} \quad (21)$$

where $\varphi_{\mu}^{(1,2)}$ and $\varphi_{\nu}^{(1,2)}$ are given by

$$\varphi_{\mu}{}^{(1,2)}(\theta_i) = \left| \frac{\mu^{(1)}(\theta_i) - \mu^{(2)}(\theta_i)}{2} \right| \quad (22)$$

$$\varphi_{\nu}{}^{(1,2)}(\theta_i) = \left| \frac{1 - \nu^{(1)}(\theta_i)}{2} - \frac{1 - \nu^{(2)}(\theta_i)}{2} \right| \quad (23)$$

What we want to define is the dissimilarity, so before being used to construct the dissimilarities of evidence, the original similarities defined in [20], [22] and [23] are converted to dissimilarities as illustrated in (16), (18) and (21), respectively. The dissimilarities we propose are all bounded by 0 and 1. It is easy to prove that when $m_1(.) = m_2(.)$ (i.e. when the bba's are totally similar) then $d_F$, $d_{IF1}$, $d_{IF2}$ and $d_{IF3}$ give zero value. When the dissimilarity reaches its maximum value 1 then the two bba's are considered as totally dissimilar.

## V. Simulation results

To verify the rationality of the proposed new dissimilarities between bba's, some numerical examples are provided. In each example, $d_J$, $d_T$, $d_F$, $d_{IF1}$, $d_{IF2}$ and $d_{IF3}$ are compared.

### A. Example 1

For the FOD $\Theta = \{\theta_1, \theta_2, \theta_3\}$ satisfying Shafer's model, the bba $m_1(.)$ is listed in Table I and other six bba's are listed in Table II. We calculated the dissimilarities between $m_1(.)$ and $m_i(.)$, $i = 2, ..., 7$. See results in Figure 1. For $m_1(.)$, it has relatively large mass assignment value for the focal element $\{\theta_2\}$. Then intuitively, for $m_i(.)$, $i = 2, ..., 7$ listed in Table II, if the mass assignment for $\{\theta_2\}$ is relative large, the dissimilarity between $m_1(.)$ and $m_i(.)$ should be relatively small. As illustrated in Figure 1, all the dissimilarities used here are rational. $d_{IF1}$ has a

Table I
BBA $m_1(.)$

| Focal el.\ bba | $m_1(.)$ |
|---|---|
| $\theta_1$ | 0.1 |
| $\theta_2$ | 0.8 |
| $\theta_3$ | 0.1 |
| $\theta_1 \cup \theta_2$ | 0 |
| $\theta_2 \cup \theta_3$ | 0 |
| $\theta_1 \cup \theta_3$ | 0 |
| $\theta_1 \cup \theta_2 \cup \theta 3$ | 0 |

Table II
BBA's $m_i(.), i = 2, \dots, 7$

| Focal el.\ bba's | $m_2(.)$ | $m_3(.)$ | $m_4(.)$ | $m_5(.)$ | $m_6(.)$ | $m_7(.)$ |
|---|---|---|---|---|---|---|
| $\theta_1$ | 0.8 | 0 | 0 | 0 | 0 | 0 |
| $\theta_2$ | 0 | 0.8 | 0 | 0 | 0 | 0 |
| $\theta_3$ | 0 | 0 | 0.8 | 0 | 0 | 0 |
| $\theta_1 \cup \theta_2$ | 0 | 0 | 0 | 0.8 | 0 | 0 |
| $\theta_2 \cup \theta_3$ | 0 | 0 | 0 | 0 | 0.8 | 0 |
| $\theta_1 \cup \theta_3$ | 0 | 0 | 0 | 0 | 0 | 0.8 |
| $\theta_1 \cup \theta_2 \cup \theta 3$ | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |

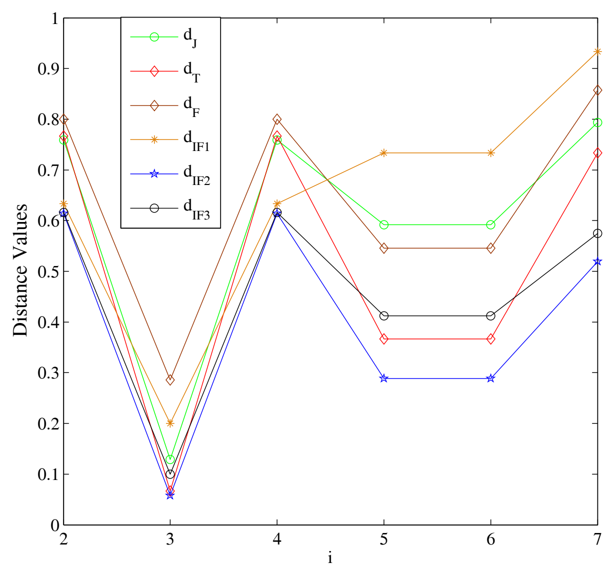


Figure 1. Dissimilarities between $m_1$ and $m_i, i = 2, ..., 7$

different trend when compared with other dissimilarities at $m_5(.)$ and $m_6(.)$. When compared with the value at $m_3(.)$, the dissimilarity value of $d_{IF1}$ at $m_4(.)$ and $m_5(.)$ increase while for other types of dissimilarity, the corresponding values decrease. For $m_5$ and $m_6$, the focal elements containing $\theta_2$ ( $\theta_1 \cup \theta_2$ and $\theta_2 \cup \theta_3$ ) is 0.8, it should be more rational if the dissimilarity value at $m_5(.)$ and $m_6(.)$ decrease.

### B. Example 2

This example was proposed in [6]. Let $\Theta$ be a FOD with 20 elements satisfying Shafer's model (i.e. all elements of $\Theta$ are truly exhaustive and exclusive). For notation convenience, we use 1, 2, etc. to denote element $\theta_1$, element $\theta_2$, etc. in the FOD. The first bba $m_1(.)$ is defined as: $m_1(\{2,3,4\}) = 0.05$, $m_1(\{7\}) = 0.05$, $m_1(\Theta) = 0.1$, and $m_1(A) = 0.8$ for some other subset $A$ of $\Theta$. The second bba used is $m_2(\{1,2,3,4,5\}) = 1$. We consider 20 cases where $A$ increments one more element at a time staring from Case 1 with $A = \{1\}$ and ending with Case 20 with $A = \{1,2,3,...,20\} = \Theta$. The different dissimilarities between $m_1(.)$ and $m_2(.)$ calculated for all 20 cases are shown in Figure 2.

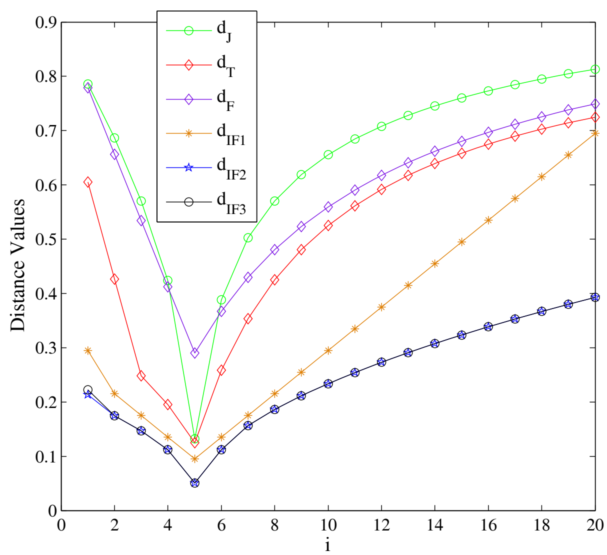


Figure 2. Dissimilarities between $m_1(.)$ and $m_2(.)$.

In this example, all the dissimilarities tested present a similar behavior. When $A = \{1,2,3,4,5\}$, all the dissimilarities used reach their minimum values.

### C. Example 3

This example was proposed in [16]. Let $\Theta = \{\theta_1, ..., \theta_n\}$ be a FOD satisfying Shafer's model. There are three bba's defined on $\Theta$ defined as follows:

$$
\begin{aligned}
m_1(\{\theta_1\}) &= m_1(\{\theta_2\}) = \cdots = m_1(\{\theta_n\}) = 1/n; \\
m_2(\Theta) &= 1; \\
m_3(\{\theta_k\}) &= 1, \text{ for some } k \in \{1, ..., n\}.
\end{aligned}
$$

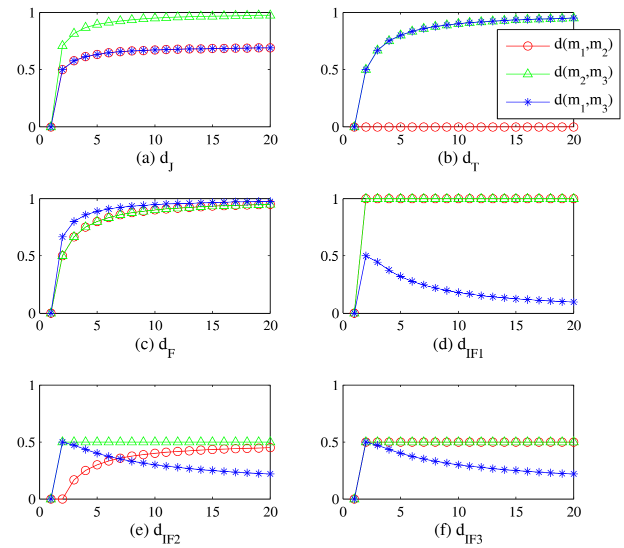


Figure 3. Dissimilarities between $m_1(.)$, $m_2(.)$ and $m_3(.)$.

The behaviors of the different dissimilarities of evidence with the increase of $n$ for $n = 1, \ldots, 20$ are shown in Figure 3. The vertical axis represents the value of dissimilarity while the horizontal axis represents the value of $n$. In this example, $m_3(.)$ is absolutely confident in $\theta_k$ and it is significantly different from both $m_1(.)$ and $m_2(.)$. $m_1(.)$ is rather different from $m_2(.)$ even if they represent both two different uncertain sources. The source $m_2(.)$ is actually a vacuous belief assignment which represent truly the full ignorance on the real state of the nature. The source $m_1(.)$ is much more specific than $m_2(.)$ since it is a Bayesian belief assignment. It turns out that $m_1(.)$ corresponds actually to nothing but a "probabilistic" fully ignorant (random) source having uniform probability mass function (pmf). As one sees in figure 3(a), Jousselme's distance cannot discriminate well the difference between these two very different cases for dealing efficiently with the specificity of the information because $d_J(m_1, m_2) = d_J(m_1, m_3) = \sqrt{\frac{1}{2}(1 - \frac{1}{n})}$. For Tessem's distance, one gets $d_T(m_1, m_2) = 0$ thus it cannot discriminate $m_1(.)$ and $m_2(.)$ as shown in Figure 3(b). For the new defined dissimilarities of evidence based on fuzzy sets theory, some of them also cannot discriminate all the three BOE's: $m_1(.)$, $m_2(.)$, $m_3(.)$, but as shown in Figure 3(e) the dissimilarity $d_{IF2}$ can discriminate all the three BOE's pretty well.

### D. Example 4

This example was proposed in [6]. Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$ be a FOD satisfying Shafer's model. We consider the following three bba's defined on $\Theta$:

$$m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = 1/3;$$
$$m_2(\{\theta_1\}) = m_2(\{\theta_2\}) = m_2(\{\theta_3\}) = 0.1, m_2(\Theta) = 0.7;$$
$$m_3(\{\theta_1\}) = m_3(\{\theta_2\}) = 0.1, m_2(\theta_3) = 0.8.$$

The values of the different dissimilarities between $m_1(.)$ and $m_2(.)$, and between $m_1(.)$ and $m_3(.)$ are given in Table III.

Table III
EXAMPLE 4: RESULTS BASED ON DIFFERENT DISSIMILARITIES OF EVIDENCE.

| Dissimilarity types \ values | $d(m_1, m_2)$ | $d(m_1, m_3)$ |
|---|---|---|
| $d_J$ | 0.4041 | 0.4041 |
| $d_T$ | 0 | 0.4667 |
| $d_F$ | 0.5833 | 0.6364 |
| $d_{IF1}$ | 0.7000 | 0.3111 |
| $d_{IF2}$ | 0.1167 | 0.3300 |
| $d_{IF3}$ | 0.3500 | 0.3300 |

In this example, one sees that it is impossible to take a rational decision from $m_1(.)$ because all masses of singletons are equal. Same problem occurs with $m_2(.)$ because this second source has a very high value of mass assignment to the total ignorance and the mass assignment values of singletons are also the same. $m_1(.)$ and $m_2(.)$ correspond to two very different situations in term of the specificity of their informational content but they yield the same problem for decision-making. $m_3(.)$ assigns its largest mass assignment to $\theta_3$. Intuitively, it seems reasonable to consider $m_1(.)$ and $m_2(.)$ more closer than $m_1(.)$ and $m_3(.)$ since $m_1(.)$ and $m_2(.)$ yields the same indeterminate choice in decision-making because of the ambiguity in choice among the singletons in the FOD. Using Jousselme's distance, one obtains $d_J(m_1, m_2) = d_J(m_1, m_3) = 0.4041$ which is not very satisfactory for such case because it means that the dissimilarities between $m_1(.)$ and $m_2(.)$ is the same as between $m_1(.)$ and $m_3(.)$ which is obviously not acceptable, nor convincing. Based on the results of Table III, one sees that when using the dissimilarities of $d_T$, $d_F$, $d_{IF2}$, one gets $d(m_1, m_2) < d(m_1, m_3)$ which is more reasonable. However for Tessem's distance, one gets $d_T(m_1, m_2) = 0$ which is not rational (intuitively acceptable) or at least very questionable.

According to the above four simple numerical examples, one sees that the new defined dissimilarities $d_F$ and $d_{IF2}$ based on fuzzy sets theory always present an acceptable behavior with respect to other dissimilarities presented in this paper.

## VI. CONCLUSIONS

In this work, some new dissimilarities of evidence based on fuzzy membership functions and intuitionist fuzzy sets have been proposed. Similarly to the definition of distances or dissimilarities based on lossy probabilistic transformations of bba's, here the basic belief assignments are first transformed into FMFs or intuitionist fuzzy sets, then the new dissimilarities of evidence are indirectly defined using the classical similarity or dissimilarity measures proposed in fuzzy sets theory. Some numerical examples were given in this paper to show the behavior of the dissimilarities in very different cases. We have shown that at least two new dissimilarities presented in this paper ($d_F$ and $d_{IF2}$) have a rational behavior, contrariwise to other tested dissimilarities. It should be noted that all these new dissimilarities are also based on lossy fuzzy sets transformations, like with lossy probability transformations. Therefore there exists a loss of information when transforming bba's into FMF's or intuitionist fuzzy sets. To try to improve these new dissimilarity measures, it is necessary to define better (possibly lossless) transformations in future research works.

In all the four new definitions of dissimilarities in evidence theory, it can be easily proved that they are non-negative, non-degeneracy and symmetric. And their values are all belonging to [0,1]. It should be noted that till now whether the new proposed dissimilarities satisfy the triangle inequality have not been theoretically proved. Strictly speaking, until they are proved to satisfy all the requirements for being a distance measure, they can be called only the "pseudo-distance" but not "distance". Not only for our proposed definitions but also all the other available distances of evidence, they all have the problem of the theoretical strictness. Although they might not be totally strict distances, to some extent they still can be used to represent the dissimilarity or difference between BOEs. To accomplish the theoretical proof for the strictness of the new proposed dissimilarities of evidence strictness and to propose more strict definitions of dissimilarity of evidence are both important works in future.


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