A Simple Method for Determining Specific Yield from Pumping Tests

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GROUND-WATER HYDRAULICS

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CONTENTS

ILLUSTRATIONS

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ABSTRACT

A simpler solution which greatly reduces the time necessary to compute the specific yield by the pumping-test method of Remson and Lang (1955) is presented. The method consists of computing the volume of dewatered material in the cone of depression and comparing it with the total volume of discharged water. The original method entails the use of a slowly converging series to compute the volume of dewatered material. The solution given herein is derived directly from Darcy's law.

INTRODUCTION

A pumping-test method to determine the specific yield of a water-table aquifer was presented by Remson and Lang (1955). The method involves the determination of the volume of dewatered material in the cone of depression during the course of a pumping test. The specific yield is then determined by comparing the volume of dewatered material with the total volume of discharged water.

The calculation of the volume of dewatered material requires the solution of an exponential series that converges very slowly and is, therefore, time consuming. The example presented by Remson and Lang needed 60 terms of the series and required more than 8 hours of computation. This paper presents a more easily evaluated equation for rapidly computing the volume of dewatered material in the cone of depression.

The work was carried out under the supervision of Allen Sinnott, district geologist, as part of the investigation of the ground-water resources of New Jersey in cooperation with the New Jersey Department of Conservation and Economic Development. The senior author, a hydrologist-geophysicist with the British Guiana Geological Survey, participated in this study as a foreign trainee under the program sponsored by the International Cooperation Administration.

THEORY

As pointed out by Remson and Lang, it may not be possible to apply the standard formulas to data from a pumping test in a shallow water-table aquifer because of the slow drainage of the aquifer material during the test and (or) because of a varying rate of discharge. However, the general equilibrium formula can be applied if a pumping rate Q is constant for a long enough period so that the cone of depression reaches approximate equilibrium form and is declining only very slowly. The condition of approximate equilibrium is described by Wenzel (1942, p. 98–99),

. . . as pumping continues, a hydraulic gradient that is essentially an equilibrium gradient will be established close to the pumped well, and water will be transmitted to the well through the water-bearing material in approximately the amount that is being pumped. The decline of the water table and the resulting unwatering of material in this area will then be much slower.

The assumptions used in the development of the general equilibrium formula and those used by Remson and Lang also apply here. The following is quoted from Remson and Lang (1955, p. 322). "Although the water table continues to decline slowly, the assumption that steady-state conditions have been reached involves only a slight error no greater than that resulting from such a cause as fluctuation in pump discharge." The following paragraph, adapted from Wenzel (1942, p. 77), describes the requisite conditions of the test:

An isotropic and homogeneous water-bearing bed of infinite areal extent is assumed to rest on a relatively impervious formation. The discharging well, equipped with a pump, is fully screened to the bottom of the water-bearing material. It is assumed that water movement from the outer radius of the screen

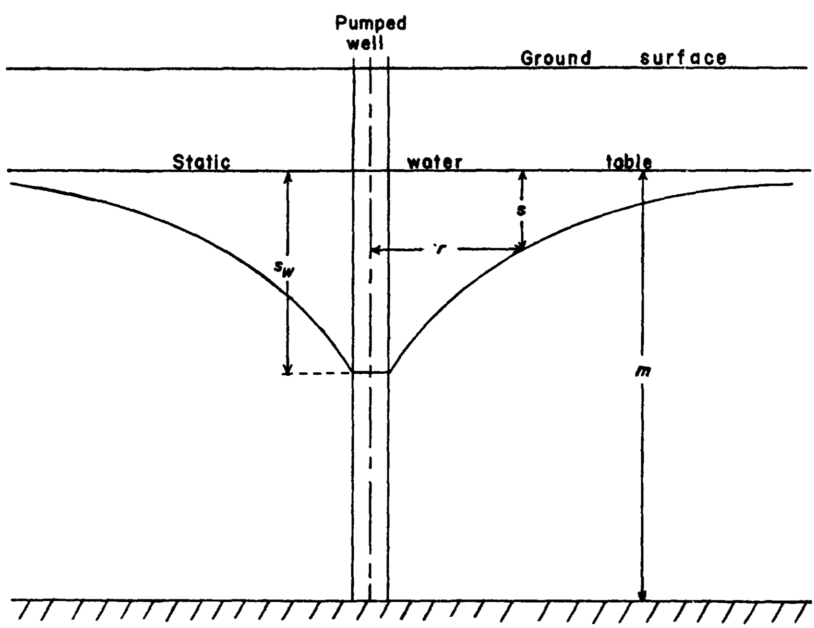

FIGURE 11. Diagram showing drawdown of the water table in the vicinity of a pumped well.

to the pump intake occurs without loss of head or with a head loss that is negligible compared with the drawdown in the well. The water table before pumping, and the underlying impervious bed, are assumed to be horizontal. It is assumed also that there is no recharge to the aquifer during the test and that all the water pumped is removed from storage.

Figure 11 shows the shape of the cone of depression in the vicinity of a well pumping from a water-table aquifer.

The following symbols or nomenclature are used in the mathematical derivations in this report:

Q=the discharge rate of the pumped well in gallons per day

P=the field coefficient of permeability of the aquifer in gallons per day per square foot under a unit hydraulic gradient and at the prevailing water temperature

r=the horizontal distance from the axis of the pumped well to a point on the cone of depression, in feet

s=the drawdown at distance r, in feet

s_w=the drawdown just outside the screen of the pumped well, in feet

m=the thickness of the zone of saturation before pumping or the height of the static water table above the aquifer bottom, in feet

$T=Pm$=the coefficient of transmissibility of the aquifer in gallons per day per foot. It is the flow through a vertical strip of the aquifer 1 foot wide and extending the saturated height of the aquifer, at unit hydraulic gradient.

From Darcy's law:

$$Q=2\pi rP\left(-\frac{ds}{dr}\right)(m-s) \tag{1}$$

Therefore

$$\frac{dr}{r}=-\frac{2\pi P}{Q}(m-s)ds=-a(m-s)ds \tag{2}$$

where

$$a=\frac{2\pi P}{Q}$$

Integrating,

$$ln\,r=-ams+\frac{as^2}{2}+lnB \tag{3}$$

where B is the constant of integration.

Then

$$r=Be^{-ams+as^2/2} \tag{4}$$

Equation 4 describes the cone of depression when it has reached virtually an equilibrium shape or position.

The volume of dewatered material in cubic feet, V, within the cone of depression is

$$V=\pi\int_0^{s_w} r^2\,ds \tag{5}$$

the limits of integration being chosen at zero drawdown (for example, the extent of the cone at equilibrium) and the drawdown outside the screen of the pumped well. The value for r in equation 4 may be substituted in equation 5 to give

$$V=\pi B^2\int_0^{s_w} e^{-2ams+as^2}\,ds \tag{6}$$

Observe in equation 6 that the exponent of e may be written in the equivalent form $[-2ams(1-s/2m)]$.

Because $s/2m$ is generally very small compared to unity, it may be neglected; therefore equation 6 becomes

$$V\approx\pi B^2\int_0^{s_w} e^{-2ams}\,ds \tag{7}$$

$$V\approx\frac{\pi B^2}{-2am}\,[e^{-2ams}]_0^{s_w} \tag{8}$$

$$\approx\frac{\pi B^2}{2am}\left[1-\frac{1}{e^{2ams_w}}\right] \tag{9}$$

For values found during field pumping tests

$$2ams_w>1$$

Hence $\frac{1}{e^{2ams_w}}$ is very small and can be neglected. Thus equation 9 becomes

$$V\approx\frac{\pi B^2}{2am} \tag{10}$$

From equation 4 $B=re^{ams-as^2/2}$

Substituting the value for B in equation 10 there follows

$$V\approx\frac{\pi r^2 e^{2ams-as^2}}{2am} \tag{11}$$

If the exponent of e is again modified, in the manner shown in writing equation 7, it follows that equation 11 may be written in the form

$$V=\frac{\pi r^2 e^{2ams}}{2am} \tag{11a}$$

In equation 11a the volume of dewatered material is expressed in terms of permeability, pumping rate, horizontal distance, drawdown, and aquifer thickness. In field practice, it is often necessary to make pumping tests, using wells that only partly penetrate the aquifer or for which incomplete data are available. Thus, it may not be possible to determine the coefficient of permeability, P, or the full aquifer thickness, m. Under such circumstances, equation 10 cannot be used to determine the volume of dewatered material in the cone of depression. However, if the drawdown, s, at the point of observation is small compared to suspected thickness of the zone of saturation, the thickness may be assumed to remain uniform and transmissibility, T, may be used in place of the unknown permeability and aquifer thickness ($T=Pm$). Several standard ground-water formulas permit the direct determination of the coefficient of transmissibility. Therefore, equation 11a may be further modified by substituting therein the equivalents $2\pi\ P/Q$ for a, and T for the product Pm, which yields

$$V=\frac{\pi r^2 e^{4\pi Ts/Q}}{\frac{4\pi I}{Q}}$$

$$=\frac{Qr^2e^{4\pi Ts/Q}}{4T} \qquad (12)$$

Taking the logarithm of both sides of equation 12 produces

$$\log V=\log \frac{Qr^2}{4T}+\frac{4\pi Ts}{Q}\log e$$

or

$$\log V=\log \frac{Qr^2}{4T}+\frac{5.45Ts}{Q} \qquad (13)$$

The specific yield is the volume of water pumped during the test divided by the gross volume of dewatered material within the cone of depression.

$$S=\frac{Qt}{7.48V} \qquad (14)$$

where S=specific yield

Q=the average discharge rate of the pumped well, in gallons per day

t=the time, in days, since pumping began

V=the volume of dewatered material, in cu. ft., determined from either equation 11a or 13.

APPLICATION TO TEST DATA FROM KEARNEY, NEBRASKA

The formulas derived herein were applied to field data from a pumping test near Kearney, Nebr. (Wenzel, 1942, p. 127–131). The same data were used by Remson and Lang (1955, p. 324). The data used are

$m = 48$ ft
$Q = 1{,}584{,}000$ gpd
s after 24 hr$=3.97$ ft for $r=50$ ft
s after 24 hr$=2.99$ ft for $r=100$ ft
$P=4{,}100$ gpd per square foot
$T=Pm=1.96\times10^5$ gpd per foot

The specific yield computed for the drawdown at the 50-foot distance was 9 percent whereas its value for the drawdown at the 100-foot distance was 10 percent, which is the same value that was computed by Remson and Lang using the lengthy series.

The formulas derived herein may be used only with data from an equilibrium pumping test and the test should be long enough to permit the greatest possible dewatering in the cone of depression without it being affected by recharge.

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