Spring 2016

Trigonometry: An Overview of Important Topics

Lauren Johnson
Governors State University

Trigonometry

An Overview of Important Topics

By Lauren Johnson

B.A. in Mathematics

Governors State University

May 2011

Submitted to the Graduate Faculty of

Governors State University

College of Arts and Sciences

In Partial Fulfillment of the Requirements

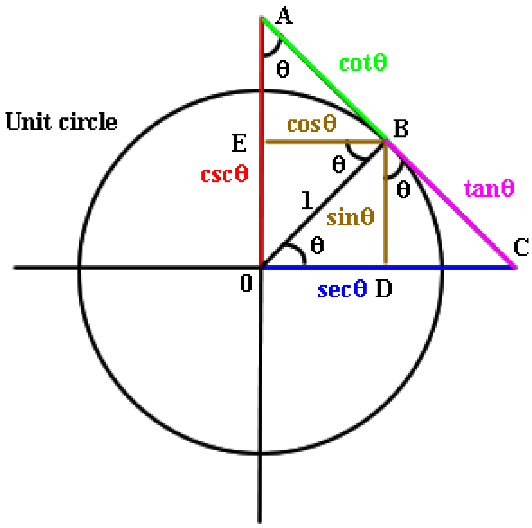
For the degree of Master of Science with a major in Mathematics

May 5th 2016

Abstract

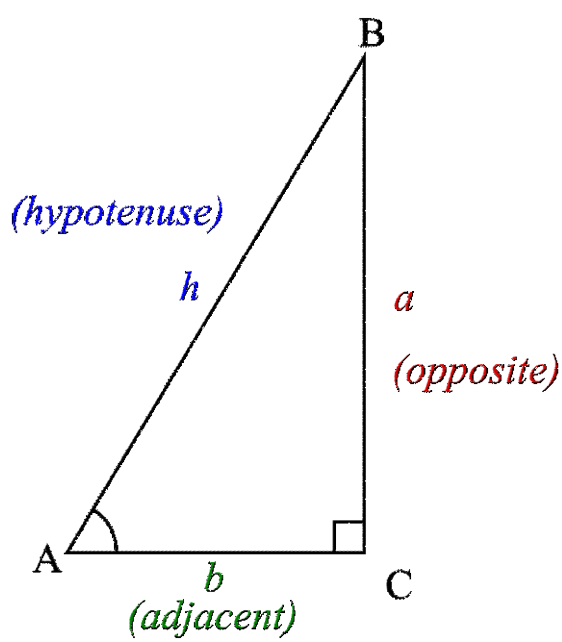
The purpose of this project was to help students achieve a better understanding of Trigonometry, in order to better prepare them for future Calculus courses. The project is a tutorial that walks the student through important Trigonometry topics. The topics range from, but are not limited to, finding the measure of an angle to analyzing the graphs of Trigonometric Functions. Each topic allows an opportunity for the student to assess their understanding by working through practice problems and checking their solutions.

At any university, incoming students will have a wide variety of mathematical backgrounds. These students differ in age, in the education they received, and their attitudes towards math. This tutorial was designed to support all students by equipping them with the knowledge they need to succeed.



Trigonometry

An Overview of Important Topics



Contents

Trigonometry – An Overview of Important Topics

So I hear you're going to take a Calculus course? Good idea to brush up on your Trigonometry!!

Trigonometry is a branch of mathematics that focuses on relationships between the sides and angles of triangles. The word trigonometry comes from the Latin derivative of Greek words for triangle (trigonon) and measure (metron). Trigonometry (Trig) is an intricate piece of other branches of mathematics such as, Geometry, Algebra, and Calculus.

In this tutorial we will go over the following topics.

- Understand how angles are measured
    - Degrees
    - Radians
    - Unit circle
    - Practice
        - Solutions
- Use trig functions to find information about right triangles
    - Definition of trig ratios and functions
    - Find the value of trig functions given an angle measure
    - Find a missing side length given an angle measure
    - Find an angle measure using trig functions
    - Practice
        - Solutions
- Use definitions and fundamental Identities of trig functions
    - Fundamental Identities
    - Sum and Difference Formulas
    - Double and Half Angle Formulas
    - Product to Sum Formulas
    - Sum to Product Formulas
    - Law of Sines and Cosines
    - Practice
        - Solutions

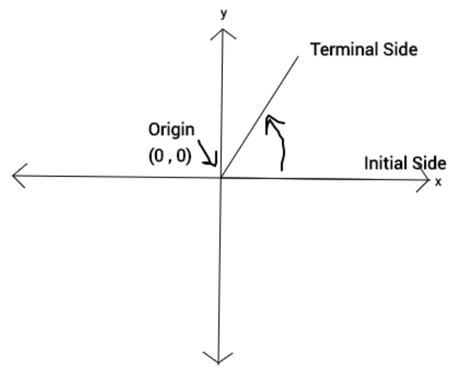
- Understand key features of graphs of trig functions
    - Graph of the sine function
    - Graph of the cosine function
    - Key features of the sine and cosine function
    - Graph of the tangent function
    - Key features of the tangent function
    - Practice
        - Solutions

Back to Table of Contents.

UNDERSTAND HOW ANGLES ARE MEASURED

Since Trigonometry focuses on relationships of sides and angles of a triangle, let's go over how angles are measured…

Angles are formed by an initial side and a terminal side. An initial side is said to be in standard position when it's vertex is located at the origin and the ray goes along the positive x axis.



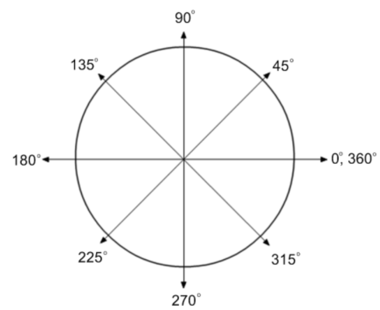
An angle is measured by the amount of rotation from the initial side to the terminal side. A positive angle is made by a rotation in the counterclockwise direction and a negative angle is made by a rotation in the clockwise direction.

Angles can be measured two ways:

1. Degrees
2. Radians

Degrees

A circle is comprised of 360°, which is called one revolution

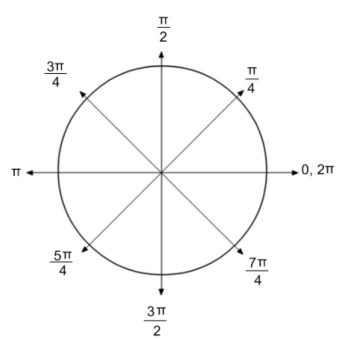


Degrees are used primarily to describe the size of an angle.

The real mathematician is the radian, since most computations are done in radians.

Radians

1 revolution measured in radians is 2π, where π is the constant approximately 3.14.



How can we convert between the two you ask?

Easy, since $360° = 2\pi$ radians (1 revolution)

Then, $180° = \pi$ radians

So that means that $1° = \frac{\pi}{180}$ radians

And $\frac{180}{\pi}$ degrees = 1 radian

Example 1

Convert 60° into radians

$60 \cdot (1 \text{ degree}) \ \frac{\pi}{180} = 60 \cdot \frac{\pi}{180} = \frac{60\pi}{180} = \frac{\pi}{3}$ radian

Example 2

Convert (-45°) into radians

$-45 \cdot \frac{\pi}{180} = \frac{-45\pi}{180} = -\frac{\pi}{4}$ radian

Example 3

Convert $\frac{3\pi}{2}$ radian into degrees

$\frac{3\pi}{2} \cdot (1 \text{ radian}) \frac{180}{\pi} = \frac{3\pi}{2} \cdot \frac{180}{\pi} = \frac{540\pi}{2\pi} = 270°$

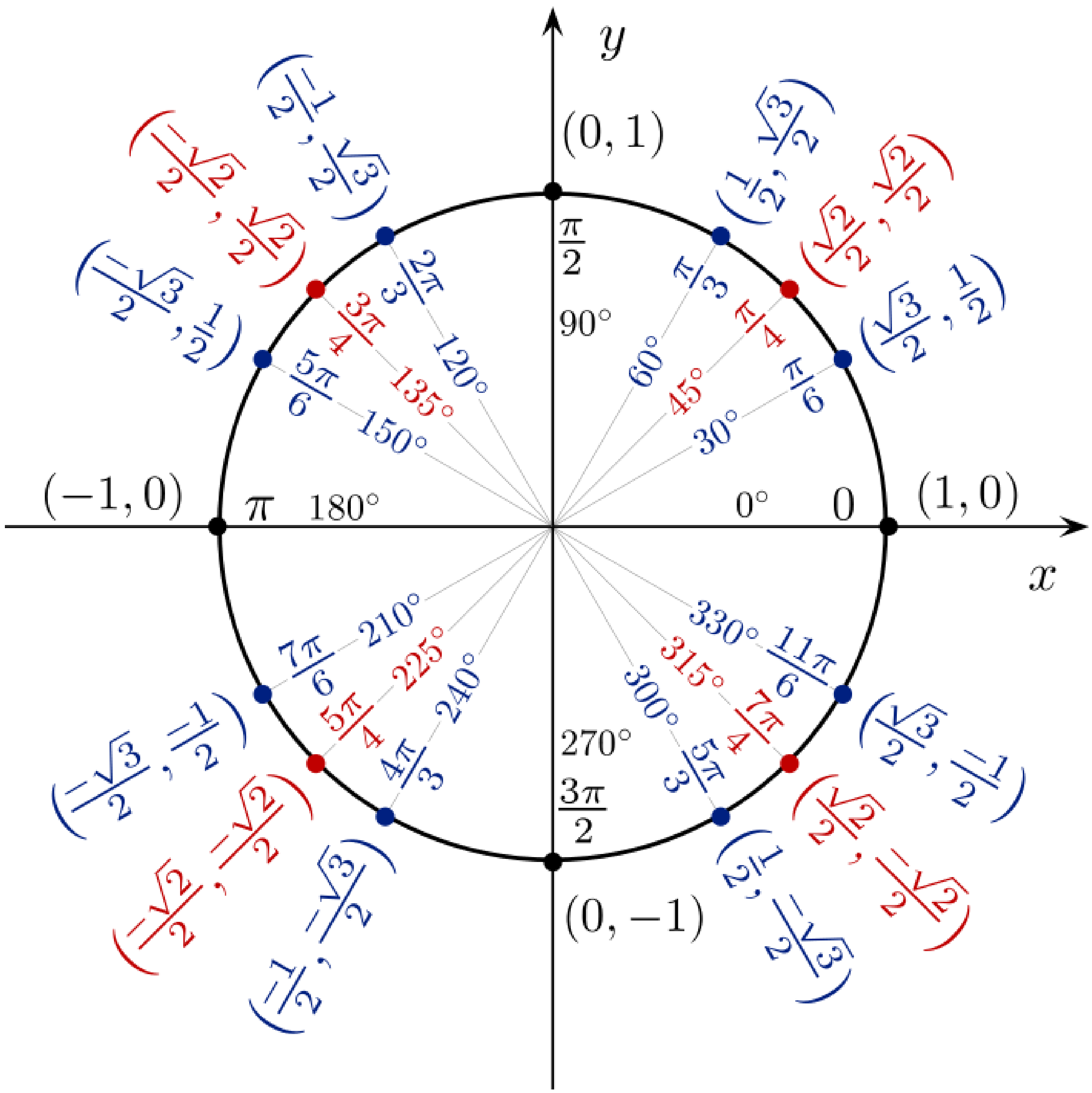
Example 4

Convert $-\frac{7\pi}{3}$ radian into degrees

$$-\frac{7\pi}{3} \cdot \frac{180}{\pi} = \frac{1260}{3} = 420°$$

Before we move on to the next section, let’s take a look at the Unit Circle.

Unit Circle



The Unit Circle is a circle that is centered at the origin and always has a radius of 1. The unit circle will be helpful to us later when we define the trigonometric ratios. You may remember from Algebra 2 that the equation of the Unit Circle is $x^2 + y^2 = 1$.

Need more help? Click below for a Khan Academy video

[Khan Academy video 1]

Practice Problems

Convert the following degree measures into radians

1) 45°

2) 76°

3) 510°

4) -240°

5) 0°

6) 150°

7) 40°

8) 270°

9) 120°

10) 10°

11) 50°

12) -30°

13) 6°

14) 300°

15) -24°

Convert the following radian measures into degrees

16) $\frac{7\pi}{3}$

17) $\frac{\pi}{6}$

18) $\frac{\pi}{18}$

19) $\frac{3\pi}{4}$

20) $-\frac{13\pi}{4}$

21) π

22) $\frac{7\pi}{4}$

23) $-\frac{17\pi}{4}$

24) 2π

25) $\frac{\pi}{3}$

26) $\frac{2\pi}{3}$

27) $-\frac{\pi}{4}$

28) 0

29) $\frac{5\pi}{4}$

30) 3π

Solutions

Convert the following degree measures into radians

1) $\frac{\pi}{4}$ 2) $\frac{19\pi}{45}$ 3) $\frac{17\pi}{6}$ 4) $-\frac{4\pi}{3}$ 5) 0 6) $\frac{5\pi}{6}$ 7) $\frac{2\pi}{9}$ 8) $\frac{3\pi}{2}$

9) $\frac{2\pi}{3}$ 10) $\frac{\pi}{18}$ 11) $\frac{5\pi}{18}$ 12) $-\frac{\pi}{6}$ 13) $\frac{\pi}{30}$ 14) $\frac{5\pi}{3}$ 15) $-\frac{2\pi}{15}$

Convert the following radian measures into degrees

16) 420° 17) 30° 18) 10° 19) 135° 20) -585° 21) 180° 22) 315°

23)-765° 24) 360° 25) 60° 26) 120° 27)$-45°$ 28) 0° 29) 225°

30) 540°

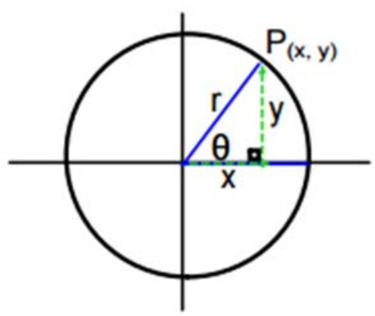
[Back to Table of Contents.]

TRIGONOMETRIC FUNCTIONS

Definitions of trig ratios and functions

In Trigonometry there are six trigonometric ratios that relate the angle measures of a right triangle to the length of its sides. (Remember a right triangle contains a 90° angle)

A right triangle can be formed from an initial side *x* and a terminal side *r*, where *r* is the radius and hypotenuse of the right triangle. (see figure below) The Pythagorean Theorem tells us that $x^2 + y^2 = r^2$, therefore $r = \sqrt{x^2 + y^2}$. θ (theta) is used to label a non-right angle. The six trigonometric functions can be used to find the ratio of the side lengths. The six functions are sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot). Below you will see the ratios formed by these functions.



$\sin\theta = \frac{y}{r}$, also referred to as $\frac{opposite\ side}{hypotenuse}$

$\cos\theta = \frac{x}{r}$, also referred to as $\frac{adjacent\ side}{hypotenuse}$

$\tan\theta = \frac{y}{x}$, also referred to as $\frac{opposite\ side}{adjacent\ side}$

These three functions have 3 reciprocal functions

$\csc\theta = \frac{r}{y}$, which is the reciprocal of $\sin\theta$

sec θ = $\frac{r}{x}$,which is the reciprocal of cos

cot θ = $\frac{x}{y}$, which is the reciprocal of tan

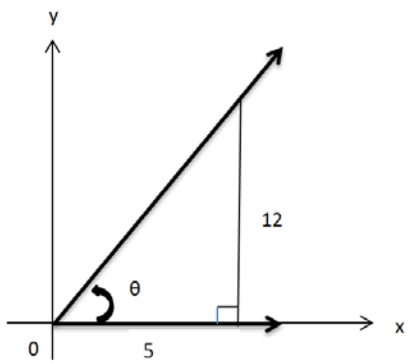
You may recall a little something called SOH-CAH-TOA to help your remember the functions!

SOH… Sine = opposite/hypotenuse

…CAH… Cosine = adjacent/hypotenuse

…TOA Tangent = opposite/adjacent

Example: Find the values of the trigonometric ratios of angle θ



Before we can find the values of the six trig ratios, we need to find the length of the missing side. Any ideas? Good call, we can use r = $\sqrt{x^2 + y^2}$ (from the Pythagorean Theorem)

r = $\sqrt{5^2 + 12^2}$ = $\sqrt{25 + 144}$ = $\sqrt{169}$ = 13

Now we can find the values of the six trig functions

sin θ = $\frac{opposite}{hyptoenuse}$ = $\frac{12}{13}$

cos θ = $\frac{adjacent}{hypotenuse}$ = $\frac{5}{13}$

tan θ = $\frac{opposite}{adjacent}$ = $\frac{12}{5}$

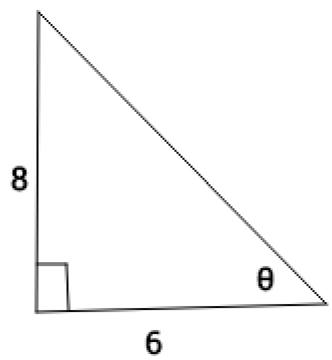
csc θ = $\frac{hyptoenuse}{opposite}$ = $\frac{13}{12}$

sec θ = $\frac{hypotenuse}{adjacent}$ = $\frac{13}{5}$

cot θ = $\frac{adjacent}{opposite}$ = $\frac{5}{12}$

Example 5

a) Use the triangle below to find the six trig ratios



First use Pythagorean Theorem to find the hypotenuse

$a^2 + b^2 = c^2$, where a and b are legs of the right triangle and c is the hypotenuse

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$100 = c^2$

$\sqrt{100} = \sqrt{c^2}$

$10 = c$

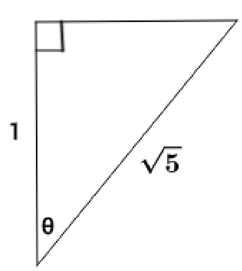
$\sin\theta = \frac{o}{h} = \frac{8}{10} = \frac{4}{5}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{5}{4}$

$\cos\theta = \frac{a}{h} = \frac{6}{10} = \frac{3}{5}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$

$\tan\theta = \frac{o}{a} = \frac{8}{6} = \frac{4}{3}$ $\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$

Example 6

Use the triangle below to find the six trig ratios



$1^2 + b^2 = (\sqrt{5})^2$

$1 + b^2 = 5$

$b^2 = 4$

$b = 2$

$\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\csc\theta = \frac{\sqrt{5}}{2}$

$\cos\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ $\sec\theta = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\tan\theta = \frac{2}{1} = 2$ $\cot\theta = \frac{1}{2}$

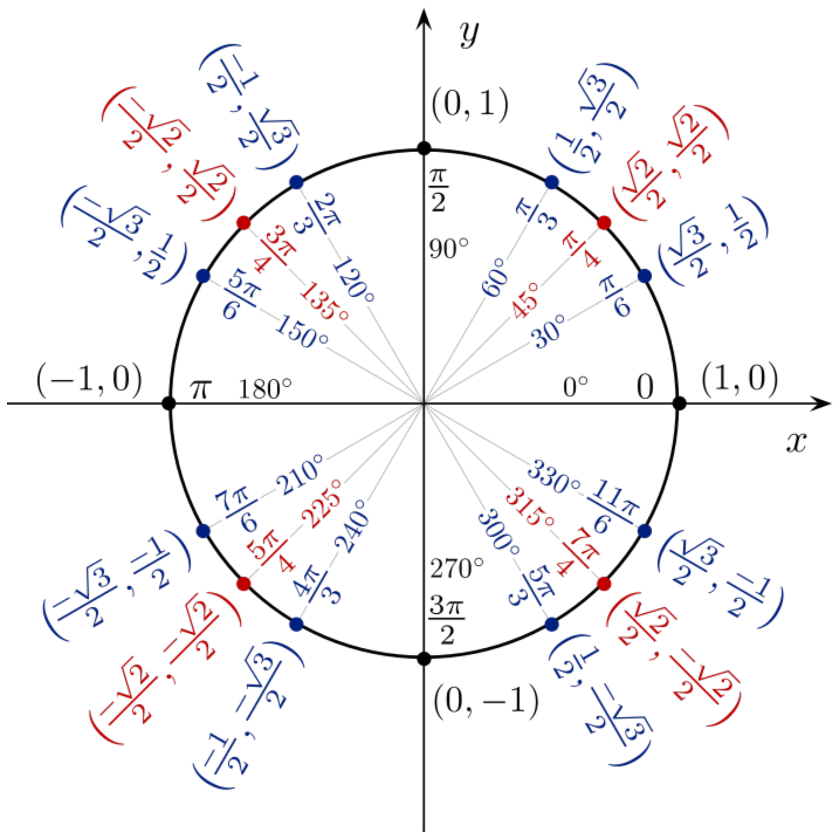
Need more help? Click below for a Khan Academy Video

Khan Academy video 2

Find the value of trig functions given an angle measure

Suppose you know the value of θ is 45°, how can this help you find the values of the six trigonometric functions?

First way: You can familiarize yourself with the unit circle we talked about.



An ordered pair along the unit circle (x, y) can also be known as (cos θ, sin θ), since the r value on the unit circle is always 1. So to find the trig function values for 45° you can look on the unit circle and easily see that sin 45° = $\frac{\sqrt{2}}{2}$, cos 45° = $\frac{\sqrt{2}}{2}$

With that information we can easily find the values of the reciprocal functions

csc 45° = $\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$, sec 45° = $\sqrt{2}$

We can also find the tangent and cotangent function values using the quotient identities

$\tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$\cot 45° = 1$

Example 7

Find $\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$

Example 8

Find $\tan\left(\frac{\pi}{6}\right) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

Example 9

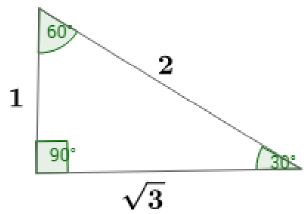
Find $\cot 240° = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

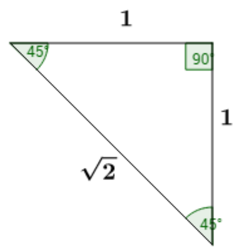
Using this method limits us to finding trig function values for angles that are accessible on the unit circle, plus who wants to memorize it!!!

Second Way: If you are given a problem that has an angle measure of 45°, 30°, or 60°, you are in luck! These angle measures belong to special triangles.

If you remember these special triangles you can easily find the ratios for all the trig functions.

Below are the two special right triangles and their side length ratios





How do we use these special right triangles to find the trig ratios?

If the θ you are given has one of these angle measures it's easy!

Example 10

Find $\sin 30°$

$\sin 30° = \frac{1}{2}$

Example 11

Find $\cos 45°$

$\cos 45° = \frac{\sqrt{2}}{2}$

Example 12

Find $\tan 60°$

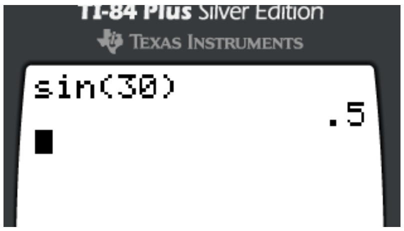
$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

Third way: This is not only the easiest way, but also this way you can find trig values for angle measures that are less common. You can use your TI Graphing calculator.

First make sure your TI Graphing calculator is set to degrees by pressing mode

Next choose which trig function you need

After you choose which function you need type in your angle measure



Example 13

$\cos 55° \approx 0.5736$

Example 14

$\tan 0° = 0$

Example 15

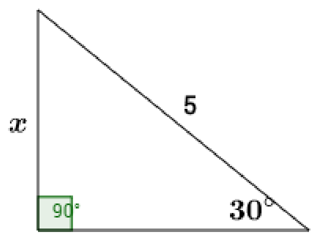
$\sin 30° = 0.5$

Find a missing side length given an angle measure

Suppose you are given an angle measure and a side length, can you find the remaining side lengths?

Yes. You can use the trig functions to formulate an equation to find missing side lengths of a right triangle.

Example 16



First we know that $\sin\theta = \frac{o}{h}$, therefore $\sin 30 = \frac{x}{5}$

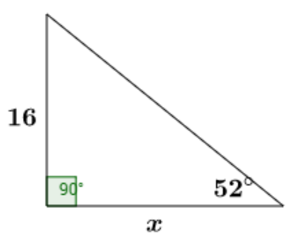
Next we solve for x, $5 \cdot \sin 30 = x$

Use your TI calculator to compute $5 \cdot \sin 30$,

And you find out $x = 2.5$

Let's see another example,

Example 17



We are given information about the opposite and adjacent sides of the triangle, so we will use tan

$$\tan 52 = \frac{16}{x}$$

$$x = \frac{16}{\tan 52}$$

$$x \approx 12.5$$

Need more help? Click below for a Khan Academy video

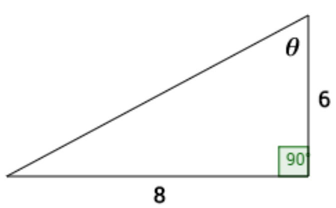
[Khan Academy video 3]

Find an angle measure using trig functions

Wait a minute, what happens if you have the trig ratio, but you are asked to find the angles measure? Grab your TI Graphing calculator and notice that above the sin, cos, and tan buttons, there is sin^{-1}, cos^{-1}, tan^{-1}. These are your inverse trigonometric functions, also known as arcsine, arccosine, and arctangent. If you use these buttons in conjunction with your trig ratio, you will get the angle measure for θ!

Let's see some examples of this.

Example 18



We know that $\tan\theta = \frac{8}{6}$

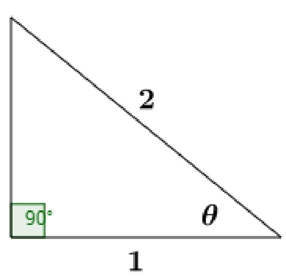
So to find the value of θ, press 2nd tan on your calculator and then type in (8/6)

$$tan^{-1}\left(\frac{8}{6}\right) \approx 53.13$$

$$\theta \approx 53.13°$$

How about another

Example 19



We are given information about the adjacent side and the hypotenuse, so we will use the cosine function

$$\cos\theta = \frac{1}{2}$$

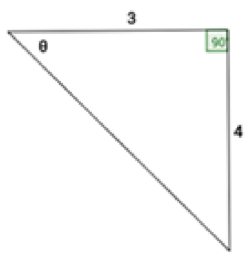
$$cos^{-1}\left(\frac{1}{2}\right) = 60$$

$$\theta = 60°$$

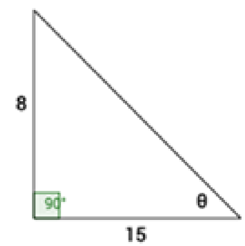
Practice Problems

Find the value of the six trigonometric functions

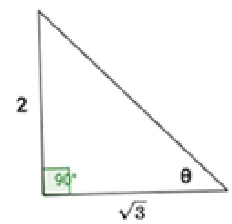
1)



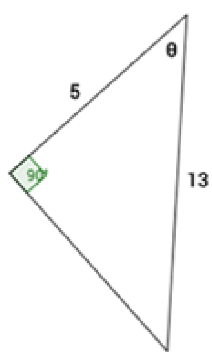
2)



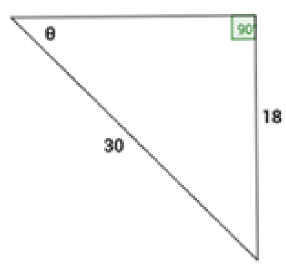
3)



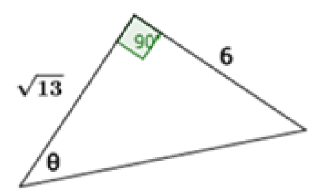
4)



5)



6)



Use the definition of the trig ratios to find the trig function indicated

7) Given: $\cos\theta = \frac{4}{5}$, find $\tan\theta$

8) Given: $\csc\theta = \frac{25}{7}$, find $\sec\theta$

9) Given: $\sin\theta = \frac{2}{3}$, $and\ \cos\theta = \frac{\sqrt{5}}{3}$, find $\cot\theta$

10) Given: $\cos\theta = \frac{\sqrt{3}}{2}$, find $\tan\theta$

Use the Unit Circle to find the values of the trig functions

11) $\cos 45°$ 12) $\sin 30°$ 13) $\sin \frac{3\pi}{4}$ 14) $\tan \frac{7\pi}{6}$

15) $\sec(-90°)$ 16) $\cot(-45°)$ 17) $\csc 150°$ 18) $\sin 270°$

19) $\cos \frac{5\pi}{4}$ 20) $\tan \frac{11\pi}{6}$

Use the special triangles (30-60-90 and 45-45-90) to find the values of the trig functions

21) $\cos 30°$ 22) $\sin 60°$ 23) $\csc 45°$ 24) $\cot 45°$

25) $\sin 30°$ 26) $\sec 45°$ 27) $\tan 60°$ 28) $\cos 45°$

Use your graphing calculator to compute the following trig values. Round to four decimal places

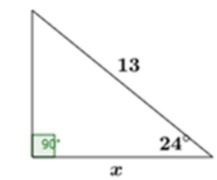
29) $\tan 43°$ 30) $\sin 13°$ 31) $\cos 120°$ 32) $\cot 79°$

33) $\sec 30°$ 34) $\cot 240°$ 35) $\sin 0$ 36) $\csc 10°$

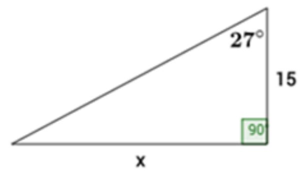
37) $\sin 30°$ 38) $\sin -330°$ 39) $\tan -36°$ 40) $\cos 135°$

Use the trig functions to find the missing side lengths. Round to the nearest hundredth.

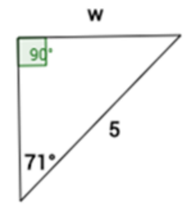
41)



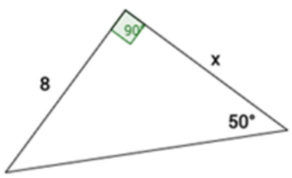
42)



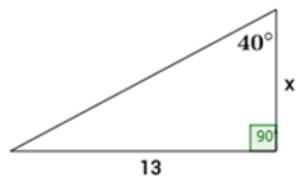
43)



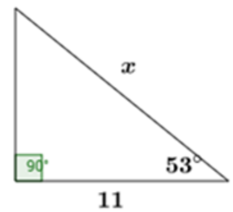
44)



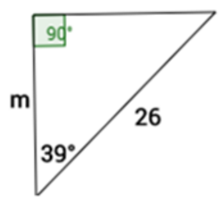
45)



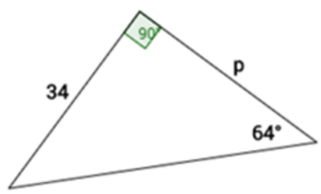
46)



47)



48)



Use the trig functions to find the value of θ. Round to the nearest degree.

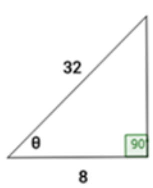
49) $\cos\theta = \frac{1}{2}$

50) $\tan\theta = \frac{30}{50}$

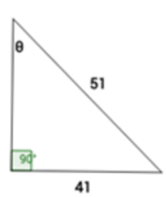
51) $\sin\theta = \frac{6}{7}$

52) $\cos\theta = \frac{42}{48}$

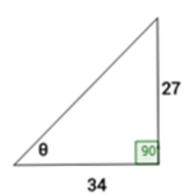
53)



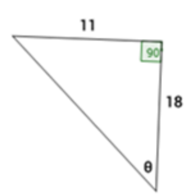
54)



55)



56)



57) $\cos\theta = \frac{48}{59}$

58) $\tan\theta = \frac{20}{17}$

59) $\sin\theta = \frac{2}{3}$

60) $\sin\theta = \frac{1}{3}$

Solutions

Find the value of the six trigonometric functions

1) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

2) $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$, $\tan\theta = \frac{8}{15}$, $\csc\theta = \frac{17}{8}$, $\sec\theta = \frac{17}{15}$, $\cot\theta = \frac{15}{8}$

3) $\sin\theta = \frac{2\sqrt{7}}{7}$, $\cos\theta = \frac{\sqrt{21}}{7}$, $\tan\theta = \frac{2\sqrt{3}}{3}$, $\csc\theta = \frac{\sqrt{7}}{2}$, $\sec\theta = \frac{\sqrt{21}}{3}$, $\cot\theta = \frac{\sqrt{3}}{2}$

4) $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$, $\csc\theta = \frac{13}{12}$, $\sec\theta = \frac{13}{5}$ $\cot\theta = \frac{5}{12}$

5) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$

6) $\sin\theta = \frac{6}{7}$, $\cos\theta = \frac{\sqrt{13}}{7}$, $\tan\theta = \frac{6\sqrt{13}}{13}$, $\csc\theta = \frac{7}{6}$, $\sec\theta = \frac{7\sqrt{13}}{13}$, $\cot\theta = \frac{\sqrt{13}}{6}$

Use the definition of the trig ratios to find the trig function indicated

7) $tan\ \theta = \frac{3}{4}$ 8) $sec\ \theta = \frac{25}{24}$ 9) $cot\ \theta = \frac{\sqrt{5}}{2}$ 10) $tan\ \theta = \frac{\sqrt{3}}{3}$

Use the Unit Circle to find the values of the trig functions

11) $\frac{\sqrt{2}}{2}$ 12) $\frac{1}{2}$ 13) $\frac{\sqrt{2}}{2}$ 14) $\frac{\sqrt{3}}{3}$ 15) Undefined 16) -1 17) 2 18) -1 19) $-\frac{\sqrt{2}}{2}$ 20) $-\frac{\sqrt{3}}{3}$

Use the special triangles to find the values of the trig functions

21) $\frac{\sqrt{3}}{2}$ 22) $\frac{\sqrt{3}}{2}$ 23) $\sqrt{2}$ 24) 1

25) $\frac{1}{2}$ 26) $\sqrt{2}$ 27) $\sqrt{3}$ 28) $\frac{\sqrt{2}}{2}$

** Notice by the complementary angle theorem #11 and #12 have the same value because

$\cos 30 = \sin 60$

Compute the following trig values. Round to four decimal places

29) 0.9325 30) 0.225 31) -0.5 32) 0.1944

33) 1.1547 34) 0.5774 35) 0 36) 5.7588

37) 0.5 38) 0.5 39) -0.7265 40) -0.7071

Use trig functions to find missing side lengths

41) 11.88 42) 7.64 43) 4.73 44) 6.71

45) 15.49 46) 18.28 47) 20.21 48) 16.58

Use trig functions to find the value of θ

49) 60° 50) 31° 51) 59° 52) 29°

53) 76° 54) 54° 55) 38° 56) 31°

57) 36° 58) 50° 59) 42° 60) 19°

Back to Table of Contents.

USING DEFINITIONS AND FUNDAMENTAL IDENTITIES OF TRIG FUNCTIONS

Fundamental Identities

Reciprocal Identities

sin θ = 1/(csc θ)

csc θ = 1/(sin θ)

cos θ = 1/(sec θ)

sec θ = 1/(cos θ)

tan θ = 1/(cot θ)

cot θ = 1/(tan θ)

Quotient Identities

tan θ = (sin θ)/(cos θ)

cot θ = (cos θ)/(sin θ)

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad$ $\sec(-\theta) = \sec\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Complementary Angle Theorem

If two acute angles add up to be 90°, they are considered complimentary.

The following are considered cofunctions:

sine and cosine $\quad$ tangent and cotangent $\quad$ secant and cosecant

The complementary angle theorem says that cofunctions of complimentary angles are equal.

Example 20) $\sin 54° = \cos 36°$

How can we use these identities to find exact values of trigonometric functions?

Follow these examples to find out! Examples 21-26

21) Find the exact value of the expression

$\sin^2 30° + \cos^2 30°$

Solution: Since $\sin^2\theta + \cos^2\theta = 1$, therefore $\sin^2 30° + \cos^2 30° = 1$

22) Find the exact value of the expression

$$\tan 45° - \frac{\sin 45°}{\cos 45°}$$

Solution: Since $\left(\frac{\sin 45°}{\cos 45°}\right) = \tan 45°$, therefore $\tan 45° - \tan 45° = 0$

23) $\tan 35° \cdot \cos 35° \cdot \csc 35°$

Solution: $\frac{\sin 35°}{\cos 35°} \cdot \frac{\cos 35°}{1} \cdot \frac{1}{\sin 35°} = 1$

24) $\tan 22° - \cot 68°$

Solution: $\tan 22° = \cot 68°$, therefore $\cot 68° - \cot 68° = 0$

25) $\cot\theta = -\frac{2}{3}$, find $\csc\theta$, where θ is in quadrant II

Solution: Pick an identity that relates cotangent to cosecant, like the Pythagorean identity $1 + \cot^2\theta = \csc^2\theta$.

$$1 + \left(-\frac{2}{3}\right)^2 = \csc^2\theta$$

$$1 + \frac{4}{9} = \csc^2\theta$$

$$\frac{13}{9} = \csc^2\theta$$

$$\sqrt{\frac{13}{9}} = \csc\theta$$

$$\frac{\sqrt{13}}{3} = \csc\theta$$

The positive square root is chosen because csc is positive in quadrant II

26) Prove the following identity is true

$$\cot\theta \cdot \sin\theta \cdot \cos\theta = \cos^2\theta$$

Solution: $\frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{1} \cdot \frac{\cos\theta}{1} = \cos^2\theta$

Need more help? Click below for a Khan Academy video

Khan Academy video 4

Sum and Difference Formulas

In this section we will use formulas that involve the sum or difference of two angles, call the sum and difference formulas.

Sum and difference formulas for sines and cosines

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

How do we use these formulas?

Example 27 Find the exact value of $\cos 105°$

Well we can break 105° into 60° and 45° since those values are relatively easy to find the cosine of.

Therefore $\cos 105° = \cos(60° + 45°)$= $\cos 60°\cos 45° - \sin 60°\sin 45°$

Using the unit circle we obtain,

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{1}{4}(\sqrt{2} - \sqrt{6})$

Example 28 Find the exact value of $\sin 15°$

$= \sin(45° - 30°)$

$= \sin 45°\cos 30° - \cos 45°\sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$

Sum and difference formulas for tangent

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Example 29 Find the exact value of $\tan 75°$

$\tan 75° = \tan(45° + 30°)$

$= \frac{\tan 45°+\tan 30°}{1-\tan 45° \tan 30°} = \frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}$ (rationalize the denominator)

$$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{12+6\sqrt{3}}{6} = 2+\sqrt{3}$$

Example 30 Find $\tan\left(\frac{7\pi}{12}\right)$

$\tan\frac{7\pi}{12} = \tan\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right) = \tan\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$= \frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{3}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{\pi}{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$

Cofunction Identities

$\cos(90° - \theta) = \sin\theta$ $\quad$ $\sec(90° - \theta) = \csc\theta$

$\sin(90° - \theta) = \cos\theta$ $\quad$ $\csc(90° - \theta) = \sec\theta$

$\tan(90° - \theta) = \cot\theta$ $\quad$ $\cot(90° - \theta) = \tan\theta$

Example 31

Find $\cos 30°$

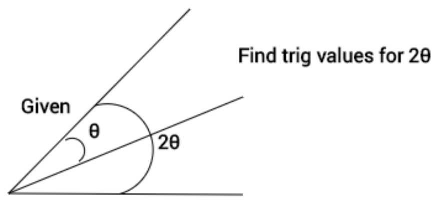
$$\cos 30° = \sin(90° - 30°) = \sin 60° = \frac{\sqrt{3}}{2}$$

Need more help? Click below for a Khan Academy video

Khan Academy video 5

Double and Half Angle Formulas

Below you will learn formulas that allow you to use the relationship between the six trig functions for a particular angle and find the trig values of an angle that is either half or double the original angle.



Double Angle Formulas

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Half Angle Formulas

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ $\quad\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$ $\quad\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

Lets see these formulas in action!

Example 32 Use the double angle formula to find the exact value of each expression

$\sin 120°$

$$\sin 120° = \sin 2(60°) = 2\sin 60° \cdot \cos 60° = \frac{\sqrt{3}}{2}$$

Example 33

$\tan\theta = \frac{5}{12}$ $and\ \pi < \theta < \frac{3\pi}{2}$, $Find \cos 2\theta$

First we need to find what the $\cos\theta$ is. We know that $\tan\theta$ is opposite leg over adjacent leg, so we need to find the hypotenuse since cos is adjacent over hypotenuse. We can use $r = \sqrt{12^2 + 5^2} = 13$ to find the length of the hypotenuse. Now we know the $\cos\theta = \frac{12}{13}$. Now use the double angle formula to find $\cos 2\theta$.

$$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{5}{13}\right)^2 = 1 - 2\left(\frac{25}{169}\right) = 1 - \frac{50}{169} = \frac{119}{169}$$

We take the positive answer since θ is in the third quadrant making the ratio a negative over a negative.

Now lets try using the half angle formula

Example 34

$\cos 15°$

$$\cos 15° = \cos\frac{30}{2} = \pm\sqrt{\frac{1 + \cos 30}{2}} = \pm\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

Choose the positive root

Example 35

$\cos\theta = -\frac{4}{5}$ and $90° < \theta < 180°$. $Find \sin\frac{\theta}{2}$

First we use the Pythagorean Theorem to find the third side

$4^2 + x^2 = 5^2$

$x^2 = 9$

$x = 3$

$\sin\theta = \frac{3}{5}$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}} = \pm\sqrt{\frac{\frac{9}{5}}{2}} = \pm\sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

Since sin is positive in the third quadrant we take the positive answer

Need more help? Click below for a Khan Academy video

Khan Academy video 6

Product to Sum Formulas

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

Example 36 Use the product-to-sum formula to change $\sin 75° \sin 15°$ to a sum

$$\sin 75° \sin 15° = \frac{1}{2}[\cos(75° - 15°) - \cos(75° + 15°)] = \frac{1}{2}[\cos 60° - \cos 90°]$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0\right] = \frac{1}{4}$$

Sum to Product Formulas

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

Example 37 Use the sum-to-product formula to change $\sin 70° - \sin 30°$ into a product

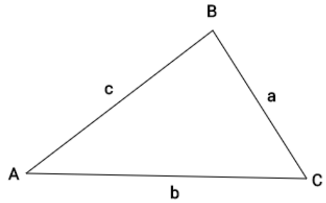
$$\sin 70° - \sin 30° = 2\cos\left(\frac{70° + 30°}{2}\right)\sin\left(\frac{70° - 30°}{2}\right) = 2\cos 50° \cdot \sin 20°$$

Law of Sines and Cosines

These laws help us to find missing information when dealing with oblique triangles (triangles that are not right triangles)

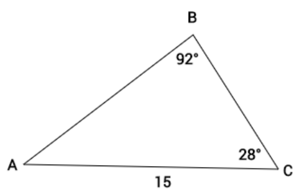
Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



You can use the Law of Sines when the problem is referring to two sets of angles and their opposite sides.

Example 38 Find the length of AB. Round your answer to the nearest tenth.



Since we are given information about an angle, the side opposite of that angle, another angle, and missing the side opposite of that angle, we can apply the Law of Sines.

$$\frac{\sin 92°}{15} = \frac{\sin 28°}{AB}$$

Multiply both sides by the common denominator in order to eliminate the fractions. We do this so that we can solve for the unknown. This gives us,

$$\sin 92 \cdot AB = \sin 28 \cdot 15$$

Then we can divide by $\sin 92$. When we do this we find $AB = 7$

Law of Cosines

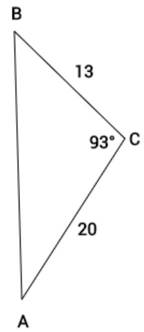
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

You can use the Law of Cosines when the problem is referring to all three sides and only one angle.

Example 39 Find the length of AB. Round to the nearest tenth.



Since all three sides of the triangle are referred to and information about one angle is given, we can use the Law of Cosines.

Since AB is opposite of <C, we will call it c and use the following formula,

$c^2 = a^2 + b^2 - 2ab\cos c$

$c^2 = 13^2 + 20^2 - 2(13)(20)\cos 93°$

$c^2 = 596$

$c = 14$

Practice Problems

1-4 Find the exact value of the expression

1) $\cos^2 30 + \cos^2 60$ 2) $\cot 45 - \tan 45$ 3) $\sin^2 53 + \cos^2 53$ 4) $\cot 20 \cdot \tan 20$

5-6 Use the given information to find the exact value

5) $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, where θ is in quadrant 1. Find $\tan\theta$

6) $\tan\theta = -\frac{4}{5}$, where θ is in quadrant 4. Find $\sec\theta$

7-12 Verify the identity

7) $\tan\theta \cdot \sin\theta \cdot \cos\theta = \sin^2\theta$ 8) $\frac{\tan\theta}{\sec\theta} = \sin\theta$ 9) $\tan^2\theta\cos^2\theta + \cot^2\theta\sin^2\theta = 1$

10) $\frac{\cot\theta}{\csc\theta} = \cos\theta$ 11) $\sec\theta\sin\theta = \tan\theta$ 12) $1 - \tan^2(-\theta) = \sec^2\theta$

Use sum and difference formulas to find exact values

13) $\tan\frac{5\pi}{12}$ 14) $\cos 15°$ 15) $\sin 75°$ 16) $\cos(105°)$ 17) $\sin\frac{7\pi}{12}$

Determine what the original trig function and angle measure were based on formula given

18) $\cos 40°\cos 50° - \sin 40°\sin 50°$ 19) $\sin 15°\cos 30° + \cos 15°\sin 30°$

Use the cofunction identities to rewrite the expressions, then find the values

20) $\sin 120°$ 21) $\tan 60°$ 22) $\cos 45°$ 23) $\sin 60°$

Use the Double Angle Formula to find the exact values

24-27 Find the value of θ given 2θ

24) $\tan 60°$ 25) $\sin\frac{4\pi}{3}$ 26) $\cos 300°$ 27) $\tan 120°$

28-31 Use the double angle formula to find the exact value of the trig function

28) $\sin\theta = -\frac{7}{25}$ and $\frac{3\pi}{2} < \theta < 2\pi. Find \cos 2\theta$

29) $\tan\theta = \frac{3}{4}$ and $\pi < \theta < \frac{3\pi}{2}. Find \tan 2\theta$

30) $\sin\theta = \frac{2\sqrt{2}}{3}$ and $\frac{\pi}{2} < \theta < \pi. Find \sin 2\theta$

31) $\cos\theta = \frac{1}{3}$ and $0° < \theta < 90°. Find \sin 2\theta$

32-33 See if you can use the double angle formula to work backwards. ($\boldsymbol{Given\ 2\theta, find\ \theta}$)

32) $\cos 2\theta = \frac{3}{4}$ and $0° < \theta < 90°. Find \sin\theta$

33) $\cos 2\theta = \frac{3}{5}$ and $0° < \theta < 90°. Find \sin\theta$

34-37 Find the value of $\boldsymbol{\theta}$ given $\boldsymbol{\frac{\theta}{2}}$

34) $\tan 75°$

35) $\sin 112.5°$

36) $\cos\frac{5\pi}{12}$

37) $\cos 30°$

38-41Use the half angle formula to find the exact value of the trig function

38) $\cos\theta = -\frac{15}{17}$ and $180° < \theta < 270°. Find \tan\frac{\theta}{2}$

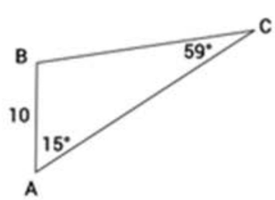
39) $\tan\theta = -\frac{7}{24}$ and $\frac{3\pi}{2} < \theta < 2\pi. Find \sin\frac{\theta}{2}$

40) $\sin\theta = -\frac{3}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi.$ $Find \tan\frac{\theta}{2}$

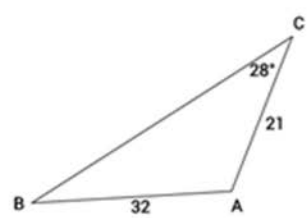
41) $\tan\theta = 2$ and $0° < \theta < \frac{\pi}{2}. Find \sin\frac{\theta}{2}$

Use the Law of Sines to find missing information about oblique triangles

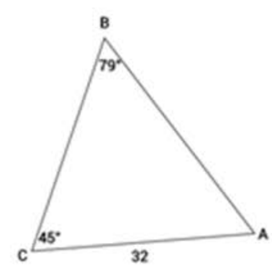
42) Find BC



43) Find < B

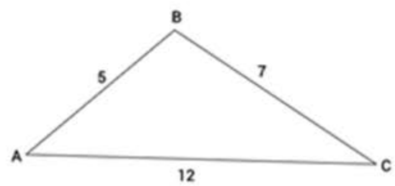


44) Find AB

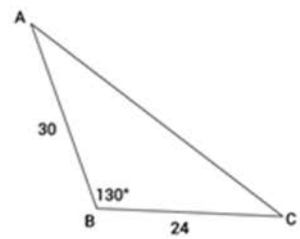


Use the Law of Cosines to find missing information about oblique triangles

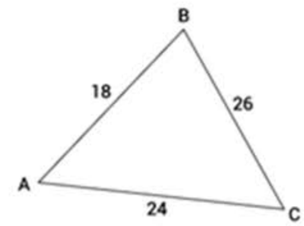
45) Find < B



46) Find AC



47) Find < C



Solutions

1) 1 2) 0 3) 1 4) 1 5) $\frac{\sqrt{3}}{3}$ 6) $\frac{\sqrt{41}}{5}$ 7) $\frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\sin\theta}{1} \cdot \frac{\cancel{\cos\theta}}{1}$ 8) $\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}$

9) $\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{1} + \frac{\cos^2\theta}{\cancel{\sin^2\theta}} \cdot \frac{\cancel{\sin^2\theta}}{1} = \sin^2\theta + \cos^2\theta = 1$ 10) $\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \cos\theta$ 11) $\frac{1}{\cos\theta} \cdot \sin\theta = \tan\theta$ 12) $1 + \tan^2\theta$

13) $2 + \sqrt{3}$ 14) $(\sqrt{6} + \sqrt{2})/4$ 15) $(\sqrt{2} + \sqrt{6})/4$ 16) $(\sqrt{2} - \sqrt{6})/4$ 17) $(\sqrt{6} + \sqrt{2})/4$

18) $\cos 90°$ 19) $\sin 45°$ 20) $\frac{\sqrt{3}}{2}$ 21) $\sqrt{3}$ 22) $\frac{\sqrt{2}}{2}$ 23) $\frac{\sqrt{3}}{2}$

24) $\sqrt{3}$ 25) $-\frac{\sqrt{3}}{2}$ 26) $\frac{1}{2}$ 27) $-\sqrt{3}$ 28) $\frac{527}{625}$ 29) $\frac{24}{7}$ 30) $-\frac{4\sqrt{2}}{9}$ 31) $\frac{4\sqrt{2}}{9}$ 32) $\frac{\sqrt{2}}{4}$ 33) $\frac{\sqrt{5}}{5}$ 34) $2 + \sqrt{3}$

35) $\frac{\sqrt{2+\sqrt{2}}}{2}$ 36) $\frac{\sqrt{2-\sqrt{3}}}{2}$ 37) $\frac{\sqrt{3}}{2}$ 38) $-\frac{5}{3}$ 39) $-\frac{\sqrt{2}}{10}$ 40) $-\frac{1}{3}$ 41) $\frac{\sqrt{50-10\sqrt{5}}}{10}$

42) 3 43) 17.9° 44) 23.1 45) 180° 46) 49 47) 42°

Back to Table of Contents.

UNDERSTAND KEY FEATURES OF GRAPHS OF TRIG FUNCTIONS

In this section you will get a brief introduction to the graphs of the three main trig functions, sine, cosine, and tangent. This section will not go over how to actually graph these functions, but will go over how to identify key features of the graphs of each function.

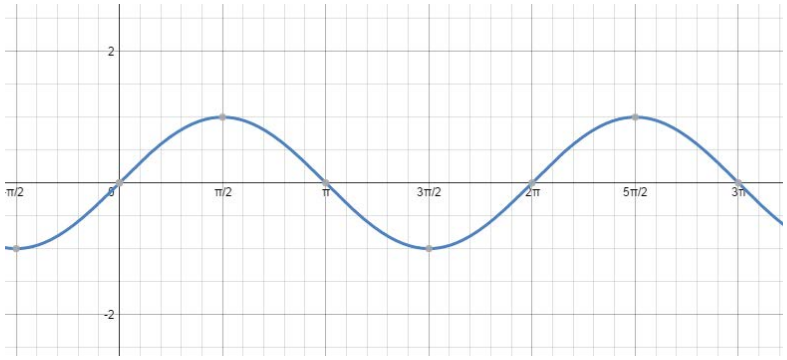
The graphs of sine and cosine are considered periodic functions, which basically means their values repeat in regular intervals known as periods.

A periodic function is a function f such that

$$f(x) = f(x + np)$$

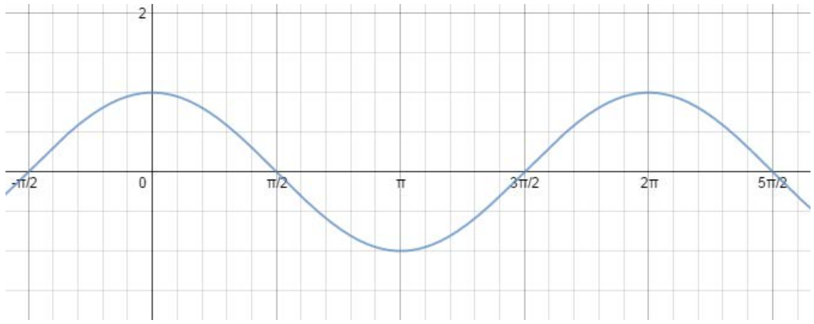
We talked about the fact that one revolution of the unit circle is 2π radians, which means that the circumference of the unit circle is 2π. Therefore, the sine and cosine function have a period of 2π.

Graph of the sine function $(y = sin\ x)$



- If you notice, the range of the sine function is [-1, 1] and the domain is (-∞, ∞)
- Also notice that the x-intercepts are always in the form nπ. Where n is an integer
- This is an odd function because it is symmetric with respect to the origin
- The period is 2π because the sine wave repeats every 2π units

Graph of the cosine function $(y = cos\ x)$



- If you notice, the range of the cosine function is [-1, 1] and the domain is (-∞, ∞)
- This is an even function because it is symmetric with respect to the y axis therefore for all cos(-x) = cos (x)
- The period is also 2π because the cosine wave repeats every 2π units

Key features of the sine and cosine function

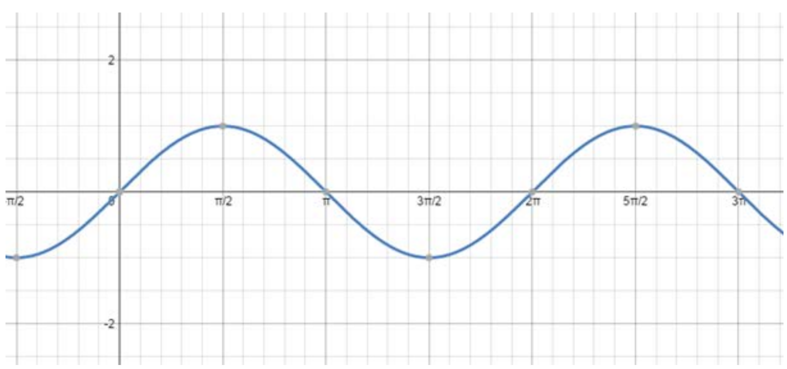
Amplitude measures how many units above and below the midline of the graph the function goes. For example, the sine wave has an amplitude of 1 because it goes one unit up and one unit down from the x-axis.

$$Y = a \sin x$$

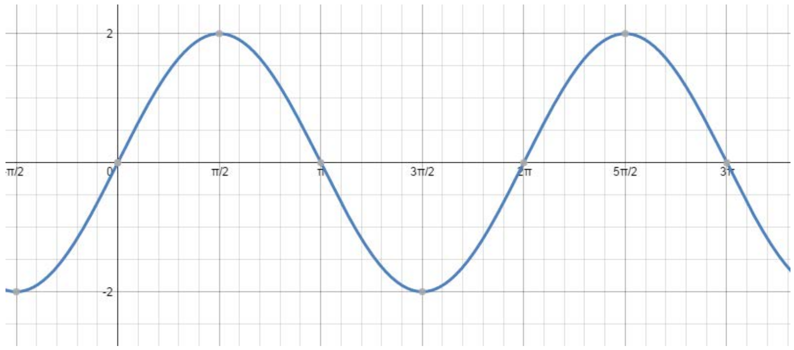
AMPLITUDE:

a is the amplitude. The graph of $y = a \sin x$ and $y = a \cos x$, where $a \neq 0$ will have a range of $[-|a|, |a|]$

Below is the graph of $y = \sin x$



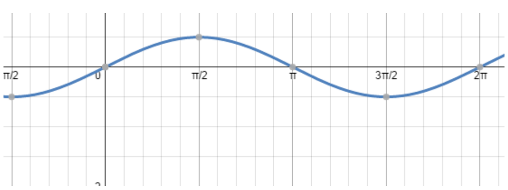
What happens if you change the amplitude to 2? Below is the graph of y = 2 sin x.

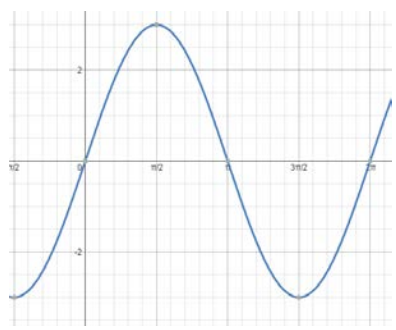


You see how the graph stretched up right? Now the range is [-2, 2] instead of [-1, 1].

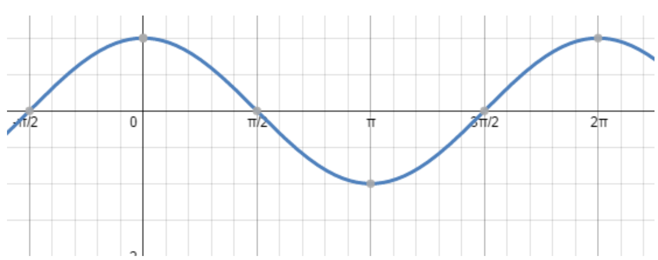
What do you think would happen if you changed the amplitude to ½? Or 3?

Check it out below

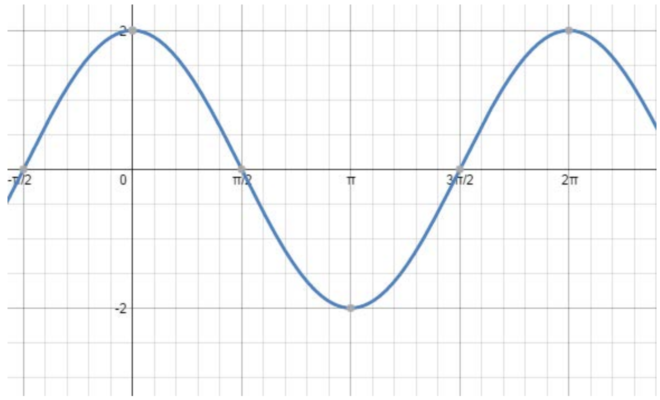




Does this happen with cosine as well? Recall the graph of y = cos x



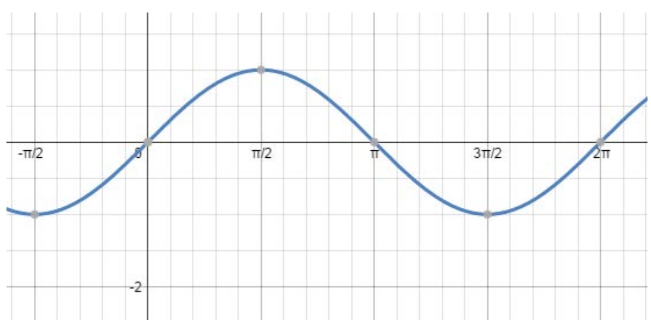
Now lets take a look at y = 2 cos x



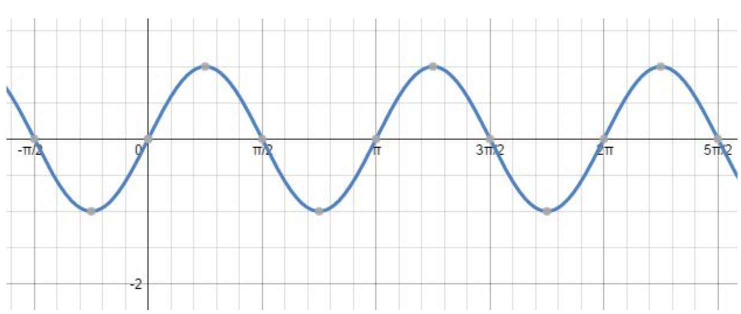
We can conclude that the amplitude vertically stretches or shrinks both the sine and cosine graphs. Notice that when the amplitude was changed the function still repeats every 2π units, therefore the amplitude does not affect the period of the function.

PERIOD

Again lets look at the graph of y = sin x



Notice how the graph changes when we change the function to y = sin 2x



Did you notice that the sine wave repeats every π units now instead of every 2π units? This means the function is finishing its cycle twice as fast, which means its period is half as long. If you consider the function y = a sin bx, the b value affects the period of the function. It will horizontally stretch or squish the graph.

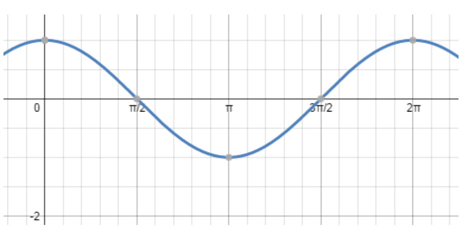
Think about what the graph would look like if you changed it to y = sin 0.5x.

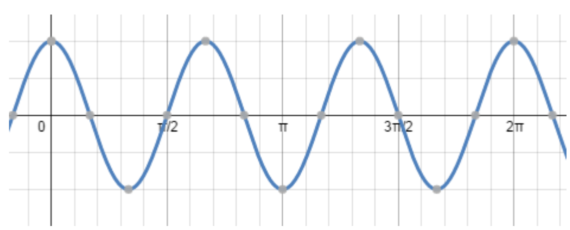
There is a general formula used to find the period (ω) of a sine or cosine function

$$\omega = \frac{2\pi}{|b|}$$

You will notice that the sine and cosine functions are affected the same by changes made in the equations, so changing the b value will have the same effect on the cosine function.

Just to show you, below are the graphs of y = cos x and y = cos 3x

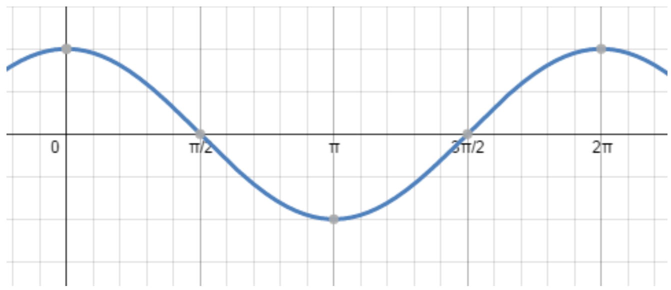




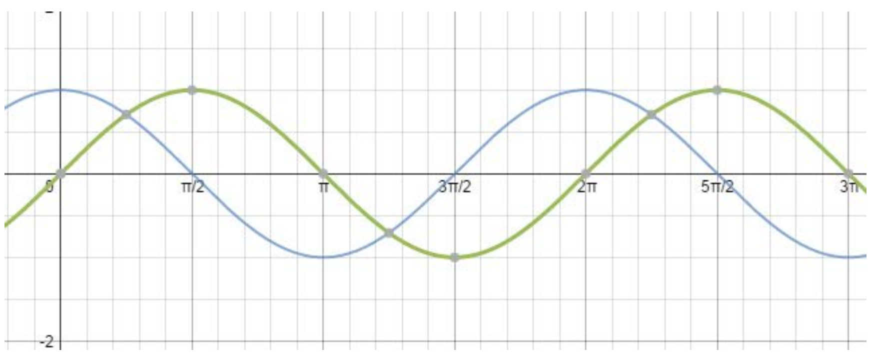
PHASE SHIFT

Not only can the graphs be contracted or stretched vertically and horizontally, but they can also be shifted left and right and up and down. First we will focus on shifting the graph left and right.

Since the sine function has been getting all the action, lets look at the cosine function y = cos x



If we change the equation to y = $\cos\left(x - \frac{\pi}{2}\right)$, watch what happens to the graph.

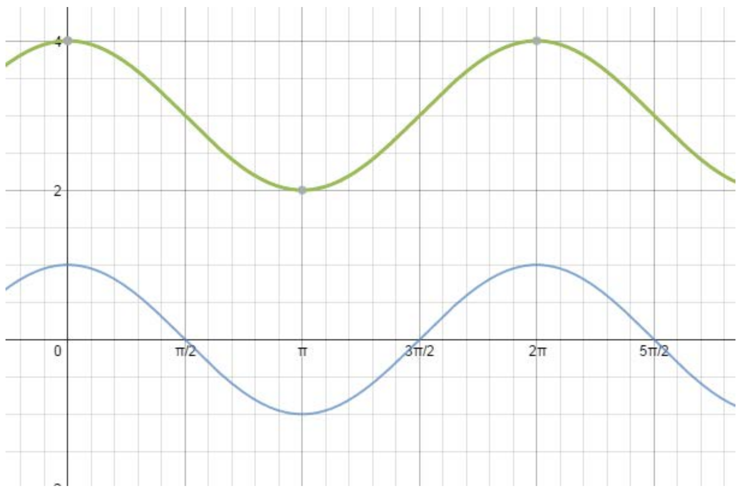


The blue wave is the original y = cos x and the green wave is the y = $\cos\left(x - \frac{\pi}{2}\right)$. You will notice that the graph was shifted $\frac{\pi}{2}$ units to the right. If the equation had been y = $\cos\left(x + \frac{\pi}{2}\right)$, the graph would have shifted $\frac{\pi}{2}$ units to the left. When the graph is shifted to the left or right it is called a "phase shift". If you consider the equation y = a cos (bx – c), the phase shift can be found by taking c/b. Again the sine function is affected the same.

VERTICAL SHIFT

The last way we can alter the sine and cosine functions is by making a vertical shift. Lets take a look at what happens to the function when we change it to

y = cos x + 3 (Notice the 3 is not in parenthesis)



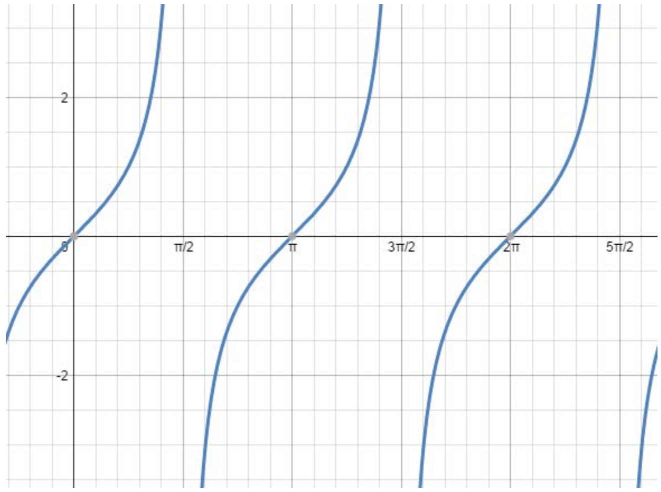
The blue wave is the original y = cos x and the green wave is the function

y = cos x + 3. When you add a number at the end you shift the graph up that many units and if you subtract a number at the end you shift the graph down that many units. When the function is written in the form y = a cos (bx – c) + d, d controls whether the function will be shifted up or down.

Need more help? Click below for a Khan Academy video

Khan Academy video 7

Graph of the tangent function $(y = tan\ x)$



- If you notice, the range of the tangent function is $(-\infty, \infty)$ and the domain is $\left\{x | x \neq n\pi + \frac{\pi}{2}, where\ n\ is\ any\ integer\right\}$
- The x intercepts are always in the form of $n\pi$
- The period is π
- The tangent will be zero wherever the numerator (sine) is zero
- The tangent will be undefined wherever the denominator (cosine) is zero
- The graph of the tangent function has vertical asymptotes at values of x in the form of $x = n\pi + \frac{\pi}{2}$
- Since the graph is symmetrical about the origin, the function is an odd function

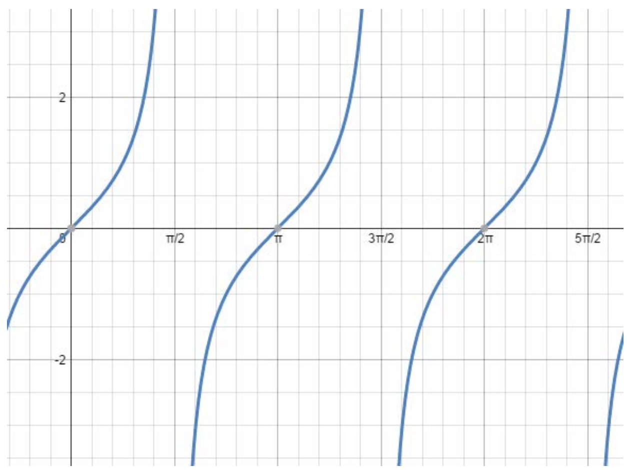
Key features of the tangent function

AMPLITUDE

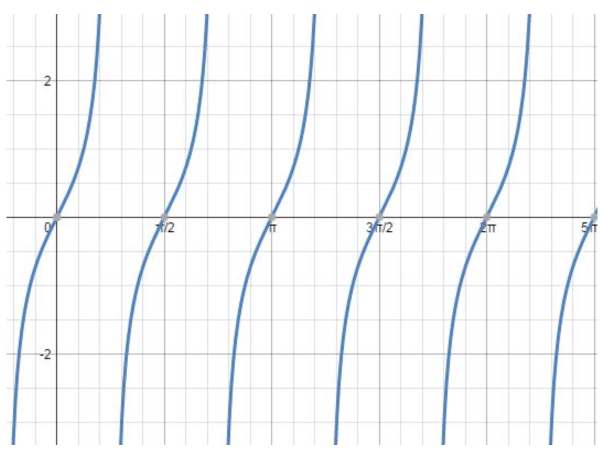
Amplitude does not apply to the tangent function since there aren't any minimum or maximum values.

PERIOD

Consider the tangent function in the form $y = a \tan bx$. To determine the period use $\frac{\pi}{b}$. Below is the graph of $y = \tan x$, notice that the tangent function repeats every π units.



Now look if we graph the function $y = \tan 2x$

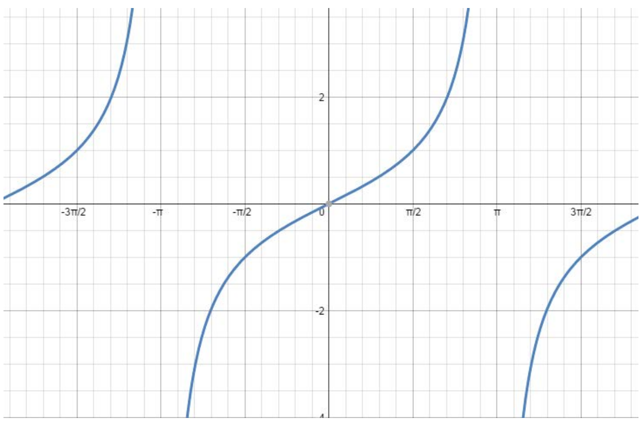


Notice now that the function repeats every $\frac{\pi}{2}$ units.

VERTICAL ASYMPTOTES

The tangent function has something called vertical asymptotes, which are invisible vertical lines that the function approaches, but never crosses. To find the two consecutive vertical asymptotes of a tangent function you can solve the two equations $bx = -\frac{\pi}{2}$ and $bx = \frac{\pi}{2}$

Consider the graph of the function $y = \tan 0.5x$

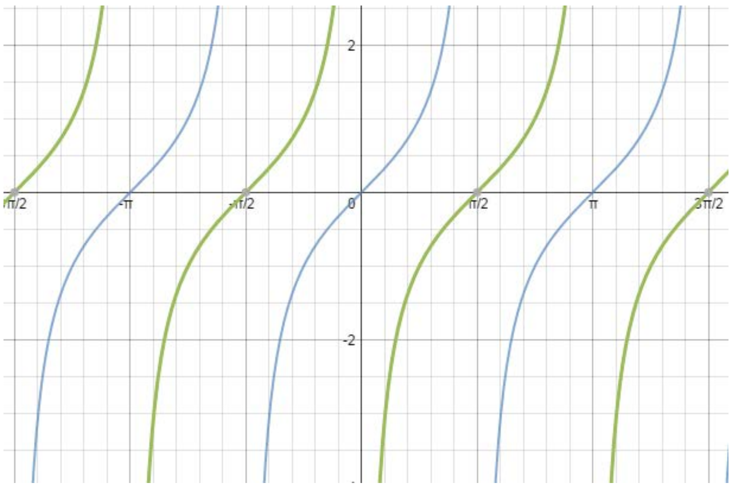


You can see from the graph that there are vertical asymptotes at $-\pi$ and π. You could have also used the equations to obtain this information.

$0.5x = -\frac{\pi}{0.5} = -\pi$ and $0.5x = \frac{\pi}{0.5} = \pi$

PHASE SHIFT

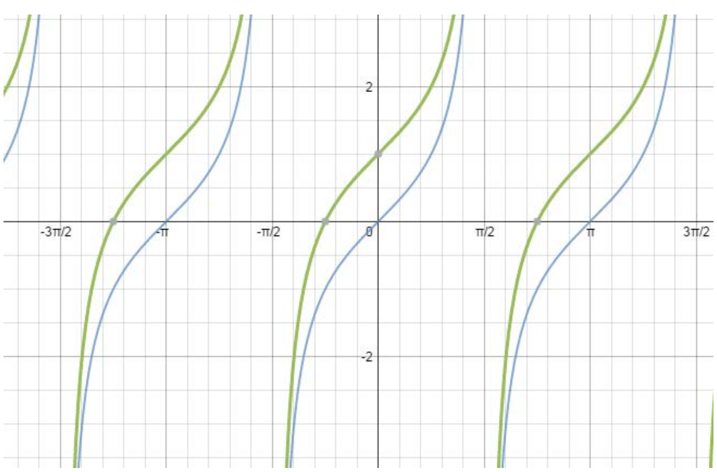
The phase shift for the tangent works the same way as the sine and cosine function. Consider the graph of the function $y = \tan\left(x - \frac{\pi}{2}\right)$



The blue is the function $y = \tan x$ and the green is the function $y = \tan\left(x - \frac{\pi}{2}\right)$. Notice that the functions appear the same except $y = \tan\left(x - \frac{\pi}{2}\right)$ is shifted $\frac{\pi}{2}$ units to the right.

VERTICAL SHIFT

Again this works the same for tangent as it did for sine and cosine. Consider the graph of the function $y = \tan x + 1$



Again the function graphed in blue is the function $y = \tan x$ and the function graphed in green is $y = \tan x + 1$. As you can see the only difference between the two graphed functions is that the function $y = \tan x + 1$ is shifted up one unit.

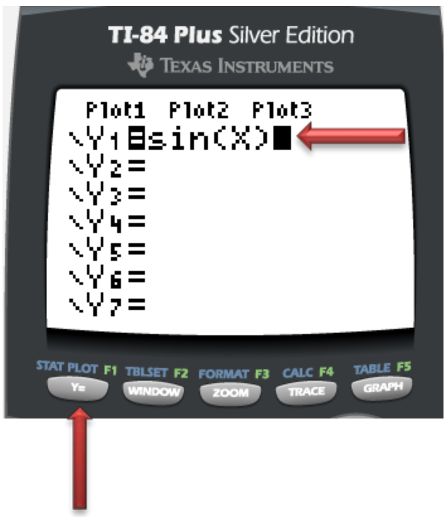
Need more help? Click below for a Khan Academy video

Khan Academy video 8

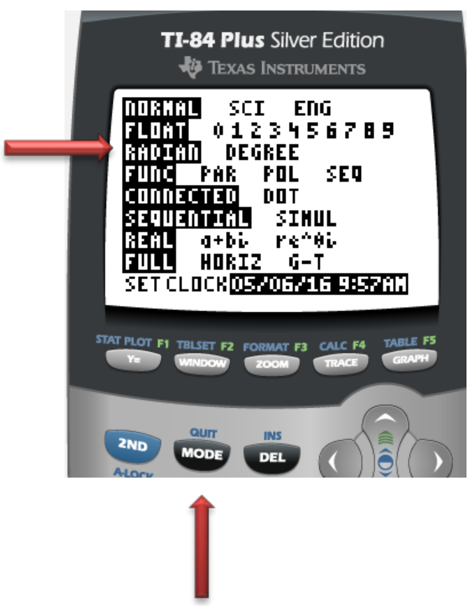
Graphing Trigonometric Function using Technology

Graphing a function in your **TI-Graphing Calculator** is easy!

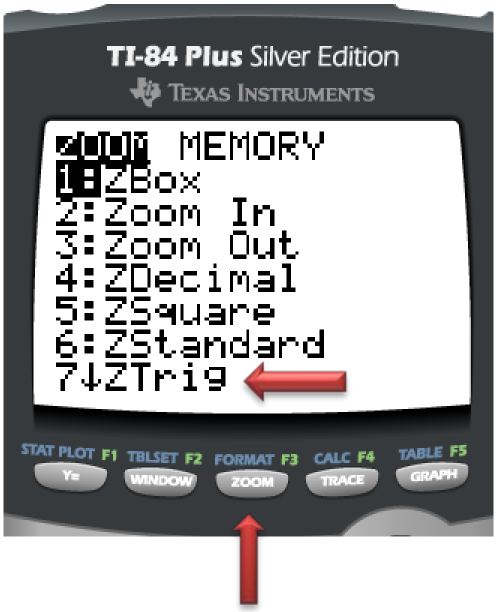
First type your function into y1



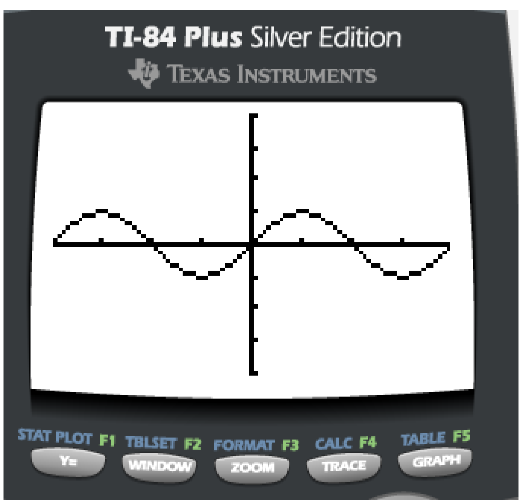
Next you need to change the mode on your calculator to radians. To do this first press MODE and then scroll down and select Radians.



Then press ZOOM and choose option 7 ZTRIG



Once you choose option 7, your function should start to graph!

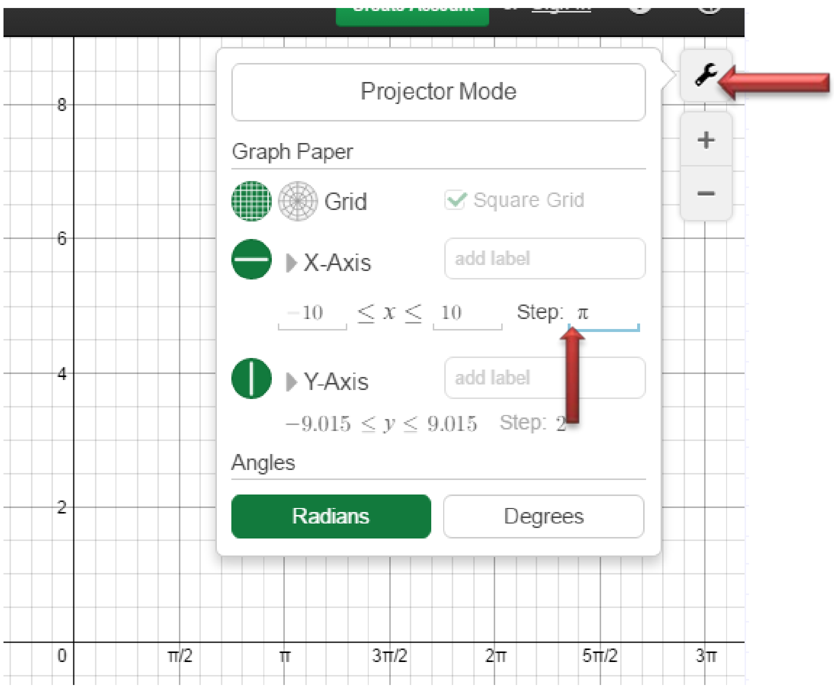


Desmos is another way to use technology to graph Trig functions

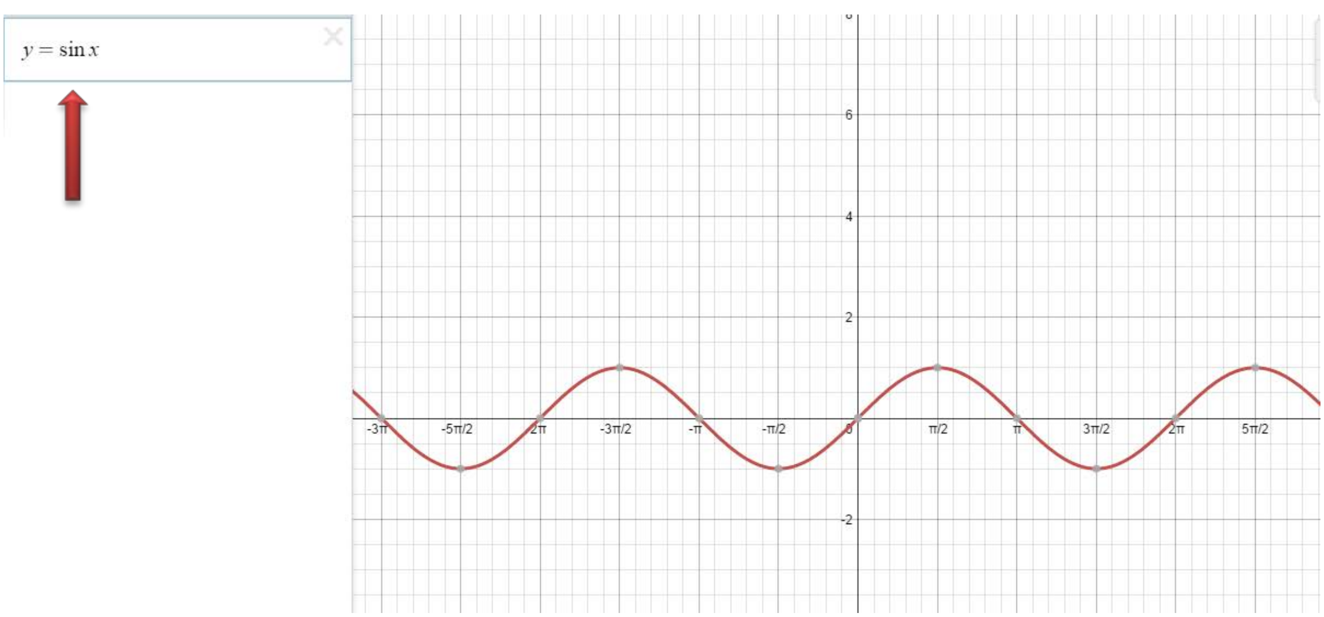
Go to https://www.desmos.com/

Click Start Graphing

In the upper right corner select the tools and type pi in for the step



Now you are ready to type in your function!



Practice Problems

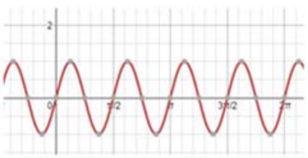
Use what you know about the graphs of the sine and cosine function to match the equations to the graphs

1) $y = \sin 4x$ 2) $y = \sin x + 4$ 3) $y = \sin\left(x + \frac{\pi}{4}\right)$ 4) $y = \cos\left(x + \frac{\pi}{4}\right)$

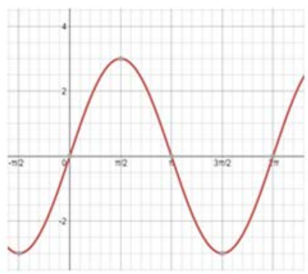
5) $y = \cos\left(x - \frac{\pi}{4}\right)$ 6) $y = -2 + \sin x$ 7) $y = -2 + \cos x$ 8) $y = 3\sin x$

9) $y = 2\cos x$ 10) $y = \cos 2x$ 11) $y = 2\sin 2x$ 12) $y = 3\sin x + 1$

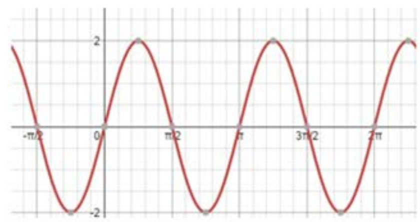
A)



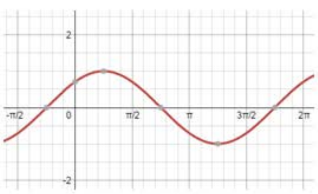
B)



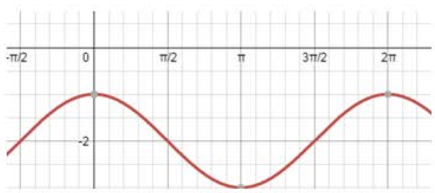
C)



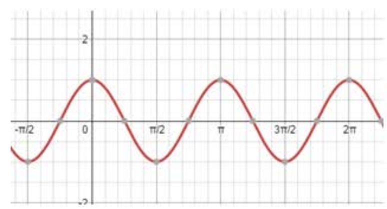
D)



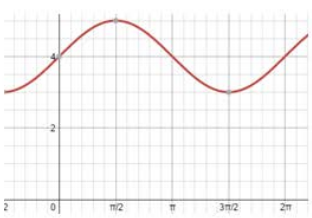
E)



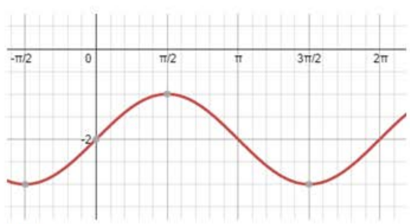
F)



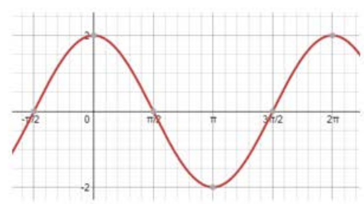
G)



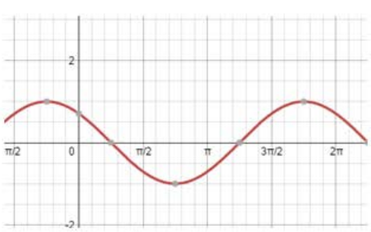
H)



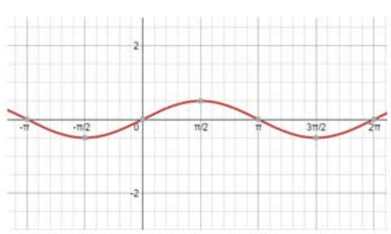
I)



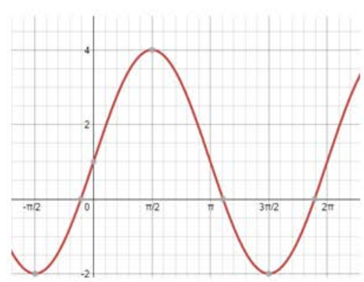
J)



K)



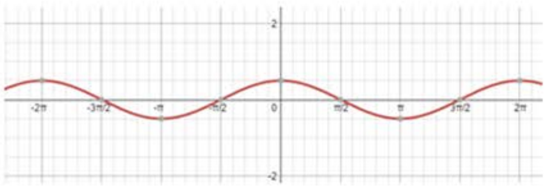
L)



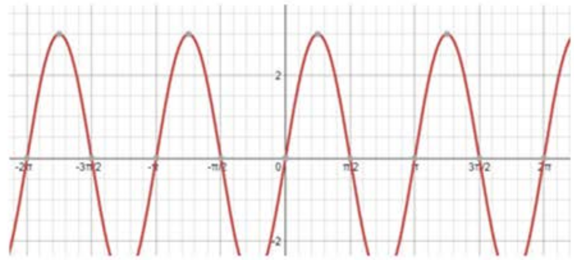
Determine the amplitude and period for each function

13) $y = \sin 4x$ 14) $y = 3\cos(-2x)$ 15) $y = 4\cos x$ 16) $y = 3\sin 2x$

17)



18)

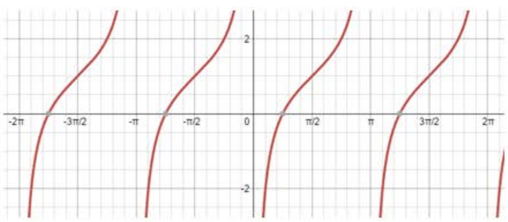


Use what you know about the graphs of the tangent function to match the equations to the graphs

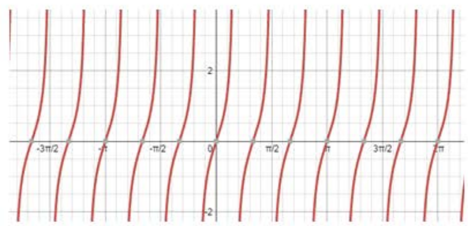
19) $y = \tan x$ 20) $y = \tan 2x$ 21) $y = \tan 3x$ 22) $y = \tan\left(x - \frac{\pi}{2}\right)$

23) $y = \tan x + 2$ 24) $y = \tan\left(x + \frac{\pi}{2}\right) + 1$

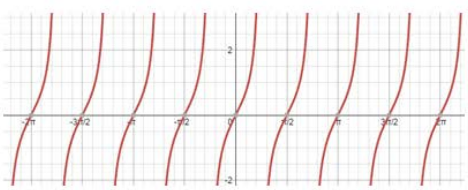
A)



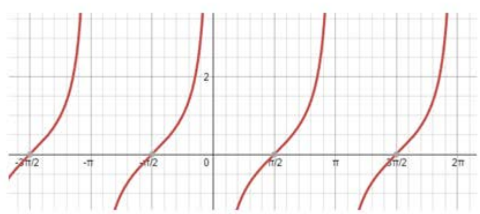
B)



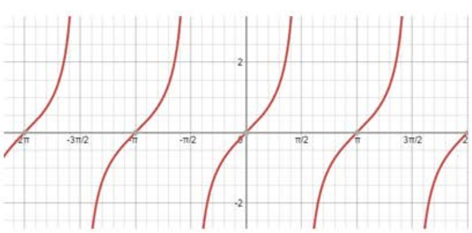
C)



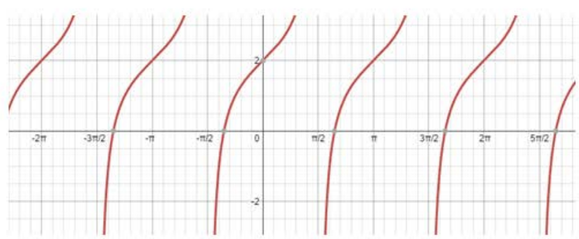
D)



E)



F)



Solutions

Match sine and cosine function to their graph

1) A 2) G 3) K 4) J 5) D 6) H 7) E 8) B 9) I 10) F 11) C 12) L

13) Amplitude: 1
Period: $\frac{\pi}{2}$

14) Amplitude: 3
Period: π

15) Amplitude: 4
Period: 2π

16) Amplitude: 3
Period: π

17) Amplitude: 0.5
Period: 2π

18) Amplitude: 3
Period: π

Use what you know about the graphs of the tangent function to match the equations to the graphs

19) E 20) C 21) B 22) D 23) F 24) A

Back to Table of Contents.