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Trapezoids to Triangles

Pattern blocks are often used in math class during geometry lessons to support concepts of shape, angle, tessellations, and symmetry. They are also used with fraction study (e.g., Lanius 2007; NCTM 2009a). Using pattern blocks to construct other larger geometric shapes can help students understand the relationships among various shapes. Algebraic relationships and functions emerge when students count blocks and measure figures. The activities in this article support the development of such geometric vocabulary as *perimeter*, *area*, *sequence*, and *nonstandard measurement*.

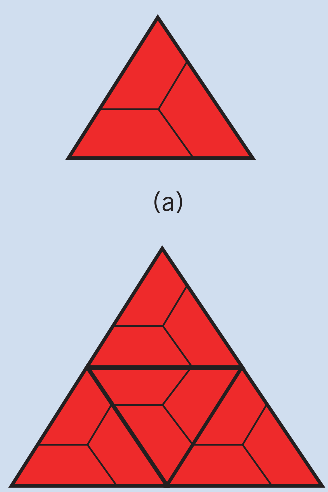
TRIANGLE SEQUENCES

Give each pair of students **activity sheet 1** and a bag of thirty red pattern blocks; include one green triangle pattern block as a nonstandard measuring tool (see **fig. 1**). Ask students to make the smallest possible triangle using the red trapezoid pattern blocks. Then discuss how the angles of the trapezoid block form the angles of an equilateral triangle: The 60-degree angles form the triangle vertices and the 120-degree angles form the central angle (see **fig. 2a**).

The smallest possible triangle uses

Fig. 1 The red trapezoid block is used to generate new shapes. The green triangle block is used as a nonstandard unit of measure.

Fig. 2 Triangles (a) and (b) are made with tessellations of the trapezoid shapes; (a) uses three, and (b) uses twelve.



Edited by **Gwen Johnson**, gjohnson@coedu.usf.edu, Secondary Education, University of South Florida, and **James Dogbey**, jdogbey@mail.usf.edu, Secondary Education, University of South Florida. This department's classroom-ready activities may be reproduced by teachers. Teachers are encouraged to submit manuscripts in a format similar to this department based on successful activities from their own classroom. Of particular interest are activities focusing on the NCTM's Content and Process Standards and Curriculum Focal Points as well as problems with a historical foundation. Send submissions by accessing **mtms.msubmit.net**.

three trapezoid blocks. The perimeter of the three-triangle can be measured using the length of a side of the green triangle pattern block as a nonstandard linear unit. In a similar way, the area of the three-triangle can be measured using the area of the green nonstandard block (Reynolds and Lillard 2006; Reynolds, Lillard, and Trowell 2008). Both the perimeter and the area of the three-triangle measure 9. Although the number of units is the same, the unit measures are in different dimensions. This situation will be discussed later in the activity.

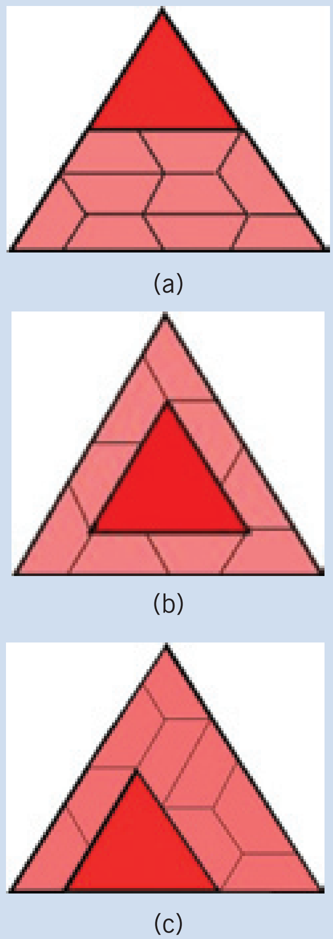
Ask students to build the next larger triangle, and allow time for them to fully explore how many trapezoids they will need. One way is to use four of the previously discussed three-triangles (see **fig. 2b**). This iterative method can also be used to build any larger triangle in a precise, predictable manner. However, students may also build on a previous triangle by adding blocks around the perimeter or in other ways (see **fig. 3a–c**). When they have found the larger twelve-triangle, ask students to record the number of trapezoid blocks, perimeter units, and area units in the table on **activity sheet 1**. Ask them to continue building the next larger triangle. They may preserve their triangles by drawing them on triangle graph paper (available at http://www.8gr.org/free-docs/isometric-graph-paper.pdf). These drawings can be displayed in the classroom or reserved for the extension activities.

Students will continue building larger and larger triangles, measuring their perimeter and area, and recording these numbers in the table. Help students look for a pattern in the data in each column by thinking about what *stays the same* and what *changes* within each column of data. Encourage them to use this thinking to create a formula for each column that will predict what will happen with the pattern for the *n*th triangle. Discuss why the numbers 3 and 9 occur in the formulas, and why the variable *n* has an exponent of 1 in the perimeter formula but an exponent of 2 in the area formula. This will help students begin to distinguish between measures of perimeter and area and between constant and growing sequences. Eventually, they will be able to differentiate between linear and quadratic data.

REPEATING TILE SEQUENCES

After completing **activity sheet 1** and finding the formulas for the *n*th triangle, students can work on the "Repeating Tile Sequences" table on **activity sheet 2**. Ask students to view their twelve-triangle drawings from the previous activity, looking for embedded three-triangles. Challenge them to build a twelve-triangle with no embedded three-triangles. Examine the student designs carefully, as three-triangles can be embedded in several ways in a twelve-triangle, which will be examined in **activity sheet 2**. Students may want to record their efforts on triangle graph paper to see all the ways that a three-triangle can appear in a twelve-triangle.

Fig. 3 Several ways are possible to obtain the next larger size of triangle after starting with three trapezoids.



Introduce the concept of *rep-tiles*, which are shapes that can be joined to make larger replicas of themselves (NCTM 2009b). Ask students to use one trapezoid block as a model and create the next larger trapezoid that is similar (see **fig. 4a**). It is important for students to understand that rep-tiles are geometrically similar to each other but constructed with different sizes. This means that corresponding angle measures are congruent, and corresponding edge lengths are in proportion. Other trapezoids can be built

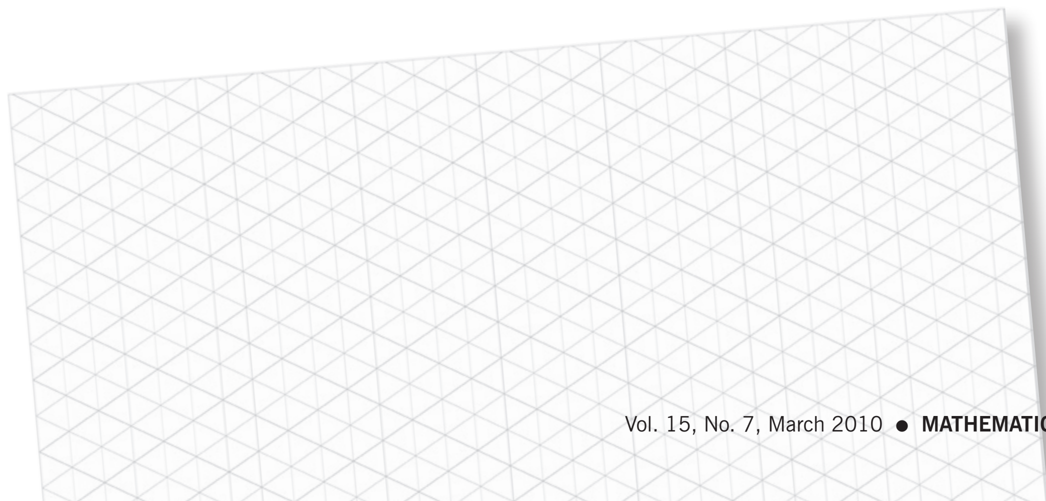
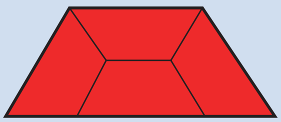
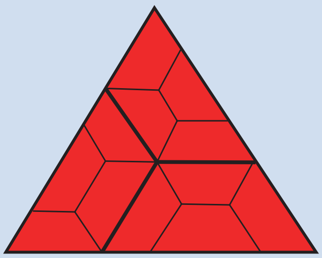


Fig. 4 Using the trapezoid pattern block, students can construct a larger trapezoid, which can, in turn, be used to construct triangles.



(a)



(b)

Table 1 These triangle sequences are using trapezoid pattern blocks.

Triangle	Total Number of Trapezoid Blocks	Perimeter Units	Area Units
1	$3 = 3(1)^2$	$9 = 9(1)$	$9 = 9(1)^2$
2	$12 = 3(2)^2$	$18 = 9(2)$	$36 = 9(2)^2$
3	$27 = 3(3)^2$	$27 = 9(3)$	$81 = 9(3)^2$
4	$48 = 3(4)^2$	$36 = 9(4)$	$144 = 9(4)^2$
5	$75 = 3(5)^2$	$45 = 9(5)$	$225 = 9(5)^2$
. . .	. . .	. . .	. . .
n	$3(n)^2$	$9(n)$	$9(n)^2$

that are not rep-tiles for the trapezoid pattern block, so students should carefully check corresponding measurements. (For more examples of rep-tiles, see http://mathworld.wolfram.com/Rep-Tile.html.)

Once students have created the larger rep-tile, ask them to use it as a building block to create the twelve-triangle (see **fig. 4b**) and even larger triangles. Notice that this time the central angle is determined by six 60-degree vertices of the trapezoid pattern block, rather than three 120-degree angles.

A new sequence is formed by considering how the rep-tiles increase in size. The first figure is a single trapezoid; the next larger rep-tile contains four trapezoids; and the third rep-tile contains nine trapezoids. As students enter the total number of trapezoids for each figure in the table on the "Rep-tile Sequence" activity sheet, they should begin to recognize the set of perfect square numbers. These numbers are created by multiplying a whole number by itself: $1 \times 1 = 1$, $2 \times 2 = 4$, $3 \times 3 = 9$, and so on.

Students may find another pattern by considering how many trapezoids will need to be added to make the next rep-tile. Begin with one trapezoid, add three more trapezoids, and then add five more. This growing pattern will continue by adding consecutive odd numbers of trapezoids. If desired, stop and point out that the *sum* of the growing pattern numbers is always a perfect square number—in fact, it is the square of the figure number. Take this opportunity to connect *perfect square* numbers with the geometric model of a square. Often students do not realize that the area of a square with whole-number sides is always a perfect square number, hence, the name.

Students may then record the perimeter and area of each rep-tile as in the original activity. Discuss the patterns in each column, considering what changes and what stays the same. Help students create formulas that will predict what will happen to the pattern in the nth rep-tile.

CONCLUSION

This trapezoid-to-triangle activity, discussing both triangle sequences and repeating tile sequences, can be used in math class to introduce, support, or formatively assess the learning of the concepts of such geometric vocabulary as perimeter, area, sequence, and nonstandard measurement. The sequence formulas derived during the activities can also be represented using function format or graphed on the coordinate plane.

These activities provide bridges among concrete, visual, and abstract thinking as students model with manipulatives, record on dot paper, and look for patterns leading to formulas. The multistep approach helps build logical reasoning, encourages the formation of conjectures, and provides support for developing simple proof. The activity is approachable for all levels of students. The "Repeating Tile Sequences" activity can also challenge students of differing abilities.

SOLUTIONS TO THE ACTIVITY SHEETS

Triangle Sequences

See **table 1**.

Repeating Tile Sequences

See **table 2**.

REFERENCES

Lanius, Cynthia. "No Matter What Shape Your Fractions Are In." 2007. http://math.rice.edu/~lanius/Patterns/index.html.

Table 2 These rep-tile sequences are using trapezoid pattern blocks.

Rep-tile	Total Number of Trapezoids	Number of Trapezoids Added	Perimeter Units	Area Units
1	$1 = 1^2$	1	$5 = 5(1)$	$3 = 3(1)^2$
2	$4 = 2^2$	3	$10 = 5(2)$	$12 = 3(2)^2$
3	$9 = 3^2$	5	$15 = 5(3)$	$27 = 3(3)^2$
4	$16 = 4^2$	7	$20 = 5(4)$	$48 = 3(4)^2$
5	$25 = 5^2$	9	$25 = 5(5)$	$75 = 3(5)^2$
⋮	⋮	⋮	⋮	⋮
n	n^2	n^2	$5(n)$	$3(n)^2$

National Council of Teachers of Mathematics (NCTM). "Pattern Block Fractions." *Illuminations: Resources for Teaching Math.* 2009a. http://illuminations.nctm.org.

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Reynolds, A., and E. Lillard. "Making Number Sense through the Investigations of Trapezoids." Workshop at the 2006 NCTM Annual Meeting, St. Louis, April 2006.

Reynolds, A., E. Lillard, and S. Trowell. "Investigations with a Trapezoid." Workshop at the 2008 NCTM Annual Meeting, Salt Lake City, April 2008.

Wolfram MathWorld. "Rep-Tile." http://mathworld.wolfram.com/Rep-Tile.html.

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Name ______________________________

TRIANGLE SEQUENCES

1. Use only red trapezoid pattern blocks to construct the smallest possible triangle without any overlaps or holes. Record your answers in the first line of the table below.

2. Use the edge of the green triangle pattern block as a linear unit. What is the perimeter of this smallest triangle?

3. Use the green triangle pattern block as an area unit. What is the area of this smallest triangle?

4. Build the next larger possible triangle. What are the perimeter and area of this triangle?

5. Continue building larger triangles and entering the measurement data in the table.

6. Look for a pattern in the data in each column. What changes, and what stays the same? Create a formula that will predict what will happen with the *n*th triangle.

Triangle	Total Number of Trapezoid Blocks	Perimeter Units	Area Units
1			
2			
3			
4			
5			
⋮	⋮	⋮	⋮
n			

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activity sheet 2

Name ______________________________

REPEATING TILE SEQUENCES

A *rep-tile* is a shape that can be tiled to make larger replicas of itself.

1. Use only red trapezoid pattern blocks to generate a larger trapezoid.

2. Use the green pattern block for unit measures, and complete the first row of the table below.

3. Find larger rep-tiles, and complete the table below.

4. Look for a pattern in the data in each column. What changes, and what stays the same? Create a formula that will predict what will happen with the *n*th trapezoid.

Rep-tile	Total Number of Trapezoids	Number of Trapezoids Added	Perimeter Units	Area Units
1				
2				
3				
4				
5				
⋮	⋮		⋮	⋮
n				