

# Application of Proposed Improved Relay Tuning for Design of Optimum PID Control of SOPTD Model

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## ABSTRACT

The coupled tank liquid level control system approximated by the second order plus time delay (SOPTD) model was considered and the proposed improved relay tuning method was applied to calculate the corrected ultimate gain ( $K_u$ ). It was found that the error in calculation of ultimate gain by proposed methods has been reduced considerably than that of conventional method. The corrected  $K_u$  was used to estimate optimum PID parameters by Ziegler Nichols method. The closed loop response was obtained using optimum values of PID parameters and compared with that of conventional method. It was found that the Integral Time Absolute Error (ITAE) has been decreased and robustness has been increased by proposed method I & II as compared to that of conventional method. Hence the proposed improved relay tuning method gives better performance.

**Keywords:** Improved Relay Tuning, SOPTD, optimization, Simulink.

## 1 INTRODUCTION

Control plays a vital role in the chemical plants with respect to economical performance, safety and operability. In a typical chemical plant there are hundreds of PID feedback loops. The PID controller is by far the most dominating form of feedback in use today. More than 90% of all control loops are PID. Although the PID controller has always been very important, practically it has only received moderate interest. They are often poorly tuned because the choice of PID controller parameters requires professional knowledge by the user. One of the most common approaches to tune a controller automatically is to use a relay as a feedback controller to the process during tuning. Astrom [1] have suggested the use of an ideal (on-off) relay to generate a sustained oscillation of the controlled variable and to get the ultimate gain ( $K_u$ ) and the ultimate frequency ( $\omega_u$ ) directly from the relay experiment. The relay feedback method has become very popular because, it is time efficient as compared to the conventional method. The relay height ( $h$ ), the amplitude ( $a$ ) and the period of oscillation ( $P_u$ ) are noted from the sustained oscillation of the system output. The ultimate gain ( $K_u$ ) and ultimate frequency ( $\omega_u$ ) are calculated from the principal harmonics approximation as given by equation;

$$K_u = \frac{4h}{\pi a} \quad (1)$$

$$\omega_u = \frac{2\pi}{P_u} \quad (2)$$

The use of relay testing for identifying a transfer function model has suggested by Luyben [2]. Since only  $K_u$  and  $\omega_u$  are available, additional information such as the steady state gain, or the time delay should be a known priori in order to fit a typical transfer function model such as unstable FOPTD. The above equations assume that, the higher order harmonics are neglected. A method of identifying a FOPTD unstable model based on the shape of the response of the

process using a symmetric relay has been proposed by Thyagarajan and Yu [3]. In this method, the output response is aligned with the input response by shifting to the left. Then, the time to peak amplitude, the peak amplitude and the period of oscillation are noted. The time delay is considered as the time to the peak value. It is to be noted that, for higher order systems, the recorded time to peak value from the response will not match with that of the actual time delay of the process. Then it was reported by Li, Eskinat and Luyben [4] that the models identified by the symmetry relay auto tune method gives error as high as 27 to -18% in the value of  $K_u$  for stable FOPTD systems. Recently Srinivasan and Chidambaram [5] have proposed a method of considering higher order harmonics, to explain the report error of 27 to -18% in  $K_u$  calculations for stable systems. An improved method by incorporating the higher order harmonics has been proposed by Sathe and Chidambaram [6] to explain the error in the  $K_u$  calculation.

In this paper, a proposed method I & II have been suggested and applied to minimize the error in  $K_u$  calculation and evaluate the corrected ultimate gain ( $K_u$ ). This corrected value was used to determine the optimum values of PID parameters and robust control design.

## 2 PROPOSED METHOD

### 2.1 Proposed Method-I

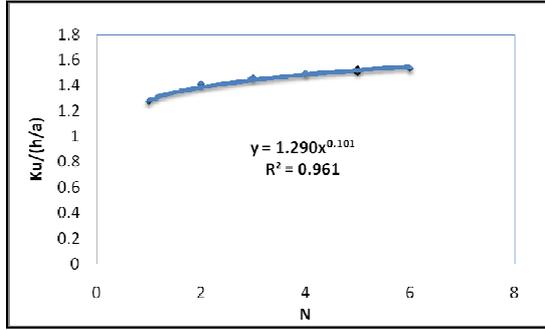
Sathe and Chidambaram [6] proposed an improved relay tuning method by incorporating the higher order harmonics to explain the error in the  $K_u$  calculation. The relay equation is given as;

$$y(t) = a[1 + (1/9) + (1/25) + (1/49) + (1/81) + \dots] \quad (3)$$

Assuming the various number of harmonics ( $N$ ) from 1 to 6 in the relay equation, the relation between ultimate gain ( $K_u$ ) and the ratio of relay amplitude to process amplitude ( $h/a$ ) in relay experiment ; was developed. The summary is presented in Table 1 and a graph of  $N$  v/s  $K_u/(h/a)$  was plotted as shown in Fig. 1.

**Table 1:** correlation between Ku and N

N	Ku/(h/a)
1	1.27
2	1.41
3	1.46
4	1.49
5	1.51
6	1.53

**Figure 1:** Correlation between Ku and N

The co-relation obtained from the plot is;

$$K_u = 1.29 (N)^{0.1} \left( \frac{h}{a} \right) \quad (4)$$

With  $R^2=0.961$  (Regression coefficient).

The new relation obtained to calculate ultimate gain (Ku) was proposed and used in this paper.

## 2.2 Proposed Method-II

The Taguchi's robust parameter design is used to determine the optimum levels of factors and to minimize the error in calculation of ultimate gain (Ku). Taguchi's method is based on statistical and sensitivity analysis for determining the optimal setting of parameters to achieve robust performance [7]. In setting up a framework for robust design, the classifications of the quantities at play in the design task are given. Relay parameters are considered as design variables. The error in ultimate gain (Ku) has been considered as performance functions to represent the performance of the design. The mean and the variance are combined into a single performance measure known as the Signal-to-Noise (S/N) ratio [8]. The target value of y (error in Ku), that is, quality variable is zero. In this situation, S/N Ratio (SNR) is defined as follows;

$$SNR = -10 \log \left( \frac{1}{n} \sum_{i=1}^n y_i^2 \right) \quad (5)$$

The optimum values obtained for relay parameters are used for calculation of ultimate gain (Ku) and consequently the optimum PID parameters

## 3. MATERIALS AND METHOD

A coupled tank liquid level system having the following SOPTD transfer function model was considered;

$$G_p(s) = \frac{0.2}{2.17s^2 + 3.17s + 1} e^{-0.3s} \quad (6)$$

Then the Simulink diagram for relay tuning was prepared as shown in Fig. 1. The relay experiments were carried out for different relay height. The amplitude (a), period of oscillations (Pu) was noted and ultimate gain was calculated. Taguchi's Orthogonal Array (OA) has been used for analysis to study the effects of parameters based on statistical analysis of experiments. It is a matrix of numbers arranged in rows and columns where each row represents the level of the factors in each run and each column represents a specific factor that can be changed from each run. The design of Relay parameters such as relay height (h) and number of harmonics (N) from equation (4) has been carried out using proposed method. These two parameters and five levels of each parameter were considered for the analysis, therefore an orthogonal array of L25 was chosen for the analysis. The physical values of the relay height were chosen as 0.2, 0.4, 0.6, 0.8 & 1.0 and the values of the N were chosen as 1, 2, 3, 4 & 5. With these physical values, simulation experiments were conducted as per L25 OA. Objective of this analysis is to improve the relay performance by minimizing the error in calculation of Ku. Simulations were carried out and Ku was calculated using proposed method I & II and are presented in the Table 2. For identifying the optimal parameter combination, sum of the SNR of each factor and their each level values were calculated as shown in Table 2. Sum of SNR values are presented in Table 3 which gives the optimum level of each parameter. That is, the level which contains maximum value of sum of the SNR is the optimum level. The estimated optimum values of h and N by proposed method II were used for calculation of ultimate gain (Ku) and consequently the optimum PID parameters. In proposed method I, relay height (h=1) and higher order harmonic (N=5) were used in equation (4) and corrected ultimate gain (Ku) was determined.

The PID controller tuning parameters were calculated using Ziegler-Nichols [9] optimum controller parameter settings as shown in Table 3. Then using these optimum values of PID parameters, the closed loop response was studied and it was compared with that of the conventional method as shown in Fig. 4. The Integral Time Absolute Error (ITAE) values for conventional and proposed method I & II were evaluated and the robustness of the controller design was determined. The robustness [10] of controller is defined in equation (7);

$$Ms = \frac{1}{1 + \lambda(s)} \quad (7)$$

Where  $\lambda$  is a shortest distance of the response  $G_p G_c$  from point (-1, 0) on Nyquist plot. Here  $G_p$  and  $G_c$  are the transfer functions of process and controller respectively.

## 4. RESULT & DISCUSSION

In proposed method I of improved relay tuning, relay height (h=1) and higher order harmonic (N=5) were used in equation (4) and corrected ultimate gain (Ku) was determined where in proposed method II, two parameters i.e. relay height (h) and number of harmonics (N) was considered and their five different values were considered for experimentation. The experiments were conducted and the error in calculation of

ultimate gain ( $K_u$ ) was estimated. The Simulink diagram for experiment, relay response and process response are shown in Fig. 1 and Fig. 2 respectively. The SNR were calculated for each experiment and the OA is shown in Table 2. Then the sum of SNR for each parameter and at each level was calculated as shown in Table 3. The optimum values for each parameter were noted. Relay Height equal to 0.2 & 1 and No. of harmonics equal to 5 & 3 were the optimum values found by proposed method I and proposed method II respectively. The optimum values obtained for relay parameters are used for calculation of ultimate gain ( $K_u$ ) and error in  $K_u$  calculation as shown in Table 4. The % error in  $K_u$  calculation found to be 21%, 8% and 0.3% for conventional, proposed method I and proposed method II respectively. The PID parameters were calculated using Ziegler-Nichols [9] optimum controller parameter settings as shown in Table 3. Using these optimum values, the closed loop response was obtained and it was compared with that of conventional method as shown in Fig. 4. The Integral Time Absolute Error (ITAE) performance criteria were tested for the experiments. The ITAE values for conventional method, proposed method I and proposed method II is 0.13, 0.11 and 0.045 respectively. The PID algorithm is an excellent trade-off between robustness and performance. Robustness of the control was found to be 1.8, 1.9 and 2 for conventional, proposed method I and proposed method II respectively. Hence the proposed method gives better performance.

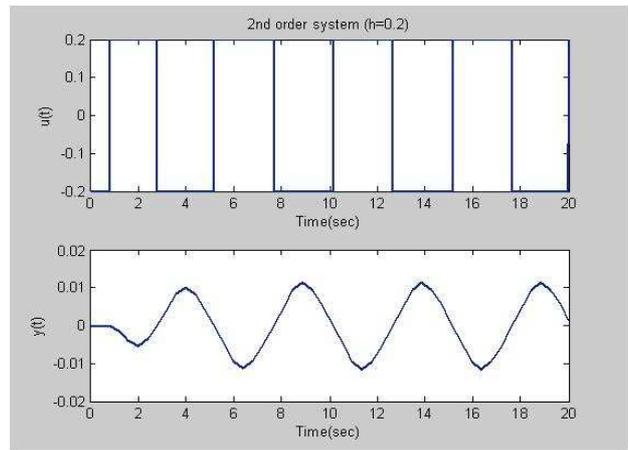


Figure 3: Relay and Process Response.

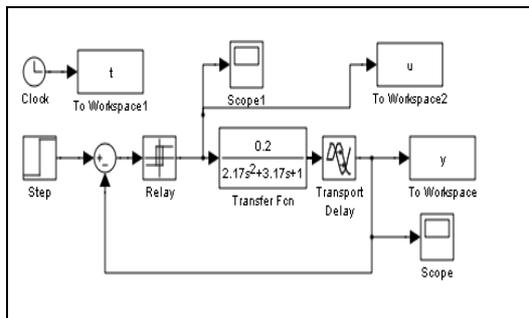


Figure 2: Simulink Diagram for Relay Experiment

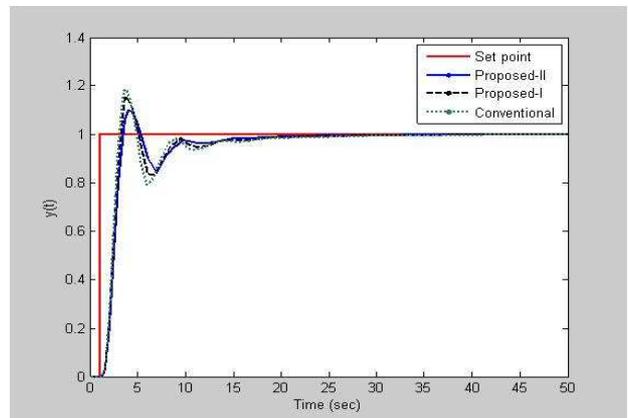


Figure 4: Closed Loop Response with optimum PID Parameters.

**Table 2:** Taguchi OA for given SOPTD system

Sr.	h	N	Ku (Cal)	Ku(Act)	%Error	SNR
1	0.2	1	25.8	28.9	10.72664	19.39072
2	0.2	2	27.65	28.9	4.32526	27.27976
3	0.2	3	28.8	28.9	0.346021	49.21796
4	0.2	4	29.63	28.9	-2.52595	31.9515
5	0.2	5	30.3	28.9	-4.84429	26.2954
6	0.4	1	23.45455	28.9	18.8424	14.49727
7	0.4	2	25.13796	28.9	13.01744	17.70949
8	0.4	3	26.17816	28.9	9.418126	20.52071
9	0.4	4	26.9422	28.9	6.774402	23.38258
10	0.4	5	27.55015	28.9	4.67075	26.61227
11	0.6	1	22.76471	28.9	21.22939	13.46125
12	0.6	2	24.39861	28.9	15.57575	16.15102
13	0.6	3	25.40822	28.9	12.0823	18.35701
14	0.6	4	26.14978	28.9	9.516331	20.43061
15	0.6	5	26.73985	28.9	7.474551	22.5283
16	0.8	1	22.93333	28.9	20.64591	13.70332
17	0.8	2	24.57934	28.9	14.95039	16.50695
18	0.8	3	25.59642	28.9	11.43106	18.83827
19	0.8	4	26.34348	28.9	8.846082	21.06498
20	0.8	5	26.93793	28.9	6.789177	23.36366
21	1	1	22.63158	28.9	21.69004	13.27479
22	1	2	24.25593	28.9	16.06946	15.87997
23	1	3	25.25963	28.9	12.59644	17.99505
24	1	4	25.99686	28.9	10.04548	19.96059
25	1	5	26.58348	28.9	8.015635	21.92124

**Table 3:** Sum of SNR for different factors and levels for Relay Tuning

Parameter	Sum of SNR at each Level					Total
	L1	L2	L3	L4	L5	
Relay Height, h	<b>154.11</b>	102.7	90.91	93.45	89.01	<b>530.1</b>
No. of harmonics, N	74.23	94.49	<b>124.9</b>	116.78	122.7	<b>530.1</b>

**Table 4:** Comparison of % error in Ku calculation

Sr. No.	Method	Ku (Act)	Ku (cal)	% Error
1	Conventional Method	28.9	22.6	21.6%
2	Proportional-I Method	28.9	26.5	8%
3	Proportional-II Method	28.9	28.8	0.3%

**Table 5:** PID parameters using Ziegler-Nichols settings

Sr. No.	Method	P	I	D
1	Conventional Relay Method	13	2.5	0.6
2	Prop-I Method	16	2.5	0.6
3	Prop-II Method	17	2.5	0.6

## 5. CONCLUSION

In this study, a proposed method of improved Relay-tuning of PID control has been suggested and applied for the coupled tank level control (SOPTD) model. By using this proposed method, the parameters are optimally and robustly adjusted with respect to the system dynamics. It was found that the % error in Ku calculation found to be 21%, 8% and 0.3% for conventional method, proposed method I and proposed method II respectively. The Integral Time Absolute Error (ITAE) has been decreased and robustness has been increased by proposed method I & II as compared to that of conventional method. Hence the proposed method gives better performance. This technique is found to be more effective than conventional tuning methods. This method can be easily extended to multi input and multi-output systems from basic single-input and single-output systems. The simple structure, robustness and ease of computation of the proposed method make it very attractive.

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