# Chapter 13
# Area of Circle

## 13.1 Circle

A circle is a plane figure bounded by one line which is called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal to another.

The point C is called the center of the circle.

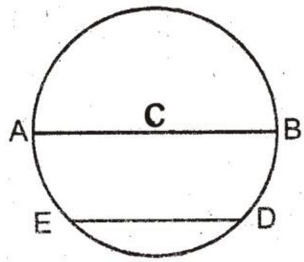



Fig. 13.1

**Radius**

A radius of a circle is a straight line drawn from the center to the circumference.

**Diameter**

A diameter of a circle is a straight line drawn through the center and terminated both ways by the circumference.

In figure AC is a radius, and AB a diameter.

Circumference of a circle = $\pi d$ or $2\pi r$

**Chord**

A chord of a circle is a straight line joining two points on the circumference.

ED is chord.

## 13.2 Area of the Circle:

If r is the radius of the circle, d is the diameter of the circle.

Then Area of circle = $\pi r^2$ or $\frac{\pi}{4}d$

Hence r = $\frac{\sqrt{A}}{\pi}$

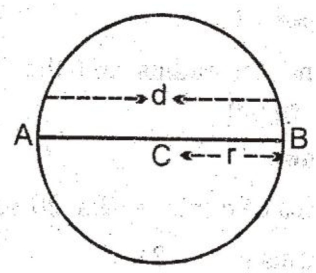



Fig. 13.2

## 13.3 Concentric Circles:

Concentric circles are such as have the same center.

**Area of the Annulus (Ring)**

Area between two concentric circles is known as annulus, for example, area of a washer, the area of cross-section of a concrete pipe. Area of the annulus = Area of outer circle – area of inner circle

$$= \frac{\pi}{4}D^2 - \frac{4}{\pi}d^2$$

$$= \frac{\pi}{4}(D^2 - d^2)$$

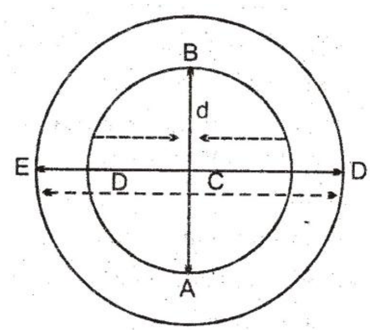


Fig. 13.3

Where D is the diameter of outer circle and d is the diameter of inner circle.

If R and r denote the radius of the outer and inner circles respectively.

Then ,Area of ring = $\pi R^2 - \pi r^2$

= $\pi(R^2 - r^2)$ square units

**Example 1:**

Find the radius and the perimeter of a circle the area of which is 9.3129 sq. cm.

**Solution:**

Area of circle = 9.3129 sq. cm.
Radius r = ?
Area of circle = $\pi r^2$
9.3129 = $(3.14)r^2$
$r^2 = \frac{9.3129}{3.14}$
$r^2 = 2.96$
r = 1.72 cm
Perimeter of the circle = $2\pi r$
= 2(3.14)(1.72) = 10.80cm.

**Example 2:**

A path 14 m wide surrounds a circular lawn whose diameter is 120 m. find the area of the path.

**Solution:**

| | | |
|---|---|---|
| Diameter of inner circle | | = 120m |
| Radius of inner circle | = r | = 60m |
| Radius of outer circle | = R | = 60 + 14 = 74m |

Area of path = $\pi ( R^2 - r^2 )$

$$= \frac{22}{7}(74^2 - 60^2) = \frac{22}{7}(1876)$$

= 5896 sq. m

**Example 3:**

A hollow shaft with 5 m internal diameter is to have the same cross-sectional area as the solid shaft of 11m diameter. Find the external diameter of the hollow shaft.

**Solution:**

Let D = diameter of solid shaft = 11m

Area of the solid shaft = $\frac{\pi}{4}(11)^2 = \frac{121}{4}\pi$

Let, d = Internal diameter of hollow shaft, d = 5m

Let, D = External diameter of hollow shaft = ?

Area of annulus $= \frac{\pi}{4}(D^2 - d^2)$

$$= \frac{\pi}{4}[(D^2 - (5)^2] = \frac{\pi}{4}(D^2 - 25)$$

But, Area of annulus = Area of solid shaft

$$\frac{\pi}{4}(d^2 - 25) = \frac{121}{4}\pi$$

$\Rightarrow$ $D^2 - 25 = 121$

$D^2 = 146$

D = 12.1m which is external diameter

## 13.4 Sector of a Circle:

A sector of a circle is a figure bounded by two radii and the arc intercepted between them. The angle contained by the two radii is the angle of the sector.

In figure <PCQ is called the angle of sector PCQ

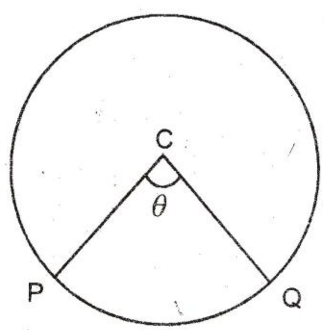


Fig. 13.4

1. Area of sector

When angle is given in degree.

Area of circle for angle $1^o = \frac{\pi r^2}{360}$

Hence, Area of sector for angle $N^o = \frac{\pi r^2}{360} \times N$

Length of arc = $l = \frac{2\pi r}{360} . N$

2. If angle <POQ is given in radian say $\theta$ radian.

Area of circle for angle $2\pi$ rad = $\pi r^2$

Area of circle for angle 1 rad = $\dfrac{\pi r^2}{2\pi} = \dfrac{1}{2}r^2$

Hence,

Area of sector for angle $\theta$ rad = $\dfrac{1}{2}r^2\theta$ --------(1)

3. Area of sector when arc $\ell$ and the radius of the circle r are given.

Since, Area of sector $= \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2}r^2\left(\dfrac{\ell}{r}\right)$ (because $\theta = \ell/r$ )

A $= \dfrac{1}{2}rl$

**Example 4:**

Find the area of the sector of the circle whose radius is 4cm and length of the arc is 9cm.

**Solution:**

Let, AOB be the sector of the circle in which

OA = OB = r = 4cm

AB = $l$ = 9cm

Area of the sector $= \dfrac{1}{2}lr$

$= \dfrac{1}{2}$ x 9 x 4

= 18 sq. cm

**Example 5:**

Find the area of the sector of the circle when the radius of the circle is 15cm and the angle at the center is $60^o$.

**Solution:**

Since, r = 15cm, and angle $\theta = 60^o$

Area of the sector $= \dfrac{\pi r^2}{360^o}$ x $60^o$

$= \dfrac{22}{7}(15)^2 \text{ x } \dfrac{1}{6} = 117.8$ sq. cm

**Example 6:**

Find the expense of paving a circular court 60cm in diameter at Rs. 3.37 per square cm. If a space is left in the center for a fountain in the shape of a hexagonal each side of which is one cm.

**Solution:**

Area of the circle = $\frac{\pi}{4}d^2$ , but d = 60cm

Area = $\frac{3.14}{4}$ x 60 x 60

= 2828.5714 sq. cm

Area of the hexagon = $\frac{na^2}{4}\text{Cot}\ \frac{180^o}{n}$

But n = 4, a = 1cm

Area of the hexagon = $\frac{6 \text{ x } (1)^2}{4}\text{Cot}\ \frac{180^o}{6}$

= $\frac{3}{2}\text{Cot } 60^o$

Area = 2.5980 sq. cm

Area of the plot which is to be paved:

= Area of the circle – area of the hexagon

= 2828.5714 – 2.5981

= 2826 sq. cm

∴ Expense = 2826 x 3.37

= 9523.5 rupees

**13.5 Area of Segment:**

A segment is a portion of circle which is cut off by a straight line not passing through the centre. The straight line AB is called the chord of the circle.

The segment smaller than a semicircle is called a minor segment and a segment greater than a semi-circle is called a major segment.

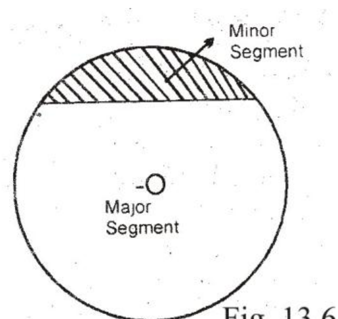



Fig. 13.6

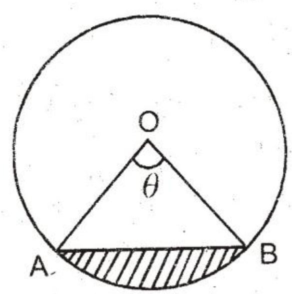



Fig. 13.7

Area of segment = Area of sector AOB $\pm$ area of $\Delta$ AOB

= $\frac{1}{2}r^2\theta \pm \frac{1}{2}r^2\text{Sin}\theta$

For major segment, positive sign is taken and for minor segment, negative sign is used.

**Area of the segment in terms of Height and Length of the Chord of the Segment:**

If "h" is the maximum height and "c" is the length of the chord of the segment, then area of the segment is given by.

Area = $\frac{h}{6c}(3h^2 + 4c^2)$ ……………. (1)

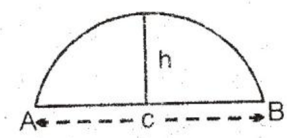


Fig. 13.8

**Length of Chord and Maximum height of arc:**

Let ACD is an arc of a circle with center 'O' and radius 'r' and ADB is the chord of length c and CD is maximum height 'h' of the segment.

(a) If 'h' and 'r' are given then 'c' can be calculated.

In right $\Delta OAD$, by Pythagoras theorem,

$(OA)^2 = (AD)^2 + (OD)^2$

$(AD)^2 = (OA)^2 - (OD)^2$

$(AD)^2 = r^2 - (r-h)^2 = r^2 - r^2 + 2rh - h^2$

AD $= \sqrt{2hr - h^2}$

$\frac{C}{2} = \sqrt{2hr - h^2}$ because AD = $\frac{1}{2}(ADB)$

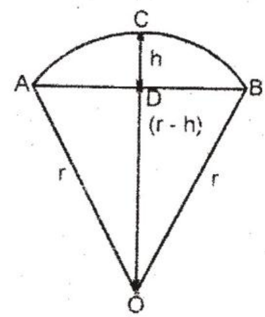


Fig. 13.9

C $= 2\sqrt{2hr - h^2}$ ……………. (i)

(b) If 'r' and 'c' are given, then 'h' can be calculated.

From (i)

Squaring (i) both sides

$C^2 = 4(2hr - h^2) \Rightarrow c^2 = 8hr \Rightarrow 4h^2$

Or $4h^2 - 8hr + c^2 = 0$

Which is the quadratic equation in 'h'

$a = 4, \; b = -8r, \; c = c^2$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8r \pm \sqrt{64r^2 - 16c^2}}{8}$$

$$h = \frac{8r \pm 8\sqrt{r^2 - \frac{c^2}{4}}}{8} = r \pm \sqrt{r^2 - \left(\frac{c}{2}\right)^2}$$

+ve sign is used for major segment and – ve sign to be taken for minor segment.

**Example 7:**

Find the area of a segment the chord of which 8cm with a height of 2cm.

**Solution:**

Since, h = 2cm, c = chord of segment = 8cm

Area of segment $= \frac{h}{6c}(3h^2 + 4c^2)$

$= \frac{2}{6(8)}(3(2)^2 + 4(8)^2) = 11.16$ sq. cm

**Example 8:**

The span of a circular arch of $90^o$ is 120cm. Find the area of the segment.

**Solution**

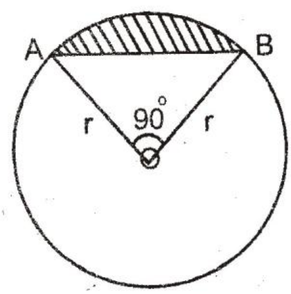


Fig. 13.10

Let O be the centre and r be the radius of the circle. Span AB = 120cm. In right angle $\Delta$AOB,

$(OA)^2 + (OB)^2 = (AB)^2$ , $r^2 + r^2 = (120)^2$

$\Rightarrow$ $2r^2 = 14400$

$r^2 = 7200$ …… (i) $\Rightarrow$ r = 84.85 cm

Now area of $\Delta$AOB $= \frac{1}{2}(OB)(OA) = \frac{1}{2}(r)\ (r)$

$= \frac{1}{2}r^2$

Area of $\Delta$AOB $= \frac{1}{2}(7200)$ by (i)

= 3600 sq. cm.

Area of sector $= \frac{\pi r^2}{360^o} \text{ x } N^o$

$= \frac{3.142 \text{ x } 7200}{360^o} \text{ x } 90^o = 5656$ sq. cm

Area of segment = Area of sector – Area of $\Delta$AOB

= 5656 – 3600 = 2056 sq. cm

**Example 9:**

The chord of an arc is 5cm and the diameter of the circle is 7cm. Find the height of the arc.

**Solution:**

Here c = 5cm

d = 7cm

r = 3.5cm

height of arc = $r \pm \sqrt{r^2 - \left(\frac{c}{2}\right)^2}$

h = $3.5 \pm \sqrt{(3.5)^2 - \left(\frac{5}{2}\right)^2}$

h = $3.5 \pm \sqrt{12.25 - 6.25}$

h = 3.5 $\pm$ 2.45

h = 3.5 $\pm$ 2.45, h = 3.5 – 2.45

h = 5.59 cm, h = 105cm

**Example 10:**

Find the chord of arc whose height is 24 cm, in a circle of radius 15 cm.

**Solution:**

Here, h = 24cm
r = 15cm

Chord of arc = $2\sqrt{2hr - h^2}$

C = $2\sqrt{2(24)(15) - (24)^2}$

C = $2\sqrt{720 - 576} = 2\sqrt{144} = 2(12)$

C = 24cm

## 13.6 Ellipse:

An ellipse is defined as the locus of a point which moves such that the sum of its distance from two fixed points remains constant.

The fixed points are the foci of the ellipse.

i.e. |PF| + |PF’| = constant

In figure, F and F′ are the two foci and D is the centre of the ellipse. AA′ is the major axis and BB′ is the minor axis, OA and OB are the semi-axes. Also 2a is the length of major axis and 2b is the length of minor axis.

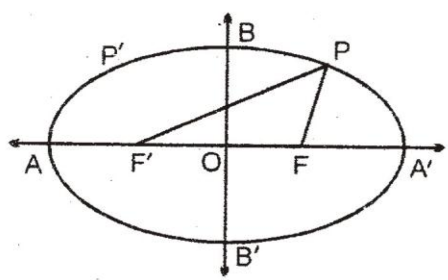



Fig. 13.11

**Area of an ellipse**

Area of an ellipse = $\pi$ ab

Where a = semi-major axis
b = semi-minor axis

and perimeter of an ellipse = $\pi$ (a + b)

**Example 11:**

It is desired to lay out a plot in the form of an ellipse. The area is 23100 sq. cm. The axes are in the ratio 3:2. Find the length of the fence required for this plot.

**Solution:**

Given area of plot in the form of ellipse = 23100 sq. cm.

Since axes are in the ratio 3:2

$\therefore \quad a = \frac{3}{2}b$

Area of plot $= \pi ab = \left(\frac{22}{7}\right) x \frac{3}{2} b \text{ x } b$

$23100 = 4.71b^2$

$b^2 = \frac{23100}{4.71} = 4904.46$

$b = 70.03\text{cm}$

$a = \frac{3}{2}(70.30) = 105\text{cm}$

Perimeter of plot $= \pi (a + b)$

$= \frac{22}{7}(70 + 105) \quad = 550 \text{ cm}$

**Example 12:**

An elliptical pipe has a major axis of 16cm and minor axis of 10cm. Find the diameter of a circular pipe that has the same area of cross-section.

**Solution:**

Major axis $= 2a = 16\text{cm}$

$a = 8\text{cm}$

Minor axis $= 2b = 10\text{cm}$

$b = 5\text{cm}$

$A = \pi ab$

$A = (3.142)(8)(5) = 125.64$ sq. cm.

Area of circle $= \pi r^2$ but $D = 2r$

Area of circle $= \pi \frac{D^2}{4}$

By given condition

Area of circle = Area of ellipse

$\pi \frac{D^2}{4} = 125.64$

$$D^2 = \frac{125.64 \times 4}{3.14} = 159.97$$

D = 12.64 cm

## Exercise 13

Q.1 The area of a semi-circle is 130 sq. cm. Find its total perimeter. Hint $P = \pi r + 2r$.

Q.2 A road 10m wide is to be made around a circular plot of 75m diameter. Find the cost of the ground needed for the road at Rs 4.00 per square meter.

Q.3 The areas of two concentric circles are 1386 sq. cm and 1886.5 sq. cm respectively. Find the width of the ring.

Q.4 The area of a circle is 154 sq. cm. Find the length of the side of the inscribed squares.

Q.5 A circular arc has a base of 4cm and maximum height 1.6cm. Find radius, length of arc and area of segment.

Q.6 The height of an arc is 7cm and its chord 42cm. Find the diameter of the circle.

Q.7 In a circle of diameter 25cm, the chord of an arc in 10cm, find its height.

Q.8 The radius of a circle is 33.5cm. Find the area of a sector enclosed by two radii and an arc 133.74 cm in length.

Q.9 The inner diameter of a circular building is 54m and the base of the wall occupies a space of 352 sq. m. Find the thickness of the wall.

Q.10 The axis of an ellipse are 40 cm and 60 cm. Find its perimeter and area.

Q.11 The sides of the triangle are 8, 21 and 25 m. find the radius of the circle whose area is equal to the area of triangle.

Q.12 The area of a sector is $76\pi$ sq.cm and angle of the sector is $70^0$. Find radius of the circle.

## Answers 13

| | | | | | |
|---|---|---|---|---|---|
| Q1. | 46.8cm | Q2. | 10684 rupees | Q3. | 23.2 sq. cm |
| Q4. | 9.89 cm | Q5. | 4.771 sq. cm; 2.05cm; 2.77cm | | |
| Q6. | 70cm | Q7. | 1.04 cm | Q8. | 2240.14 sq.cm |
| Q9. | 2m | Q10. | 157.1cm; 1885.20sq.cm | | |
| Q11. | 4.99m | Q12. | 19.77cm. | | |

## Summary

1. Area of circle A = $\pi r^2$
2. Perimeter or circumferences of circle = $2\pi r$
3. Area of Annulus (ring)

   A = $\pi(R^2 - r^2)$

   Where R = radius of outer circle , r = radius of inner circle
4. (a) Area of the sector if angle is given in degree

   Area of the sector = $\frac{\pi r^2}{360^o} \times N^o$

   Length of the arc = $\frac{2\pi r}{360^o} \times N^o$

   (b) If the angle in radian, say $\theta$ radians,

   then Area of sector = $\frac{1}{2} r^2 \theta$

   (c) If l is the length of an arc and r, radius of the circle, then area of sector.

   A = $\frac{1}{2} l r$

5. Area of segment = Area of sector AOB $\pm \frac{1}{2} r^2 \sin\theta$

   +ve sign is taken for major axis
   -ve sign is taken for major axis
6. Area of the segment in terms of Height and Length of the chord of the segment

   Area = $\frac{h}{6c}(3h^2 + 4c^2)$

   Where h = maximum height, c = length of the chord.

## Short Questions

**Q.1:** Define a circle.

**Q.2:** Define diameter of a circle

**Q.3:** Define chord of a circle.

**Q.4:** What is the area and circumference of circle .

**Q.5:** Find the radius of a circle the area of which is 9.3129 sq. cm.

**Q.6:** What are concentric circle.

**Q.7:** Define area of the Annulus (Ring).

**Q.8:** A path 14cmwide, surrounds a circular lawn whose diameter is 360 cm. Find the area of the path.

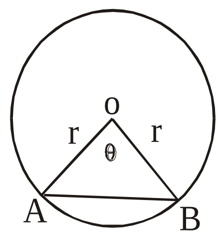
**Q.9:** Define a sector of the circle.

**Q.10:** Write the area of the sector.

**Q.11:** The minute hand of a clock is 12 cm long. Find the area which is described on the clock face between 6 A.M.to6.20A.M.

**Q.12:** Define a segment.

**Q.13:** Write the formula of Area of the minor segment and major segment when angle ‘θ’ and radius ‘r’ are given.




**Q.14:** Write the area of the segment in terms of Height and length of the chord of the segment.

**Q.15:** Find the area of a segment the chord of which 8 cm with a height of 2 cm.

**Q.16:** The area of a semi-circle is 130 sq. cm. Find its total perimeter.

## Answers

**Q5.** r= 1.72 cm. **Q8**. 16456 sq.cm **Q11.** 150.7 sq.cm.

**Q13.** Area of minor segment $= \frac{1}{2} r^2 (\theta - \sin\theta)$

Area of a major segment $= \frac{1}{2} r^2 (2\overline{\wedge} - \theta + \sin\theta)$

**Q15.** 11.16 sq. cm. **Q16.** 46.7 cm.

# Objective Type Questions

Q.1 Each questions has four possible answers. Choose the correct answer and encircle it.

___1. Area of a circle whose radius is 'a' cm is:

(a) $\pi r^2$ (b) $\pi a^2$ (c) $\pi a$ (d) $\frac{\pi}{2}a^2$

__2. Circumference of a circle whose radius is $\frac{1}{2}$ cm is equal to

(a) $2\pi$ (b) $2\pi r$ (c) $\frac{\pi r}{2}$ (d) $\frac{r}{2}$

___3. Area of a sector of $60^o$ in a circle of radius 6 cm is:

(a) $6\pi$ (b) $6\pi^2$ (c) $36\pi$ (d) $3\pi$

___4. The space enclosed between two concentric circles is called

(a) cone (b) ellipse (c) annulus (d) none of these

___5. Arc of circle with diameter 'd' is:

(a) $\frac{\pi}{2}r^2$ (b) $\frac{\pi}{2}d^2$ (c) $\frac{\pi}{4}d^2$ (d) None of these

___6. If R and r denote the radii of the outer and inner circles, then Area of annulus (ring) is:

(a) $\pi(R^2 - r^2)$ (b) $\frac{\pi}{2}(R^2 - r^2)$ (c) $\pi(R^2 + r^2)$

(d) $(R^2 - r^2)$

___7. If 2a and 2b are the major and minor axis of the ellipse, then area of ellipse is:

(a) ab (b) $\pi ab$ (c) $\frac{\pi}{2}ab$ (d) $\pi^2 ab$

___8. If 2a and 2b are the major and minor axis of the ellipse, the circumference of the ellipse is:

(a) $\pi(a + b)$ (b) $\pi(a - b)$

(c) $\pi(a + b)^2$ (d) $\pi^2(a - b)$

___9. If the area of circle is , then radius r is:

(a) 4 (b) 2 (c) 16 (d) 8

___10. If x and y the major and minor axis of the ellipse then area of ellipse is:

(a) $\frac{\pi xy}{2}$ (b) $\frac{\pi xy}{4}$ (c) $\pi xy$ (d) $\pi^2 xy$

___11. The circumference of a pulley is $\frac{440}{7}$ cm, its diameter is:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (a) | 20cm | (b) | 40cm | (c) | 80cm | (d) | 10cm |

**Answers**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Q.1 | (1) | b | (2) | a | (3) | a | (4) | c |
| | (5) | c | (6) | a | (7) | b | (8) | a |
| | (9) | a | (10) | b | (11) | a | | |