

Smarandache Sequence of Happy Cube Numbers

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Abstract : I have studied the Smarandache Happy Cube Numbers and I have got some interesting results and facts . I have discovered some open problems about the Happy Cube and Smarandache Happy Cube Numbers .

Keywords : Fixed Happy Cube Number (FHCN) , Cyclic Happy Cube Number (CHCN), Consecutive fixed happy cube numbers , General Happy Cube Numbers(GHCN), Happy numbers , Fibonacci numbers , Lucas numbers , Pell numbers , Smarandache Fixed Happy Cube Numbers (SFHCN), Reversed Smarandache Fixed Happy Cube Numbers(RSFHCN), Smarandache Cyclic Happy Cube Numbers (SCHCN), Reversed Smarandache Cyclic Happy Cube Numbers(RSCHCN), Smarandache General Smarandache Happy Cube Numbers(SGHCN), , Reversed Smarandache General Smarandache Happy Cube Numbers(RSGHCN)

Definition1 : A positive integer is called *Fixed Happy Cube Numbers* (FHCN) in case, if you are cubing its digits and adding them together one time you got the same number .

For example , $370 = 3^3 + 7^3 + 0^3$, and , $371 = 3^3 + 7^3 + 1^3$, it follows that 370

and 371 are both considered as Fixed Happy Cube Numbers (FHCN).

While it's worth notably that any permutation of the digits of the(FHCN) doesn't end with the same integer e.g. $730 \neq 3^3 + 7^3 + 0^3$. In this case , the integer called unhappy cube .

So the proposed sequence of the FHCN, is $FHCN = \{ 1, 153, 370, 371, 407, \dots \}$.

Open Problems needing answers

1. Is the sequence of the proposed FHCN finite or infinite ?
2. If it is infinite, what is the next number of 407 ?
3. What is the density of FHCN ?
4. Is there any sequence of FHCN following a definite mathematical patterns?
5. How many primes are there in FHCN ?
6. Is there FHCN and *Happy Number* at the same time ?

7. Is there relations between FHCN and the following numbers :*Happy Numbers, Fibonacci Numbers, Lucas Numbers, and Pell Numbers* ?
8. What about other bases or higher powers of FHCN?
9. We have 370 , and 371 consecutive FHCN, are there other consecutive FHCN?

Smarandache Fixed Happy Cube Number (SFHCN)

Definition2 : *Smarandache Fixed Happy Cube Number* (SFHCN) is the number formed from FHCN , as a result :

$$\text{SFHCN} = \{ 1, 1153, 1153370, 1153370371, 1153370371407, \dots \}.$$

Note the following observations :

1. 1153 is a prime number.
2. 1153370 is happy number (Because $1^2 + 1^2 + 5^2 + 3^2 + 3^2 + 7^2 + 0^2 \rightarrow 9^2 + 4^2 \rightarrow 9^2 + 7^2 \rightarrow 1^2 + 3^2 + 0^2 \rightarrow 1^2 + 0^2 \rightarrow 1$.
3. 1153370371407 . if we are squaring the digits and adding them together we get the number 153 i.e. FHCN.

Open Problems needing answers

- 1) How many prime numbers are there in SFHCN ?
- 2) How many SFHCN and *Happy Numbers* are there at the same time ?
- 3) Is there a relationship between SFHCN and FHCN numbers ?
- 4) Are there consecutive SFHCN?

Reversed Smarandache Fixed Happy Cube Number (RSFHCN)

Definition3 : *Reversed Smarandache Fixed Happy Cube Number* (RSFHCN) is the number formed from SFHCN , as a result :

$$\text{RSFHCN} = \{ 1, 1531, 3701531, 3713701531, 4073713701531, \dots \}.$$

Note the following observations :

1. 1531, and 3713701531 are bothe prime- RSFHCN .
2. 3701531 is happy – RSFHCN .

Open Problems needing answers

- 1) How many prime numbers are there in RSFHCN ?
- 2) How many RSFHCN and *Happy Number* are there at the same time ?

- 3) Is there a relationship between RSFHCN and SFHCN ?
- 4) Are there consecutive RSFHCN?

Definition4 : A positive integer is called *Cyclic Happy Cube Numbers* (CHCN), in case , if you are cubing its digits and adding them together many times you got the same number .

For example , $160 \rightarrow 1^3 + 6^3 + 0^3 \rightarrow 217 \rightarrow 2^3 + 1^3 + 7^3 \rightarrow 352 \rightarrow 3^3 + 5^3 + 2^3 \rightarrow 160$. So 160 is cyclic happy cube numbers .

Consequently the proposed CHCN, is $CHCN = \{ 55, 133, 136, 160, 217, 244, 250, 352, 919, 1459, \dots \}$.

Note that the numbers 919, and 1459 are prime numbers ,and the number 55 is Fibonacci number.

Open Problems needing answers

- 1) Is the sequence of the proposed CHCN finite or infinite ?
- 2) What is the next number of 1459 ? If exist !
- 3) What is the density of CHCN ?
- 4) Are there any sequence of CHCN following a definite mathematical patterns?
- 5) How many primes are there in CHCN ?
- 6) Is there CHCN and *Happy Number* at the same time ?
- 7) Is there a relations between CHCN and the following numbers : *Happy Numbers* , *Fibonacci Numbers* , *Lucas Numbers* , and *Pell Numbers* ?
- 8) What about other bases or higher powers of CHCN?
- 9) Are there CHCN, 2, 3,4,5 ,... etc , consecutive CHCN?
Smarandache Cyclic Happy Cube Number (SCHCN)

Definition5 : *Smarandache Cyclic Happy Cube Number* (SCHCN) is the number formed from CHCN ,
hence $SCHCN = \{ 55, 55133, 55133136, 55133136160, \dots \}$.

Open Problems needing answers

1. How many prime numbers are there in SCHCN ?
2. How many SCHCN and *Happy Number* are there at the same time ?
3. Is there a relation between SCHCN and CHCN ?

4. Are there consecutive SCHCN?
5. What is the density of SCHCN ?
6. Is there any sequence of SCHCN following a definite mathematical patterns?
7. How many prime numbers are there in SCHCN ?
8. Is there SCHCN and *Happy Number* at the same time ?
9. Is there a relation between SCHCN and the following numbers : *Happy Numbers , Fibonacci Numbers , Lucas Numbers , and Pell Numbers* ?
10. What about other bases or higher powers of SCHCN?
11. Are there SCHCN, 2, 3,4,5 ,... etc , consecutive SCHCN?

Reversed Smarandache Cyclic Happy Cube Number (RSCHCN)

Definition6 : *Reversed Smarandache Cyclic Happy Cube Number (RSCHCN)* is the number formed from SCHCN ,
 Consequently, $RSCHCN = \{ 55 , 13355 , 13613355 , 16013613355 , \dots \}$.

Open Problems needing answers

1. How many prime numbers are there in RSCHCN ?
2. How many RSCHCN and *Happy Number* are there at the same time ?
3. Is there a relation between RSCHCN and CHCN ?
4. Are there consecutive RSCHCN?
5. What is the density of RSCHCN ?
6. Is there any sequence of RSCHCN following a definite mathematical patterns?
7. How many prime numbers are there in RSCHCN ?
8. Are there RSCHCN and *Happy Number* at the same time ?
9. Is there a relation between RSCHCN and the following numbers : *Happy Numbers , Fibonacci Numbers , Lucas Numbers , and Pell Numbers* ?
10. What about other bases or higher powers of RSCHCN?
11. Are there RSCHCN, 2, 3,4,5 ,... etc , consecutive RSCHCN?

Definition 7 : If there are union between the set of the (FHCN) and (CHCN) , We will get the *General Happy Cube Numbers* (GHCN), namely ;

$GHCN = \{1, 55, 133, 136, 153, 160, 217, 244, 250, 352, 370, 371, 407, 919, 1459, \dots\}$.

Definition 8 : *Smarandache General Happy Cube Numbers* ,formed from GHCN i.e.

$SGHCN = \{1, 155, 155133, 155133136, 155133136153, \dots\}$.

Definition 9 : *Reversed Smarandache General Happy Cube Numbers* , which formed from SGHCN , i.e.

$RSGHCN = \{1, 551, 133551, 136133551, 160153136133551, \dots\}$.

All the above opened questions need answers .

Curious notes :

- 1) The digit 8 doesn't appear . So is there happy cube number has in its digits the digit 8?, or as I think it is impossible !
- 2) The sum of the digits of any General Happy Cube Number follows the pattern $\{ 1, 10, 7, 10, 9, 7, 10, 10, 7, 10, 10, 11, 9, 19, 19, \dots\}$.

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References:

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