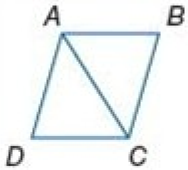
6-5 Rhombi and Squares

ALGEBRA Quadrilateral *ABCD* is a rhombus. Find each value or measure.



1. If $m\angle BCD = 64$, find $m\angle BAC$.

SOLUTION:

A rhombus is a parallelogram with all four sides congruent. So, $\overline{AB} \cong \overline{BC}$. Then, ΔABC is an isosceles triangle. Therefore, $m\angle BAC = m\angle BCA$. If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. So,

$m\angle BCA = \frac{1}{2}(m\angle BCD) = 32$.

Therefore, $m\angle BAC = 32$.

ANSWER:

32

2. If *AB* = 2*x* + 3 and *BC* = *x* + 7, find *CD*.

SOLUTION:

A rhombus is a parallelogram with all four sides congruent. So, $AB = BC$.

$2x + 3 = x + 7$

$x = 4$

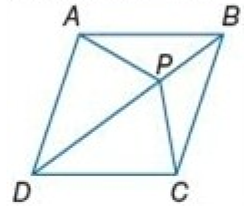
So, *AB* = 2(4) + 3 = 11.

CD is congruent to *AB*, so *CD* = 11.

ANSWER:

11

3. **PROOF** Write a two-column proof to prove that if *ABCD* is a rhombus with diagonal $\overline{DB}$, then $\overline{AP} \cong \overline{CP}$.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a rhombus with diagonal $\overline{DB}$. You need to prove $\overline{AP} \cong \overline{CP}$. Use the properties that you have learned about rhombi to walk through the proof.

Given: *ABCD* is a rhombus with diagonal $\overline{DB}$.

Prove: $\overline{AP} \cong \overline{CP}$

Proof:

Statements(Reasons)

1. *ABCD* is a rhombus with diagonal $\overline{DB}$. (Given)
2. $\angle ABP \cong \angle CBP$ (Diag. of rhombus bisects $\angle$)
3. $\overline{PB} \cong \overline{PB}$ (Refl. Prop.)
4. $\overline{AB} \cong \overline{CB}$ (Def. of rhombus)
5. $\Delta APB \cong \Delta CPB$ (SAS)
6. $\overline{AP} \cong \overline{CP}$ (CPCTC)

ANSWER:

Given: *ABCD* is a rhombus with diagonal $\overline{DB}$.

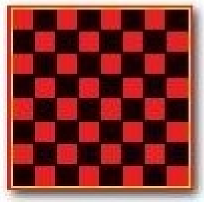
Prove: $\overline{AP} \cong \overline{CP}$

Proof:

Statements(Reasons)

1. *ABCD* is a rhombus with diagonal $\overline{DB}$. (Given)
2. $\angle ABP \cong \angle CBP$ (Diag. of rhombus bisects $\angle$)
3. $\overline{PB} \cong \overline{PB}$ (Refl. Prop.)
4. $\overline{AB} \cong \overline{CB}$ (Def. of rhombus)
5. $\Delta APB \cong \Delta CPB$ (SAS)
6. $\overline{AP} \cong \overline{CP}$ (CPCTC)

4. **GAMES** The checkerboard below is made up of 64 congruent black and red squares. Use this information to prove that the board itself is a square.



SOLUTION:

Sample answer:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given that the checkerboard is made up of 64 congruent squares. You need to prove that the board is a square. Use the properties that you have learned about squares to walk through the proof.

Since each side of the board is 8 squares in length and each of the squares is congruent, the lengths of all four sides of the board are equal. Since we know that each of the four quadrilaterals that form the corners of the board are squares, we know that the measure of the angle of each vertex of the board is 90. Therefore, the board is a square.

ANSWER:

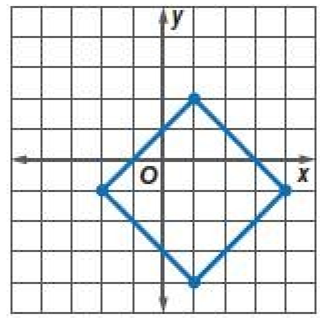
Sample answer: Since each side of the board is 8 squares in length and each of the squares is congruent, the lengths of all four sides of the board are equal. Since we know that each of the four quadrilaterals that form the corners of the board are squares, we know that the measure of the angle of each vertex of the board is 90. Therefore, the board is a square.

COORDINATE GEOMETRY Given each set of vertices, determine whether ▱QRST is a *rhombus, a rectangle,* or a *square*. List all that apply. Explain.

5. $Q(1, 2)$, $R(-2, -1)$, $S(1, -4)$, $T(4, -1)$

SOLUTION:

First graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$$QS = \sqrt{(1-1)^2 + (2-(-4))^2} = \sqrt{36} = 6$$

$$RT = \sqrt{(4-(-2))^2 + (-1-(-1))^2} = \sqrt{36} = 6$$

So, the parallelogram is a rectangle. Check whether the two diagonals are perpendicular.

QS has a slope of $\frac{6}{0}$.

RT has a slope of $\frac{0}{6}$. These slopes are opposite reciprocals.

The diagonals are perpendicular. So, it is a rhombus. Since the diagonals are both congruent and perpendicular to each other the parallelogram is a rectangle, rhombus and square.

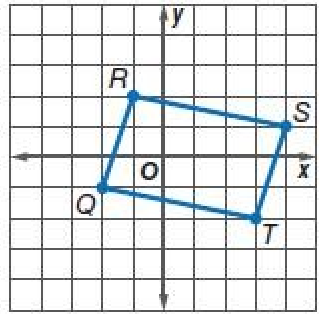
ANSWER:

Rectangle, rhombus, square; consecutive sides are $\perp$, all sides are $\cong$.

6. $Q(-2, -1)$, $R(-1, 2)$, $S(4, 1)$, $T(3, -2)$

SOLUTION:

First graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$QS = \sqrt{(4-(-2))^2 + (1-(-1))^2} = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40}$

$RT = \sqrt{(3-(-1))^2 + (-2-2)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

The diagonals are not congruent. So, the parallelogram is not a rectangle. Check whether the two diagonals are perpendicular.

$$m_{QS} = \frac{1-(-1)}{4-(-2)} = \frac{1}{3}$$

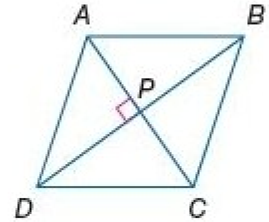
$$m_{RT} = \frac{-2-(-2)}{3-(-1)} = 0$$

The diagonals are not perpendicular. So, it is not a rhombus either.

ANSWER:

None; the diagonals are not congruent or perpendicular.

ALGEBRA Quadrilateral *ABCD* is a rhombus. Find each value or measure.



7. If $AB = 14$, find BC.

SOLUTION:

A rhombus is a parallelogram with all four sides congruent. So, $\overline{AB} \cong \overline{BC}$.
Therefore, $BC = 14$.

ANSWER:

14

8. If $m\angle BCD = 54$, find $m\angle BAC$.

SOLUTION:

A rhombus is a parallelogram with all four sides congruent. So, $\overline{AB} \cong \overline{BC}$. Then, ΔABC is an isosceles triangle. Therefore, $m\angle BAC = m\angle BCA$.
If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. So,

$$m\angle BCA = \frac{1}{2}(m\angle BCD) = 27.$$

Therefore, $m\angle BAC = m\angle BCA = 27$.

ANSWER:

27

9. If $AP = 3x - 1$ and $PC = x + 9$, find AC.

SOLUTION:

The diagonals of a rhombus bisect each other.

$3x - 1 = x + 9$

$2x = 10$

$x = 5$

Therefore, $AC = 2(5 + 9) = 28$.

ANSWER:

28

10. If $DB = 2x - 4$ and $PB = 2x - 9$, find PD.

SOLUTION:

The diagonals of a rhombus bisect each other. So,

$$2x - 4 = 2(2x - 9).$$
$$-2x = -14$$
$$x = 7$$

Therefore, $PD = PB = 2(7) - 9 = 5$.

ANSWER:

5

11. If $m\angle ABC = 2x - 7$ and $m\angle BCD = 2x + 3$, find $m\angle DAB$.

SOLUTION:

In a rhombus, consecutive interior angles are supplementary.

$$m\angle ABC + m\angle BCD = 180$$
$$2x - 7 + 2x + 3 = 180$$
$$4x = 184$$
$$x = 46$$

Each pair of opposite angles of a rhombus is congruent. So, $m\angle DAB = m\angle BCD = 2(46) + 3 = 95.$

ANSWER:

95

12. If $m\angle DPC = 3x - 15$, find x.

SOLUTION:

The diagonals of a rhombus are perpendicular to each other.

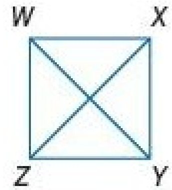
$$m\angle DPC = 90$$
$$3x - 15 = 90$$
$$3x = 105$$
$$x = 35$$

ANSWER:

35

CCSS ARGUMENTS Write a two-column proof.

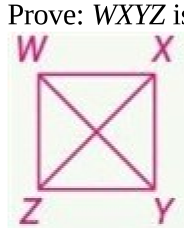
13. **Given:**
$\overline{WZ} \parallel \overline{XY}, \overline{WX} \parallel \overline{ZY}$
$\overline{WZ} \cong \overline{ZY}$
Prove: *WXYZ* is a rhombus.



SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $\overline{WZ} \parallel \overline{XY}, \overline{WX} \parallel \overline{ZY}$, and $\overline{WZ} \cong \overline{ZY}$. You need to prove that *WXYZ* is a rhombus. Use the properties that you have learned about parallelograms and rhombi to walk through the proof.

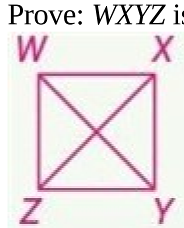
Given:
$\overline{WZ} \parallel \overline{XY}, \overline{WX} \parallel \overline{ZY}$
$\overline{WZ} \cong \overline{ZY}$
Prove: *WXYZ* is a rhombus.



Proof:
Statements(Reasons)
1. $\overline{WZ} \parallel \overline{XY}, \overline{WX} \parallel \overline{ZY}, \overline{WZ} \cong \overline{ZY}$ (Given)
2. *WXYZ* is a ▱. (Both pairs of opp. sides are ∥.)
3. *WXYZ* is a rhombus. (If one pair of consecutive sides of a are ≅, the ▱ is a rhombus.)

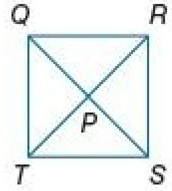
ANSWER:

Given:
$\overline{WZ} \parallel \overline{XY}, \overline{WX} \parallel \overline{ZY}$
$\overline{WZ} \cong \overline{ZY}$
Prove: *WXYZ* is a rhombus.



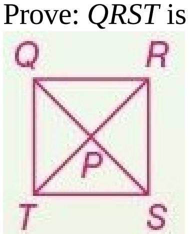
Proof:
Statements(Reasons)
1. $\overline{WZ} \parallel \overline{XY}, \overline{WX} \parallel \overline{ZY}, \overline{WZ} \cong \overline{ZY}$ (Given)
2. *WXYZ* is a ▱. (Both pairs of opp. sides are ∥.)
3. *WXYZ* is a rhombus. (If one pair of consecutive sides of a ▱ are ≅, the ▱ is a rhombus.)

14. **Given:** *QRST* is a parallelogram.
$\overline{TR} \cong \overline{QS}, m\angle QPR = 90$
Prove: *QRST* is a square.



SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *QRST* is a parallelogram; $\overline{TR} \cong \overline{QS}, m\angle QPR = 90$. You need to prove that *QRST* is a square. Use the properties that you have learned about parallelograms and squares to walk through the proof.

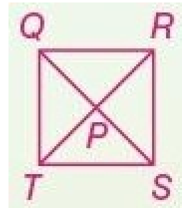
Given: *QRST* is a parallelogram;
$\overline{TR} \cong \overline{QS}, m\angle QPR = 90$
Prove: *QRST* is a square.



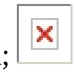
Proof:
Statements(Reasons)
1. *QRST* is a parallelogram;
$\overline{TR} \cong \overline{QS}, m\angle QPR = 90$. (Given)
2. *QRST* is a rectangle. (If the diagonals of a ▱ are ≅, the ▱ is a rectangle.)
3. $\angle QPR$ is a right angle. (Def of rt. ∠)
4. $\overline{QS} \perp \overline{TR}$ (Def. of perpendicular)
5. *QRST* is a rhombus. (If the diagonals of a ▱ are ⊥, ▱ is a rhombus.)
6. *QRST* is a square. (Thm.6.2, if a quadrilateral is a rectangle and a rhombus, then it is a square.)

ANSWER:
Given: *QRST* is a parallelogram;

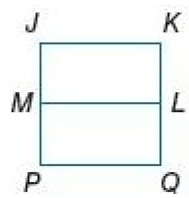
Prove: *QRST* is a square.



Proof:
Statements(Reasons)

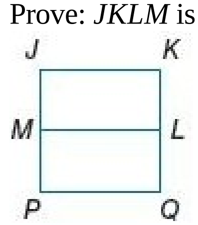
1. *QRST* is a parallelogram; . (Given)
2. *QRST* is a rectangle. (If the diagonals of a ▱ are ≅, the ▱ is a rectangle.)
3. $\angle QPR$ is a right angle. (Def of rt. ∠)
4. $\overline{QS} \perp \overline{TR}$ (Def. of perpendicular)
5. *QRST* is a rhombus. (If the diagonals of a ▱ are ⊥, ▱ is a rhombus.)
6. *QRST* is a square. (Thm.6.2, if a quadrilateral is a rectangle and a rhombus, then it is a square.)

15. **Given:** *JKQP* is a square. $\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$.
Prove: *JKLM* is a parallelogram.



SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *JKQP* is a square and $\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$. You need to prove that *JKLM* is a parallelogram. Use the properties that you have learned about parallelograms to walk through the proof.

Given: *JKQP* is a square.
$\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$.
Prove: *JKLM* is a parallelogram.

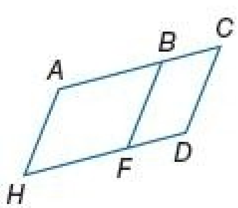


Proof:
Statements (Reasons)
1. *JKQP* is a square. $\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$. (Given)

2. *JKQP* is a parallelogram. (All squares are parallelograms.)
3. $\overline{JM} \parallel \overline{KL}$ (Def. of ▱)
4. $\overline{JP} \cong \overline{KQ}$ (Opp. Sides of ▱ are ≅.)
5. $JP = KQ$ (Def of ≅ segs.)
6. $JM = MP$, $KL = LQ$ (Def. of bisects)
7. $JP = JM + MP$, $KQ = KL + LQ$ (Seg. Add Post.)
8. $JP = 2JM$, $KQ = 2KL$ (Subst.)
9. $2JM = 2KL$ (Subst.)
10. $JM = KL$ (Division Prop.)
11. $\overline{KL} \cong \overline{JM}$ (Def. of ≅ segs.)
12. *JKLM* is a parallelogram. (If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.)

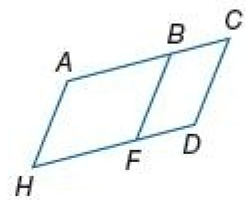
ANSWER:
Proof:
Statements (Reasons)
1. *JKQP* is a square. $\overline{ML}$ bisects $\overline{JP}$ and $\overline{KQ}$. (Given)
2. *JKQP* is a parallelogram. (All squares are parallelograms.)
3. (Def. of ▱)
4. $\overline{JP} \cong \overline{KQ}$ (Opp. sides of of ▱ are ≅.)
5. $JP = KQ$ (Def of ≅ segs.)
6. $JM = MP$, $KL = LQ$ (Def. of bisects)
7. $JP = JM + MP$, $KQ = KL + LQ$ (Seg. Add Post.)
8. $JP = 2JM$, $KQ = 2KL$ (Subst.)
9. $2JM = 2KL$ (Subst.)
10. $JM = KL$ (Division Prop.)
11. $\overline{KL} \cong \overline{JM}$ (Def. of ≅ segs.)
12. *JKLM* is a parallelogram. (If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.)

16. **Given:** *ACDH* and *BCDF* are parallelograms; $\overline{BF} \cong \overline{AB}$.
Prove: *ABFH* is a rhombus.



SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ACDH* and *BCDF* are parallelograms; $\overline{BF} \cong \overline{AB}$. You need to prove that *ABHF* is a rhombus. Use the properties that you have learned about parallelograms and rhombi to walk through the proof.

Given: *ACDH* and *BCDF* are parallelograms; $\overline{BF} \cong \overline{AB}$.
Prove: *ABHF* is a rhombus.



Proof:
Statements (Reasons)
1. *ACDH* and *BCDF* are parallelograms; $\overline{BF} \cong \overline{AB}$. (Given)
2. $\overline{CD} \cong \overline{BF}, \overline{CD} \cong \overline{AH}$ (Def. of ▱)
3. $\overline{BF} \cong \overline{AH}$ (Trans. Prop)
4. $\overline{BC} \cong \overline{FD}, \overline{AC} \cong \overline{HD}$ (Def. of ▱)
5. $AC = HD$ (Def of ≅ segs.)
6. $AC = AB + BC$, $HD = HF + FD$ (Seg. Add. Post.)
7. $AC - HD = AB + BC - HF - FD$ (Subt. Prop.)
8. $AB = HF$ (Subst.)
9. $\overline{AB} \cong \overline{HF}$ (Def. of ≅ segs.)
10. $\overline{AH} \cong \overline{BF}, \overline{AB} \cong \overline{HF}$ (Subst.)
11. *ABFH* is a rhombus. (Def. of rhombus)

ANSWER:
Proof:
Statements (Reasons)
1. *ACDH* and *BCDF* are parallelograms; $\overline{BF} \cong \overline{AB}$. (Given)
2. $\overline{CD} \cong \overline{BF}, \overline{CD} \cong \overline{AH}$ (Def. of ▱)
3. $\overline{BF} \cong \overline{AH}$ (Trans. Prop)
4. $\overline{BC} \cong \overline{FD}, \overline{AC} \cong \overline{HD}$ (Def. of ▱)
5. $AC = HD$ (Def of ≅ segs.)
6. $AC = AB + BC$, $HD = HF + FD$ (Seg. Add. Post.)
7. $AC - HD = AB + BC - HF - FD$ (Subt. Prop.)
8. $AB = HF$ (Subst.)
9. $\overline{AB} \cong \overline{HF}$ (Def. of ≅ segs.)
10. $\overline{AH} \cong \overline{BF}, \overline{AB} \cong \overline{HF}$ (Subst.)
11. *ABFH* is a rhombus. (Def. of rhombus)

17. **ROADWAYS** Main Street and High Street intersect as shown in the diagram. Each of the crosswalks is the same length. Classify the quadrilateral formed by the crosswalks. Explain your reasoning.

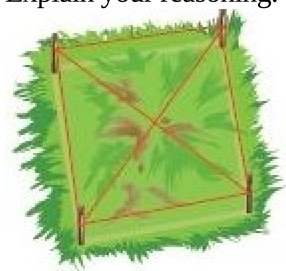
SOLUTION:

The two streets intersect at a 29 degree angle. Analyze this first to determine the measure of an angle of the quadrilateral. Then analyze the information given about the lengths of the crosswalks to categorize the quadrilateral formed.
The measure of the angle formed between the two streets is 29, and vertical angles are congruent, so the measure of one angle of the quadrilateral is 29. So the quadrilateral is not a rectangle or square. Since the crosswalks are the same length, the sides of the quadrilateral are congruent. Therefore, they form a rhombus.

ANSWER:

Rhombus; Sample answer: The measure of angle formed between the two streets is 29, and vertical angles are congruent, so the measure of one angle of the quadrilateral is 29. Since the crosswalks are the same length, the sides of the quadrilateral are congruent. Therefore, they form a rhombus.

18. **CCSS MODELING** A landscaper has staked out the area for a square garden as shown. She has confirmed that each side of the quadrilateral formed by the stakes is congruent and that the diagonals are perpendicular. Is this information enough for the landscaper to be sure that the garden is a square? Explain your reasoning.



SOLUTION:

Compare the information given to the criteria to classify a quadrilateral as a square.
Since the four sides of the quadrilateral are congruent and the diagonals are perpendicular, the figure is either a square or a rhombus. To be sure that the garden is a square, she also needs to confirm that the diagonals are congruent.

ANSWER:

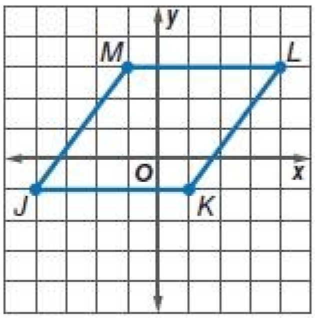
No; sample answer: Since the four sides of the quadrilateral are congruent and the diagonals are perpendicular, the figure is either a square or a rhombus. To be sure that the garden is a square, she also needs to confirm that the diagonals are congruent.

COORDINATE GEOMETRY Given each set of vertices, determine whether ▱*JKLM* is a *rhombus, a rectangle,* or a *square*. List all that apply. Explain.

19. $J(–4, –1), K(1, –1), L(4, 3), M(–1, 3)$

SOLUTION:

First graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$$JL = \sqrt{(4-(-4))^2 + (3-(-1))^2} = \sqrt{8^2+4^2} = \sqrt{64+16} = \sqrt{80}$$
$$KM = \sqrt{(-1-1)^2 + (3-(-1))^2} = \sqrt{(-2)^2+4^2} = \sqrt{4+16} = \sqrt{20}$$

The diagonals are not congruent. So, the parallelogram is not a rectangle. Check whether the two diagonals are perpendicular.

$$m_{JL} = \frac{3-(-1)}{4-(-4)} = \frac{1}{2}$$

$$m_{KM} = \frac{3-(-1)}{-1-1} = -2$$

The diagonals are perpendicular. So, it is a rhombus.

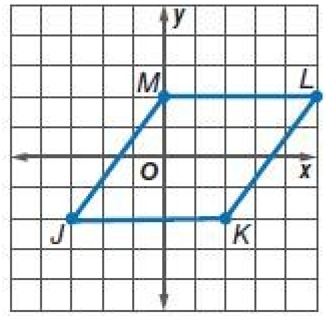
ANSWER:

Rhombus; the diagonals are ⊥.

20. $J(–3, –2), K(2, –2), L(5, 2), M(0, 2)$

SOLUTION:

First, graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$$JL = \sqrt{(5-(-3))^2 + (2-(-2))^2} = \sqrt{8^2+4^2} = \sqrt{64+16} = \sqrt{80}$$
$$KM = \sqrt{(0-2)^2 + (2-(-2))^2} = \sqrt{(-2)^2+4^2} = \sqrt{4+16} = \sqrt{20}$$

The diagonals are not congruent. So, the parallelogram is not a rectangle. Check whether the two diagonals are perpendicular.

$$m_{JL} = \frac{2-(-2)}{5-(-3)} = \frac{1}{2}$$

$$m_{KM} = \frac{2-(-2)}{0-2} = -2$$

The diagonals are perpendicular. So, it is a rhombus.

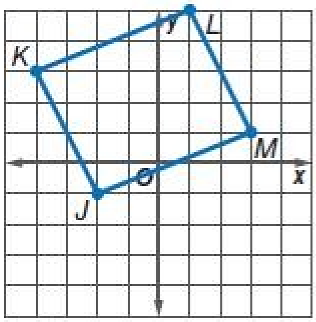
ANSWER:

Rhombus; the diagonals are ⊥.

21. $J(-2, -1), K(-4, 3), L(1, 5), M(3, 1)$

SOLUTION:

First graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$JL = \sqrt{(1-(-2))^2 + (5-(-1))^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45}$

$KM = \sqrt{(3-(-4))^2 + (1-3)^2} = \sqrt{7^2 + (-2)^2} = \sqrt{49+4} = \sqrt{53}$

The diagonals are not congruent. So, the parallelogram is not a rectangle. Check whether the two diagonals are perpendicular.

$$m_{JL} = \frac{5-(-1)}{1-(-2)} = 2$$

$$m_{KM} = \frac{1-3}{3-(-4)} = -\frac{2}{7}$$

The diagonals are not perpendicular. So, it is not a rhombus either.

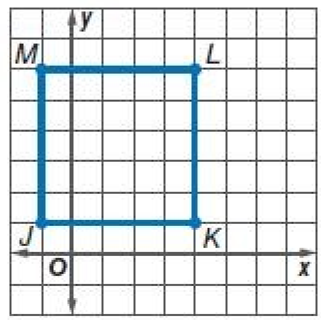
ANSWER:

None; the diagonals are not $\cong$ or $\perp$.

22. $J(-1, 1), K(4, 1), L(4, 6), M(-1, 6)$

SOLUTION:

First graph the quadrilateral.



If the diagonals of the parallelogram are congruent, then it is a rectangle. Use the Distance Formula to find the lengths of the diagonals.

$JL = \sqrt{(4-(-1))^2 + (6-1)^2} = \sqrt{5^2 + 5^2} = \sqrt{25+25} = \sqrt{50}$

$KM = \sqrt{(-1-4)^2 + (6-1)^2} = \sqrt{(-5)^2 + 5^2} = \sqrt{25+25} = \sqrt{50}$

The diagonals are congruent. So, the parallelogram is a rectangle. Check whether the two diagonals are perpendicular.

$$m_{JL} = \frac{6-1}{4-(-1)} = 1$$

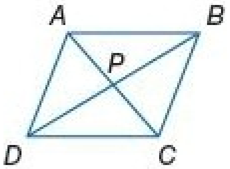
$$m_{KM} = \frac{6-1}{-1-4} = -1$$

The diagonals are perpendicular. So, it is a rhombus. Since the diagonals are both congruent and perpendicular to each other the parallelogram is a rectangle, rhombus and square.

ANSWER:

Square, rectangle, rhombus; all sides are $\cong$ and $\perp$.

***ABCD* is a rhombus. If *PB* = 12, *AB* = 15, and $m\angle ABD = 24$, find each measure.**



23. *AP*

SOLUTION:

The diagonals of a rhombus are perpendicular to each other. So, by the Pythagorean Theorem, $AP^2 = AB^2 - PB^2$.

$AP^2 = 15^2 - 12^2 = 81$

$AP = \sqrt{81} = 9$

ANSWER:

9

24. *CP*

SOLUTION:

All the four sides of a rhombus are congruent and the diagonals are perpendicular to each other.
So, by the Pythagorean Theorem, $CP^2 = BC^2 - PB^2$.

$BC = AB$. Substitute AB for BC.

$CP^2 = 15^2 - 12^2 = 81.$

$CP = \sqrt{81} = 9$

ANSWER:

9

25. $m\angle BDA$

SOLUTION:

All the four sides of a rhombus are congruent. So, ΔABD is an isosceles triangle. Then, $m\angle BDA = m\angle ABD = 24.$

ANSWER:

24

26. $m\angle ACB$

SOLUTION:

The diagonals are perpendicular to each other. So, in the right triangle *PAB*,

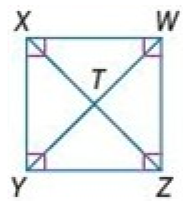
$m\angle BAP = 180 - (90 + 24) = 66.$

All the four sides of a rhombus are congruent. So, ΔABD is an isosceles triangle. Then,
$m\angle ACB = m\angle CAB = m\angle PAB = 66.$

ANSWER:

66

***WXYZ* is a square. If *WT* = 3, find each measure.**



27. *ZX*

SOLUTION:

The diagonals of a square are congruent and bisect each other.
So, *ZX* = *WY* = 2(*WT*) = 6.

ANSWER:

6

28. *XY*

SOLUTION:

The diagonals of a square are congruent and bisect each other at right angles.
So, *YT* = *XT* = *WT* = 3.

By the Pythagorean Theorem, $XY^2 = YT^2 + XT^2$.

$XY^2 = 3^2 + 3^2 = 18$

$XY = \sqrt{18}$

$XY = 3\sqrt{2}$

ANSWER:

$3\sqrt{2}$

29. $m\angle WTZ$

SOLUTION:
The diagonals of a square are perpendicular to each other.
So, $m\angle WTZ = 90$.

ANSWER:
90

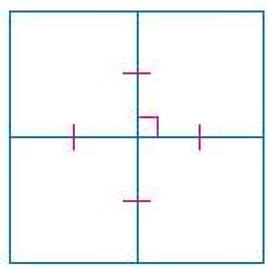
30. $m\angle WYX$

SOLUTION:
In a square, each diagonal bisects a pair of opposite angles. So, $m\angle WYX = \frac{1}{2}(m\angle XYZ) = 45$.

ANSWER:
45

Classify each quadrilateral.

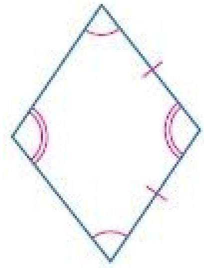
31. Refer to the photo on p. 432.



SOLUTION:
The diagonals are congruent and bisect each other at right angle. Therefore, the quadrilateral is a square.

ANSWER:
square

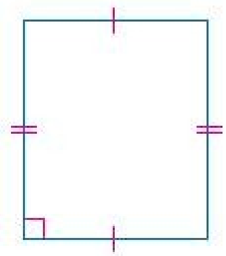
32. Refer to the photo on p. 432.



SOLUTION:
The two pairs of opposite angles are congruent and the adjacent sides are congruent. Therefore, the quadrilateral is a rhombus.

ANSWER:
rhombus

33. Refer to the photo on p. 432.



SOLUTION:
The two pairs of opposite sides are congruent and one of the angles is a right angle. Therefore, the quadrilateral is a rectangle.

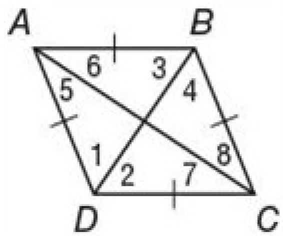
ANSWER:
rectangle

PROOF Write a paragraph proof.

34. Theorem 6.16

SOLUTION:
You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a rhombus. You need to prove that each diagonal bisects a pair of opposite angles.. Use the properties that you have learned about rhombi to walk through the proof.

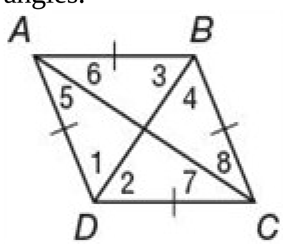
Given: *ABCD* is a rhombus.
Prove: Each diagonal bisects a pair of opposite angles.



Proof: We are given that *ABCD* is a rhombus. By definition of rhombus, *ABCD* is a parallelogram. Opposite angles of a parallelogram are congruent, so $\angle ABC \cong \angle ADC$ and $\angle BAD \cong \angle BCD$.
$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ because all sides of a rhombus are congruent. $\Delta ABC \cong \Delta ADC$ by SAS.
$\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$ by CPCTC.
$\Delta BAD \cong \Delta BCD$ by SAS. $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by CPCTC. By definition of angle bisector, each diagonal bisects a pair of opposite angles.

ANSWER:

Given: *ABCD* is a rhombus.
Prove: Each diagonal bisects a pair of opposite angles.



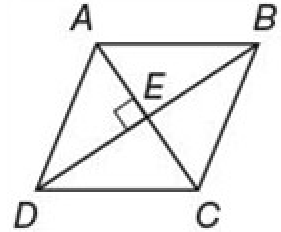
Proof: We are given that *ABCD* is a rhombus. By definition of rhombus, *ABCD* is a parallelogram. Opposite angles of a parallelogram are congruent, so $\angle ABC \cong \angle ADC$ and $\angle BAD \cong \angle BCD$.
$\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ because all sides of a rhombus are congruent. $\Delta ABC \cong \Delta ADC$ by SAS.
$\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$ by CPCTC.
$\Delta BAD \cong \Delta BCD$ by SAS. $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by CPCTC. By definition of angle bisector, each diagonal bisects a pair of opposite angles.

35. Theorem 6.17

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a parallelogram; $\overline{AC} \perp \overline{BD}$. You need to prove that *ABCD* is a rhombus. Use the properties that you have learned about rhombi to walk through the proof.

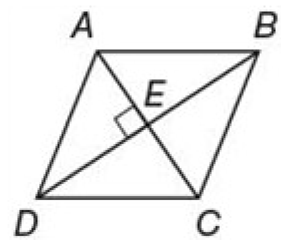
Given: *ABCD* is a parallelogram; $\overline{AC} \perp \overline{BD}$.
Prove: *ABCD* is a rhombus.



Proof: We are given that *ABCD* is a parallelogram. The diagonals of a parallelogram bisect each other, so $\overline{AE} \cong \overline{EC}$. $\overline{BE} \cong \overline{BE}$ because congruence of segments is reflexive. We are also given that $\overline{AC} \perp \overline{BD}$. Thus, $\angle AEB$ and $\angle BEC$ are right angles by the definition of perpendicular lines. Then $\angle AEB \cong \angle BEC$ because all right angles are congruent. Therefore, $\Delta AEB \cong \Delta BEC$ by SAS. $\overline{AB} \cong \overline{CD}$ by CPCTC. Opposite sides of parallelograms are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Then since congruence of segments is transitive, $\overline{AB} \cong \overline{CD} \cong \overline{BC} \cong \overline{AD}$. All four sides of *ABCD* are congruent, so *ABCD* is a rhombus by definition.

ANSWER:

Given: *ABCD* is a parallelogram; $\overline{AC} \perp \overline{BD}$.
Prove: *ABCD* is a rhombus.



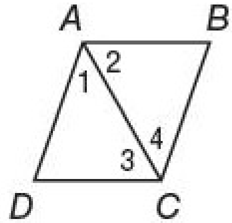
Proof: We are given that *ABCD* is a parallelogram. The diagonals of a parallelogram bisect each other, so $\overline{AE} \cong \overline{EC}$. $\overline{BE} \cong \overline{BE}$ because congruence of segments is reflexive. We are also given that $\overline{AC} \perp \overline{BD}$. Thus, $\angle AEB$ and $\angle BEC$ are right angles by the definition of perpendicular lines. Then $\angle AEB \cong \angle BEC$ because all right angles are congruent. Therefore, $\Delta AEB \cong \Delta BEC$ by SAS. $\overline{AB} \cong \overline{CD}$ by CPCTC. Opposite sides of parallelograms are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Then since congruence of segments is transitive, $\overline{AB} \cong \overline{CD} \cong \overline{BC} \cong \overline{AD}$. All four sides of *ABCD* are congruent, so *ABCD* is a rhombus by definition.

36. Theorem 6.18

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given a diagonal of a parallelogram bisects an angle of a parallelogram. You need to prove that the parallelogram is a rhombus. Use the properties that you have learned about parallelograms and rhombi to walk through the proof.

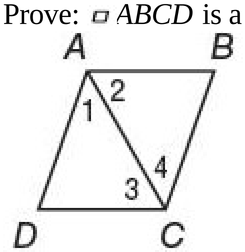
Given: *ABCD* is a parallelogram; diagonal $\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$.
Prove: ▱*ABCD* is a rhombus.



Proof: It is given that *ABCD* is a parallelogram. Since opposite sides of a parallelogram are parallel, $\overline{AB} \parallel \overline{DC}$. By definition, $\angle 2$ and $\angle 3$ are alternate interior angles of parallel sides $\overline{AB}$ and $\overline{DC}$. Since alternate interior angles are congruent, $\angle 2 \cong \angle 3$. Congruence of angles is symmetric, therefore $\angle 3 \cong \angle 2$. it is given that $\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$, so $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by definition. By the Transitive Property, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.
The sides opposite congruent angles in a triangle are congruent, therefore, $\overline{AD} \cong \overline{DC}$ and $\overline{AB} \cong \overline{BC}$. So, since a pair of consecutive sides of the parallelogram is congruent, *ABCD* is a rhombus.

ANSWER:

If a diagonal of a parallelogram bisects an angle of a parallelogram, then the parallelogram is a rhombus.
Given: *ABCD* is a parallelogram; diagonal $\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$.
Prove: ▱*ABCD* is a rhombus.



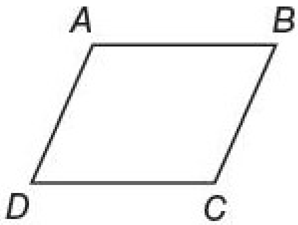
Proof: It is given that *ABCD* is a parallelogram. Since opposite sides of a parallelogram are parallel, $\overline{AB} \parallel \overline{DC}$. By definition, $\angle 2$ and $\angle 3$ are alternate interior angles of parallel sides $\overline{AB}$ and $\overline{DC}$. Since alternate interior angles are congruent, $\angle 2 \cong \angle 3$. Congruence of angles is symmetric, therefore $\angle 3 \cong \angle 2$. it is given that $\overline{AC}$ bisects $\angle DAB$ and $\angle BCD$, so $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by definition. By the Transitive Property, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.
The sides opposite congruent angles in a triangle are congruent, therefore, $\overline{AD} \cong \overline{DC}$ and $\overline{AB} \cong \overline{BC}$. So, since a pair of consecutive sides of the parallelogram is congruent, *ABCD* is a rhombus.

37. Theorem 6.19

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a parallelogram; $\overline{AB} \cong \overline{BC}$. You need to prove that *ABCD* is a rhombus. Use the properties that you have learned about rhombi to walk through the proof.

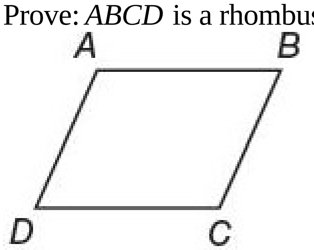
Given: *ABCD* is a parallelogram; $\overline{AB} \cong \overline{BC}$.
Prove: *ABCD* is a rhombus.



Proof: Opposite sides of a parallelogram are congruent, so $\overline{BC} \cong \overline{AD}$ and $\overline{AB} \cong \overline{CD}$. We are given that $\overline{AB} \cong \overline{BC}$. So, by the Transitive Property, $\overline{BC} \cong \overline{CD}$. So, $\overline{BC} \cong \overline{CD} \cong \overline{AB} \cong \overline{AD}$. Thus, *ABCD* is a rhombus by definition.

ANSWER:

Given: *ABCD* is a parallelogram; $\overline{AB} \cong \overline{BC}$.
Prove: *ABCD* is a rhombus.



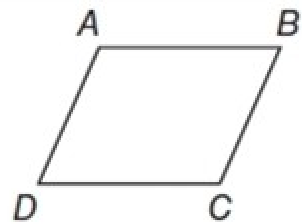
Proof: Opposite sides of a parallelogram are congruent, so $\overline{BC} \cong \overline{AD}$ and $\overline{AB} \cong \overline{CD}$. We are given that $\overline{AB} \cong \overline{BC}$. So, by the Transitive Property, $\overline{BC} \cong \overline{CD}$. So, $\overline{BC} \cong \overline{CD} \cong \overline{AB} \cong \overline{AD}$. Thus, *ABCD* is a rhombus by definition.

38. Theorem 6.20

SOLUTION:

You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given *ABCD* is a rectangle and a rhombus. You need to prove that *ABCD* is a square. Use the properties that you have learned about squares to walk through the proof.

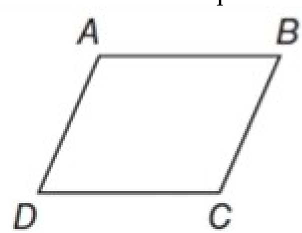
Given: *ABCD* is a rectangle and a rhombus.
Prove: *ABCD* is a square.



Proof: We know that *ABCD* is a rectangle and a rhombus. *ABCD* is a parallelogram, since all rectangles and rhombi are parallelograms. By the definition of a rectangle, $\angle A, \angle B, \angle C,$ and $\angle D$ are right angles. By the definition of a rhombus, all of the sides are congruent. Therefore, *ABCD* is a square since *ABCD* is a parallelogram with all four sides congruent and all the angles are right.

ANSWER:

Given: *ABCD* is a rectangle and a rhombus.
Prove: *ABCD* is a square.



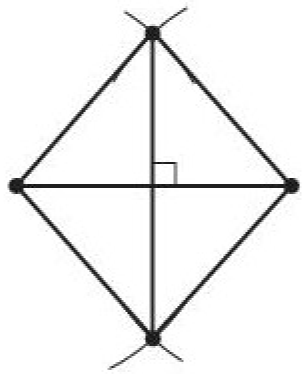
Proof: We know that *ABCD* is a rectangle and a rhombus. *ABCD* is a parallelogram, since all rectangles and rhombi are parallelograms. By the definition of a rectangle, $\angle A, \angle B, \angle C, and\ \angle D$ are right angles. By the definition of a rhombus, all of the sides are congruent. Therefore, *ABCD* is a square since *ABCD* is a parallelogram with all four sides congruent and all the angles are right.

CONSTRUCTION Use diagonals to construct each figure. Justify each construction.

39. rhombus

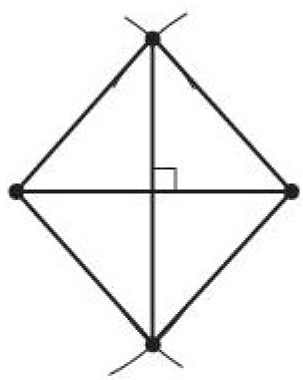
SOLUTION:

Sample answer: Construct the perpendicular bisector of a line segment. Place the compass at the midpoint of the segment. Use the same compass setting to locate a point on the perpendicular bisector above and below the segment. Connect the endpoints of the segments with the two points on the perpendicular bisector to form a quadrilateral.



Since the diagonals bisect each other, the quadrilateral is a parallelogram. If the diagonals of a parallelogram are perpendicular to each other, then the parallelogram is a rhombus. Thus, the constructed quadrilateral is a rhombus.

ANSWER:

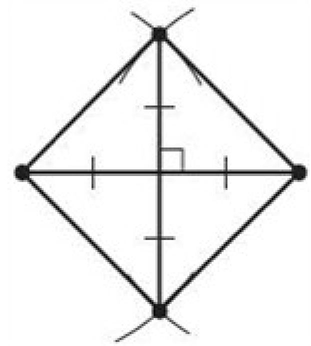


Sample answer: The diagonals bisect each other, so the quadrilateral is a parallelogram. Since the diagonals of the parallelogram are perpendicular to each other, the parallelogram is a rhombus.

40. square

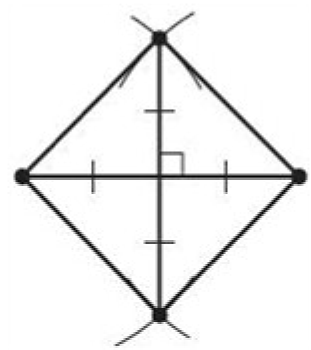
SOLUTION:

Sample answer: Construct the perpendicular bisector of a segment. Adjust the compass setting to equal the distance from the midpoint of the segment to one of its endpoints. Place the compass at the midpoint of the segment. Draw arcs that intersect the perpendicular bisector above and below the segment. Connect the two points of intersection with the endpoints of the segment to form a quadrilateral.



Since the diagonals bisect each other, the quadrilateral is a parallelogram. If the diagonals of a parallelogram are congruent and perpendicular, then the parallelogram is a square. Thus, the constructed quadrilateral is a square.

ANSWER:



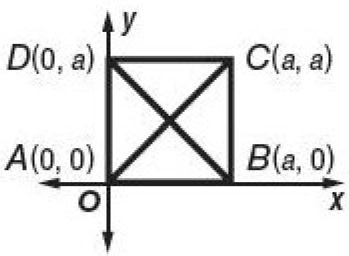
Sample answer: The diagonals bisect each other, so the quadrilateral is a parallelogram. Since the diagonals of the parallelogram are congruent and perpendicular, the parallelogram is a square.

PROOF Write a coordinate proof of each statement.

41. The diagonals of a square are perpendicular.

SOLUTION:

Begin by positioning square $ABCD$ on a coordinate plane. Place vertex A at the origin. Let the length of the bases be a units. Then the rest of the vertices are $B(a, 0)$, $C(a, a)$, and $D(0, a)$. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $ABCD$ is a square and you need to prove that $\overline{AC} \perp \overline{DB}$. Use the properties that you have learned about squares to walk through the proof.



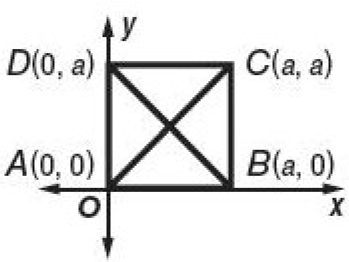
Given: $ABCD$ is a square.
Prove: $\overline{AC} \perp \overline{DB}$
Proof:

slope of $\overline{DB} = \frac{0-a}{a-0}$ or -1

slope of $\overline{AC} = \frac{0-a}{0-a}$ or 1

The slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, so they are perpendicular.

ANSWER:



Given: $ABCD$ is a square.
Prove: $\overline{AC} \perp \overline{DB}$
Proof:

slope of $\overline{DB} = \frac{0-a}{a-0}$ or -1

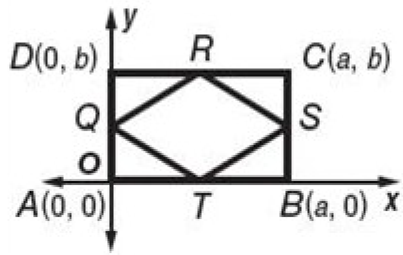
slope of $\overline{AC} = \frac{0-a}{0-a}$ or 1

The slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, so they are perpendicular.

42. The segments joining the midpoints of the sides of a rectangle form a rhombus.

SOLUTION:

Begin by positioning rectangle $ABCD$ on a coordinate plane. Place vertex A at the origin. Let the length of the bases be a units and the height be b units. Then the rest of the vertices are $B(a, 0)$, $C(a, b)$, and $D(0, b)$. You need to walk through the proof step by step. Look over what you are given and what you need to prove. Here, you are given $ABCD$ is a rectangle and Q, R, S, and T are midpoints of their respective sides.and you need to prove that $QRST$ is a rhombus. Use the properties that you have learned about rhombi to walk through the proof.



Given: $ABCD$ is a rectangle. Q, R, S, and T are midpoints of their respective sides.
Prove: $QRST$ is a rhombus.
Proof:

Midpoint Q is $\left(\frac{0+0}{2}, \frac{b+0}{2}\right)$ or $\left(0, \frac{b}{2}\right)$.

Midpoint R is $\left(\frac{a+0}{2}, \frac{b+b}{2}\right)$ or $\left(\frac{a}{2}, \frac{2b}{2}\right)$ or $\left(\frac{a}{2}, b\right)$.

Midpoint S is $\left(\frac{a+a}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{2a}{2}, \frac{b}{2}\right)$ or $\left(a, \frac{b}{2}\right)$.

Midpoint T is $\left(\frac{a+0}{2}, \frac{0+0}{2}\right)$ or $\left(\frac{a}{2}, 0\right)$.

$QR = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(b-\frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$RS = \sqrt{\left(a-\frac{a}{2}\right)^2 + \left(\frac{b}{2}-b\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$

or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$ST = \sqrt{\left(a-\frac{a}{2}\right)^2 + \left(\frac{b}{2}-0\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$QT = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(0-\frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$

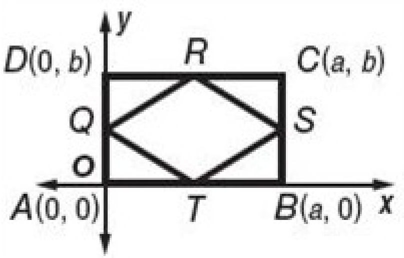
or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

QR = *RS* = *ST* = *QT*

$\overline{QR} \cong \overline{RS} \cong \overline{ST} \cong \overline{QT}$

QRST is a rhombus.

ANSWER:



Given: *ABCD* is a rectangle. *Q*, *R*, *S*, and *T* are midpoints of their respective sides.
Prove: *QRST* is a rhombus.
Proof:

Midpoint *Q* is $\left(\frac{0+0}{2}, \frac{b+0}{2}\right)$ or $\left(0, \frac{b}{2}\right)$.

Midpoint *R* is $\left(\frac{a+0}{2}, \frac{b+b}{2}\right)$ or $\left(\frac{a}{2}, \frac{2b}{2}\right)$ or $\left(\frac{a}{2}, b\right)$.

Midpoint *S* is $\left(\frac{a+a}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{2a}{2}, \frac{b}{2}\right)$ or $\left(a, \frac{b}{2}\right)$.

Midpoint *T* is $\left(\frac{a+0}{2}, \frac{0+0}{2}\right)$ or $\left(\frac{a}{2}, 0\right)$.

$QR = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(b-\frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$RS = \sqrt{\left(a-\frac{a}{2}\right)^2 + \left(\frac{b}{2}-b\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$

or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$ST = \sqrt{\left(a-\frac{a}{2}\right)^2 + \left(\frac{b}{2}-0\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$QT = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(0-\frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2}$

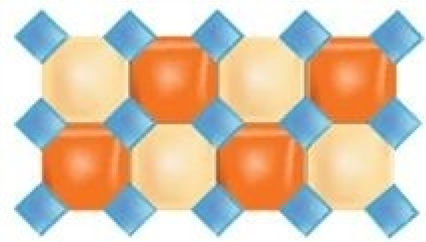
or $\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

QR = *RS* = *ST* = *QT*

$\overline{QR} \cong \overline{RS} \cong \overline{ST} \cong \overline{QT}$

QRST is a rhombus.

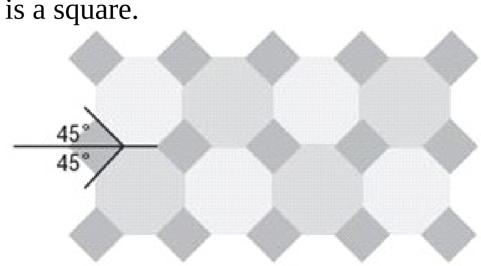
43. **DESIGN** The tile pattern below consists of regular octagons and quadrilaterals. Classify the quadrilaterals in the pattern and explain your reasoning.



SOLUTION:

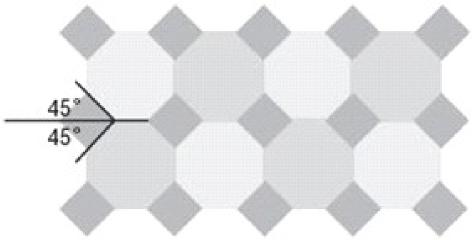
In order to classify the quadrilaterals we need information about the interior angles and the sides. It's given that each quadrilateral is formed by 4 regular octagons. We can use what we know about the exterior angles of a regular octagon as well as the sides of a regular octagon to determine which type of quadrilateral is in the pattern.

Squares; sample answer: Since the octagons are regular each side is congruent, and the quadrilaterals share common sides with the octagon, so the quadrilaterals are either rhombi or squares. The vertices of the quadrilaterals are formed by the exterior angles of the sides of the octagons adjacent to the vertices. The sum of the measures of the exterior angles of a polygon is always 360 and since a regular octagon has 8 congruent exterior angles, each one measures 45. As shown in the diagram, each angle of the quadrilaterals in the pattern measures 45 + 45 or 90. Therefore, the quadrilateral is a square.

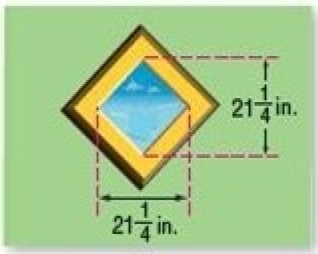


ANSWER:

Squares; sample answer: Since the octagons are regular each side is congruent, and the quadrilaterals share common sides with the octagon, so the quadrilaterals are either rhombi or squares. The vertices of the quadrilaterals are formed by the exterior angles of the sides of the octagons adjacent to the vertices. The sum of the measures of the exterior angles of a polygon is always 360 and since a regular octagon has 8 congruent exterior angles, each one measures 45. As shown in the diagram, each angle of the quadrilaterals in the pattern measures 45 + 45 or 90. Therefore, the quadrilateral is a square.



44. **REPAIR** The window pane shown needs to be replaced. What are the dimensions of the replacement pane?



SOLUTION:

The window pane is in the shape of a square. The diagonal of a square is the hypotenuse of a right triangle with two consecutive sides of the square as its legs. Let x be the length of each side of the square. So, by the Pythagorean Theorem,

$$\left(21\frac{1}{4}\right)^2 = (x)^2 + (x)^2.$$

$$\frac{7225}{16} = 2x^2$$

$$225.78125 = x^2$$

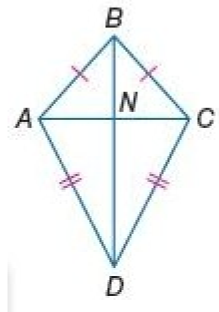
$$15 \approx x$$

Therefore, the length of each side of the square is about 15 inches.

ANSWER:

square; 15 in.

45. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the properties of kites, which are quadrilaterals with exactly two distinct pairs of adjacent congruent sides.



Kite *ABCD*

a. GEOMETRIC Draw three kites with varying side lengths. Label one kite *ABCD*, one *PQRS*, and one *WXYZ*. Then draw the diagonals of each kite, labeling the point of intersection *N* for each kite.

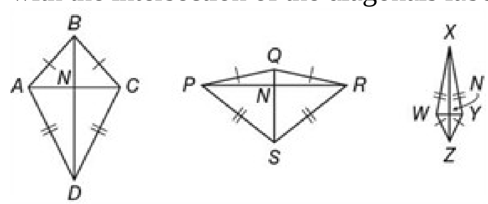
b. TABULAR Measure the distance from *N* to each vertex. Record your results in a table like the one shown.

Figure	Distance from *N* to Each Vertex Along Shorter Diagonal		Distance from *N* to Each Vertex Along Longer Diagonal	
ABCD				
PQRS				
WXYZ				

c. VERBAL Make a conjecture about the diagonals of a kite.

SOLUTION:

a. Sample answer: Draw three different kites, each with the intersection of the diagonals labeled *N*.



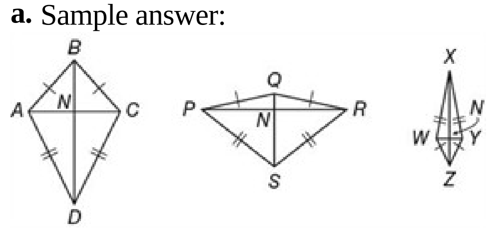
b. Use a ruler to measure each length in the table.

Figure	Distance from *N* to Each Vertex Along Shorter Diagonal		Distance from *N* to Each Vertex Along Longer Diagonal	
ABCD	0.8 cm	0.8 cm	0.9 cm	1.5 cm
PQRS	1.2 cm	1.2 cm	0.3 cm	0.9 cm
WXYZ	0.2 cm	0.2 cm	1.1 cm	0.4 cm

c. Sample answer: From the measurements recorded in the table, *N* is the midpoint of each of the short diagonals. The shorter diagonal of a kite is bisected by the longer diagonal.

ANSWER:

a. Sample answer:

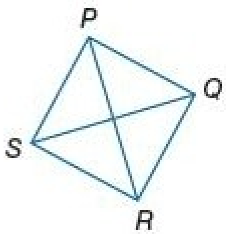


b.

Figure	Distance from *N* to Each Vertex Along Shorter Diagonal		Distance from *N* to Each Vertex Along Longer Diagonal	
ABCD	0.8 cm	0.8 cm	0.9 cm	1.5 cm
PQRS	1.2 cm	1.2 cm	0.3 cm	0.9 cm
WXYZ	0.2 cm	0.2 cm	1.1 cm	0.4 cm

c. Sample answer: The shorter diagonal of a kite is bisected by the longer diagonal.

46. **ERROR ANALYSIS** In quadrilateral *PQRS*, $\overline{PR} \cong \overline{QS}$. Lola thinks that the quadrilateral is a square, and Xavier thinks that it is a rhombus. Is either of them correct? Explain your reasoning.



SOLUTION:

The only information known is that the diagonals are congruent. Review the quadrilaterals that have congruent diagonals. Is this enough information to classify the quadrilateral?
Since they do not know that the sides of the quadrilateral are congruent, only that the diagonals are congruent, they can only conclude that the quadrilateral is a rectangle. So, neither of them are correct.

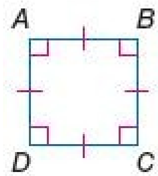
ANSWER:

Neither; sample answer: Since they do not know that the sides of the quadrilateral are congruent, only that the diagonals are congruent, they can only conclude that the quadrilateral is a rectangle.

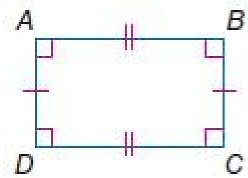
47. **CCSS ARGUMENTS** Determine whether the statement is *true* or *false*. Then write the converse, inverse, and contrapositive of the statement and determine the truth value of each. Explain your reasoning.
If a quadrilateral is a square, then it is a rectangle.

SOLUTION:

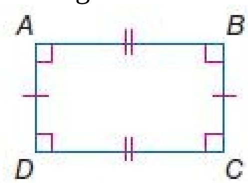
True; sample answer: A rectangle is a quadrilateral with four right angles and a square is both a rectangle and a rhombus, so a square is always a rectangle.



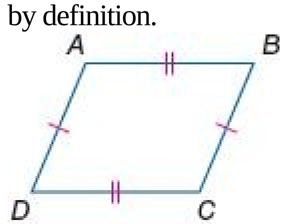
Converse: If a quadrilateral is a rectangle then it is a square. False; sample answer: A rectangle is a quadrilateral with four right angles. It is not necessarily a rhombus, so it is not necessarily a square.



Inverse: If a quadrilateral is not a square, then it is not a rectangle. False; sample answer: A quadrilateral that has four right angles and two pairs of congruent sides is not a square, but it is a rectangle.



Contrapositive: If a quadrilateral is not a rectangle, then it is not a square. True; sample answer: If a quadrilateral is not a rectangle, it is also not a square by definition.

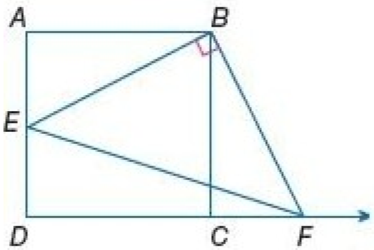


ANSWER:

True; sample answer: A rectangle is a quadrilateral with four right angles and a square is both a rectangle and a rhombus, so a square is always a rectangle.
Converse: If a quadrilateral is a rectangle then it is a square. False; sample answer: A rectangle is a quadrilateral with four right angles. It is not necessarily a rhombus, so it is not necessarily a square.
Inverse: If a quadrilateral is not a square, then it is not a rectangle. False; sample answer: A quadrilateral that has four right angles and two pairs of congruent sides is not a square, but it is a rectangle.

Contra positive: If a quadrilateral is not a rectangle, then it is not a square. True; sample answer: If a quadrilateral is not a rectangle, it is also not a square by definition.

48. **CHALLENGE** The area of square *ABCD* is 36 square units and the area of ΔEBF is 20 square units. If $\overline{EB} \perp \overline{BF}$ and $\overline{AE} = 2$, find the length of $\overline{CF}$.



SOLUTION:

Since the area of the square is 36 square units, the length of each side of the square is 6 units. All the four angles of a square are right angles. So, by the Pythagorean Theorem,

$$EB^2 = AE^2 + AB^2 = 2^2 + 6^2 = 40$$

$$EB = \sqrt{40} = 2\sqrt{10}$$

The area of ΔEBF is 20 square units. So,

$$\frac{1}{2}(EB)(BF) = 20$$

$$\frac{1}{2}\left(2\sqrt{10}\right)(BF) = 20$$

$$\sqrt{10}BF = 20$$

$$BF = \frac{20}{\sqrt{10}}$$

$$BF = \frac{20}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$

$$BF = \frac{20\sqrt{10}}{10}$$

$$BF = 2\sqrt{10}$$

$\overline{EB} \cong \overline{BF}$.

Also, we have

$\overline{BA} \cong \overline{BC}$ and $m\angle BAE = m\angle BCF = 90$.

So, by HL postulate, $\Delta BAE \cong \Delta BCF$.

AE = *CF* by CPCTC

Therefore, *CF* = 2.

ANSWER:

2

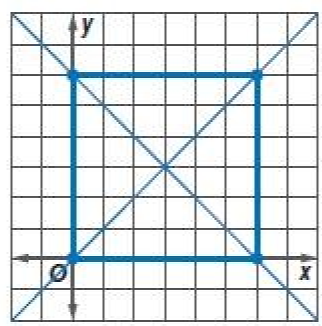
49. **OPEN ENDED** Find the vertices of a square with diagonals that are contained in the lines $y = x$ and $y = -x + 6$. Justify your reasoning.

SOLUTION:

Sample answer:

First graph the lines $y = x$ and $y = -x + 6$. There are 6 units from the origin to the *y*-intercept of $y = -x + 6$ and 6 units from the origin to the *x*-intercept of $y = -x + 6$. So, three of the vertices of a square will be at (0, 0), ((0, 6), and (6, 0). The point 6 units above (6, 0) is (6, 6).

(0, 0), (6, 0), (0, 6), (6, 6); the diagonals are perpendicular, and any four points on the lines equidistant from the intersection of the lines will be the vertices of a square.



ANSWER:

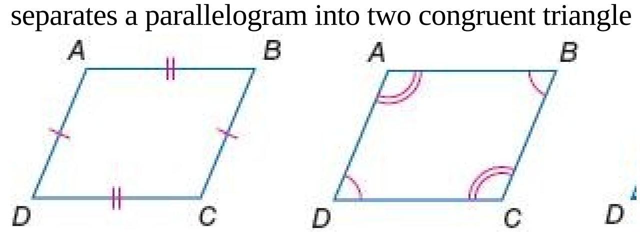
Sample answer: (0, 0), (6, 0), (0, 6), (6, 6); the diagonals are perpendicular, and any four points on the lines equidistant from the intersection of the lines will be the vertices of a square.

50. **WRITING IN MATH** Compare all of the propertie rectangles, rhombi, and squares.

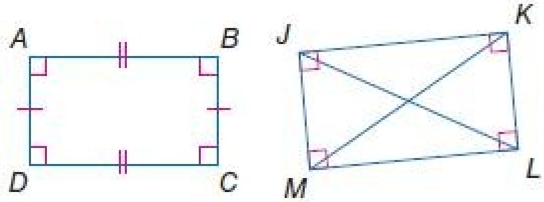
SOLUTION:

Sample answer:

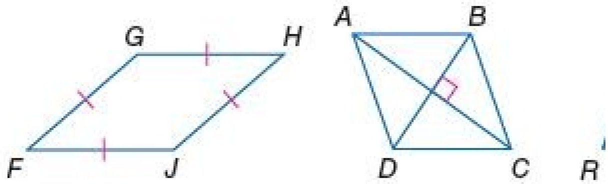
Parallelogram: Opposite sides of a parallelogram are parallelogram are congruent. The diagonals of a paral separates a parallelogram into two congruent triangle



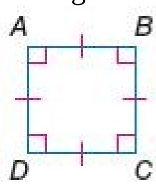
Rectangle: A rectangle has all the properties of a par
The diagonals of a rectangle are congruent.



Rhombus: A rhombus has all of the properties of a pa
congruent. The diagonals of a rhombus are perpendic



Square: A square has all of the properties of a paralle
rectangle. A square has all of the properties of a rho



ANSWER:
Sample answer:

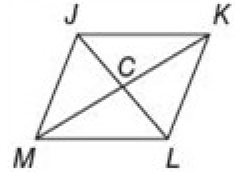
Parallelogram: Opposite sides of a parallelogram are
parallelogram are
congruent. The diagonals of a parallelogram bisect ea
parallelogram into two congruent triangles.

Rectangle: A rectangle has all the properties of a par
The diagonals of a rectangle are congruent.

Rhombus: A rhombus has all of the properties of a pa
congruent. The diagonals of a rhombus are perpendic

Square: A square has all of the properties of a paralle
rectangle. A square has all of the properties of a rho

51. *JKLM* is a rhombus. If *CK* = 8 and *JK* = 10, find *JC*.



A 4
B 6
C 8
D 10

SOLUTION:
The diagonals of a rhombus are perpendicular to each other. So, by the Pythagorean Theorem, $JC^2 = JK^2 - CK^2$.

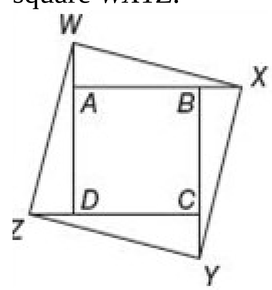
$JC^2 = 10^2 - 8^2 = 36$

$JC = \sqrt{36} = 6$

Therefore, the correct choice is B.

ANSWER:
B

52. **EXTENDED RESPONSE** The sides of square *ABCD* are extended by sides of equal length to form square *WXYZ*.



a. If *CY* = 3 cm and the area of *ABCD* is 81 cm^2, find the area of *WXYZ*.
b. If the areas of *ABCD* and *WXYZ* are 49cm^2 and 169cm^2 respectively, find *DZ*.
c. If *AB* = 2*CY* and the area of *ABCD* = *g* square meters, find the area of *WXYZ* in square meters.

SOLUTION:
a. Since the area of the square is 81 square cm, the length of each side of the square is 9 cm. Since the sides of the square *ABCD* are extended to form square *WXYZ*, *DZ* = *AW* = *BX* = *CY* = 3 cm. Area of each of the triangle is

$\frac{1}{2}(9+3)(3) = 18\text{ cm}^2.$

The area of the square *WXYZ* is the sum of the 4 congruent triangles and the area of the square.

$A_{WXYZ} = 4(18) + 81 = 153\text{ cm}^2$

b. In the right triangle *WZD*, *WZ* = 13 and *WD* = *AW* + *AD* = *DZ* + 7 = *x* + 7 where *x* is the length of the segment $\overline{DZ}$. By the Pythagorean Theorem, $x^2 = 13^2 - (x+7)^2$.

Solve the equation for *x*.

$$x^2 + x^2 + 14x + 49 = 169$$
$$2x^2 + 14x - 120 = 0$$
$$2(x^2 + 7x - 60) = 0$$
$$x^2 + 7x - 60 = 0$$
$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-60)}}{2(1)}$$
$$\approx \frac{-7 \pm 17.3}{2}$$
$$\approx -12, 5$$

Since *x* is a length, it cannot be negative. Therefore, the length *DZ* is about 5 cm.

c. If *AB* = 2*CY*, the lengths of the two legs of the right triangle *WAX* are *WA* = *CY* and *AX* = 2*CY* + *CY* = 3*CY*. Then by the Pythagorean theorem,

$WX^2 = CY^2 + (3CY)^2 = 10CY^2$.

Since *AB* = 2*CY*, $10CY^2 = 10\left(\frac{AB}{2}\right)^2 = 2.5AB^2$.

But AB^2 = the area of the square *ABCD* = *g*.

Therefore, the area of the square *WXYZ* = WX^2 = 2.5*g* square meters.

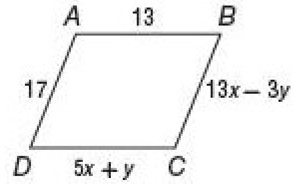
ANSWER:

a. 153 cm^2

b. 5 cm

c. 2.5*g*

53. **ALGEBRA** What values of *x* and *y* make quadrilateral *ABCD* a parallelogram?



F $x = 3, y = 2$

G $x = \frac{3}{2}, y = -1$

H $x = 2, y = 3$

J $x = 3, y = -1$

SOLUTION:

Each pair of opposite sides of a parallelogram is congruent. So, 13*x* – 3*y* = 17 and 5*x* + *y* = 13. Solve the system of two equations to find the values of *x* and *y*.

Multiply the second equation by 3 and then add to the 1st equation to eliminate the *y*-term.

3(5*x* + *y* = 13) = 15*x* + 3*y* = 39

$$\begin{aligned} 13x - 3y &= 17 \\ +\ 15x + 3y &= 39 \\ \hline 28x + 0y &= 56 \\ 28x &= 56 \\ x &= 2 \end{aligned}$$

Use the value of *x* to find the value of *y*.

$5(2) + y = 13$ Original equation.

$y = 3$ Subtract 10 from each side.

Therefore, the correct choice is H.

ANSWER:

H

54. **SAT/ACT** What is 6 more than the product of –3 and a certain number *x*?

A –3*x* – 6

B –3*x*

C –*x*

D –3*x* + 6

E 6 + 3*x*

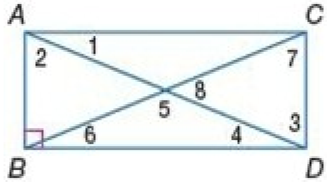
SOLUTION:

The product of –3 and *x* is –3*x*. Six more than –3*x* is –3*x* + 6. Therefore, the correct choice is D.

ANSWER:

D

Quadrilateral *ABCD* is a rectangle. Find each measure if $m\angle 1 = 38$.



55. $m\angle 2$

SOLUTION:

All the four angles of a rectangle are right angles. So, $m\angle 1 + m\angle 2 = 90.$

$m\angle 2 = 90 - 38 = 52$

ANSWER:

52

56. $m\angle 5$

SOLUTION:

The measures of angles 1 and 4 are congruent as they are alternate interior angles.
The diagonals of a rectangle are congruent and bisect each other. So, the triangle with angles 4, 5, and 6 is an isosceles triangle with $m\angle 4 = m\angle 6.$
The sum of the three angles of a triangle is 180. So,

$m\angle 5 = 180 - (m\angle 6 + m\angle 4) = 180 - (38 + 38) = 104.$

ANSWER:

104

57. $m\angle 6$

SOLUTION:

The measures of angles 1 and 4 are congruent as they are alternate interior angles.
The diagonals of a rectangle are congruent and bisect each other. So, the triangle with angles 4, 5, and 6 is an isosceles triangle with $m\angle 4 = m\angle 6.$

$m\angle 6 = m\angle 4 = m\angle 1 = 38$

ANSWER:

38

Determine whether each quadrilateral is a parallelogram. Justify your answer.

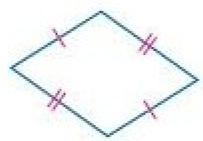
58. 

SOLUTION:

Theorem 6.5 states that parallelograms have consecutive angles that are supplementary. This figure has consecutive interior angles that are supplementary. However, no information is given about opposite sides or opposite angles. Therefore, it does not fulfill any test for parallelograms. So, the given quadrilateral is not a parallelogram.

ANSWER:

No; none of the tests for parallelograms are fulfilled.

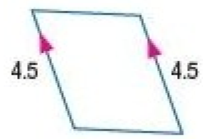
59. 

SOLUTION:

Both pairs of opposite sides are congruent. Theorem 6.9 states that if both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. Therefore, no additional information about the figure is needed to determine that this figure is a parallelogram.

ANSWER:

Yes; both pairs of opposite sides are congruent.

60. 

SOLUTION:

One pairs of opposite sides is parallel and congruent. Theorem 6.12 states that if one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. So no other information is needed to determine if it is a parallelogram. Therefore, it is a parallelogram.

ANSWER:

Yes; one pair of opposite sides is parallel and congruent.

61. **MEASUREMENT** Monifa says that her backyard is shaped like a triangle and that the lengths of its sides are 22 feet, 23 feet, and 45 feet. Do you think these measurements are correct? Explain your reasoning.

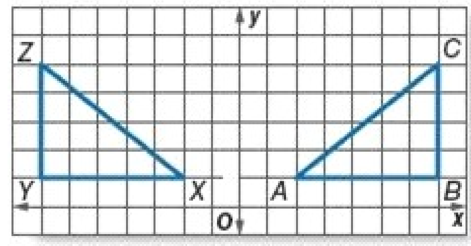
SOLUTION:

The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. Since 22 + 23 = 45, the sides of Monifa's backyard cannot be 22 ft, 23 ft and 45 ft.

ANSWER:

No; the Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. Since 22 + 23 = 45, the sides of Monifa's backyard cannot be 22 ft, 23 ft and 45 ft.

62. **COORDINATE GEOMETRY** Identify the transformation and verify that it is a congruence transformation.



SOLUTION:

ΔABC is a reflection of ΔXYZ.
Find the lengths of the sides of the two triangles.
$AB = 5$, $BC = 4$, $AC = \sqrt{41}$, $XY = 5$, $YZ = 4$, $XZ = \sqrt{41}$. Since all pairs of corresponding sides are congruent, $\Delta ABC \cong \Delta XYZ$ by SSS.

ANSWER:

ΔABC is a reflection of ΔXYZ. $AB = 5$, $BC = 4$, $AC = \sqrt{41}$, $XY = 5$, $YZ = 4$, $XZ = \sqrt{41}$. $\Delta ABC \cong \Delta XYZ$ by SSS.

Solve each equation.

63. $\frac{1}{2}(5x + 7x - 1) = 11.5$

SOLUTION:

$\frac{1}{2}(5x + 7x - 1) = 11.5$	Original equation
$5x + 7x - 1 = 2(11.5)$	Multiply each side by 2.
$5x + 7x - 1 = 23$	Simplify.
$12x - 1 = 23$	Add.
$12x = 24$	Add 1 to each side.
$x = 2$	Divide each side by 12.

ANSWER:

2

64. $\frac{1}{2}(10x + 6x + 2) = 7$

SOLUTION:

$\frac{1}{2}(10x + 6x + 2) = 7$	Original equation
$10x + 6x + 2 = 14$	Multiply each side by 2.
$10x + 6x = 12$	Subtract 2 from each side.
$16x = 12$	Add.
$x = \frac{12}{16} or \frac{3}{4}$	Divide each side by 16 and reduce fraction.

ANSWER:

$\frac{3}{4}$

65. $\frac{1}{2}(12x + 6 - 8x + 7) = 9$

SOLUTION:

$\frac{1}{2}(12x + 6 - 8x + 7) = 9$	Original equation
$12x + 6 - 8x + 7 = 18$	Multiply each side by 2.
$4x + 13 = 18$	Combine like terms.
$4x = 5$	Subtract 13 from each side.
$x = \frac{5}{4}$	Divide each side by 4.

ANSWER:

$\frac{5}{4}$