Name KEY

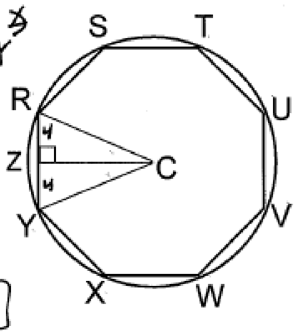
Date ____________

CC Geometry H

HW #3

1a) Name the center, a radius, a height and a central angle of the regular polygon.

Center C , radius $\overline{CR}$, height $\overline{CZ}$, cent. ∡ RCY



b) Find m∡RCY, m∡RCZ, and m∡ZRC.

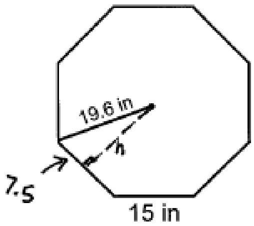
45°   22.5°   67.5°

c) If RY = 8, find CZ, to the nearest tenth.

$\frac{\tan 22.5}{1} = \frac{4}{CZ}$ $4 = CZ\tan 22.5$

$\boxed{CZ \approx 9.7 \text{ units}}$

2. Find the perimeter and area of a regular octagon with side 15 in. and radius 19.6 in. (answer to the nearest tenth)



Perimeter

$\boxed{15(8) = 120 \text{ u.}}$

$7.5^2 + h^2 = 19.6^2$

$h^2 = 327.91$

$h = \sqrt{327.91}$

$a = \frac{1}{2}ph$

$a = \frac{1}{2}(120)(\sqrt{327.91})$

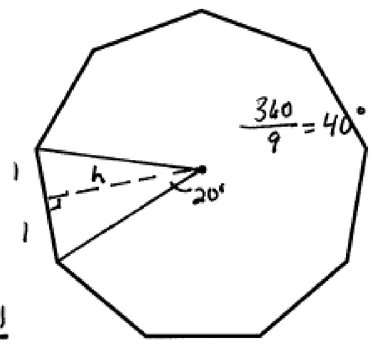
$\boxed{A \approx 1086.5 \text{ in}^2}$

3. The perimeter of a regular nonagon is 18 in. Find the area, to the nearest inch.

$A = \frac{1}{2}ph$

$A = \frac{1}{2}(18)\left(\frac{1}{\tan 20}\right)$

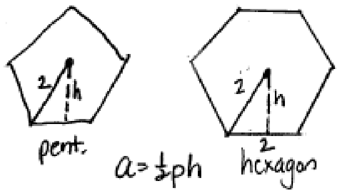
$\boxed{A \approx 25 \text{ in}^2}$



$\frac{\tan 20}{1} = \frac{1}{h}$

$\frac{h\tan 20 = 1}{\tan 20}$

$h = \frac{1}{\tan 20}$



4. True or false? $a = \frac{1}{2}ph$

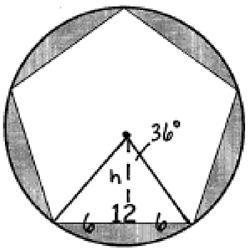
a) The area of a regular n-gon of a fixed radius r increases as n increases. true

b) The height of a regular polygon is always less than the radius. true (radius = hypotenuse)

c) The radius of a regular polygon is always less than the side length.

False, ex. in a hexagon, they are equal!

5. Approximate the area of the circle using an inscribed regular pentagon with side length 12. (nearest whole number).



$\tan 36 = \frac{6}{h}$

$\frac{h\tan 36 = 6}{\tan 36}$

$h \approx 8.258$

$A = \frac{1}{2}ph$

$A = \frac{1}{2}(5\cdot 12)(8.258)$

$A = 30(8.258)$

$\boxed{A \approx 248\ u^2}$

Mixed Review:

1. ΔABC ~ ΔDEF. AB = 8 inches, DE = 10 inches, and the area of ΔABC is 56 square inches. What is the area of ΔDEF?

$r = \frac{10}{8} = \frac{5}{4}$

area old $\cdot\, r^2$ = area new

$56 \cdot \left(\frac{5}{4}\right)^2 = \Delta DEF$

$\boxed{87.5u^2 = \Delta DEF}$

2. In ΔABC altitude $\overline{BD}$ is drawn. If AB = $4\sqrt{5}$, BD = 8, and AC = 16, is ΔABC a right triangle? Justify your answer.

① $x^2 + 8^2 = (4\sqrt{5})^2$

$x^2 + 64 = 80$

$x^2 = 16$

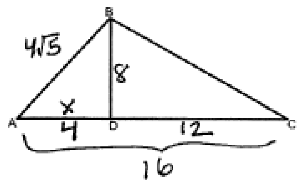
$x = 4$

② Small Δ Med. Δ

$\begin{array}{c} L1 \\ L2 \end{array} \frac{4}{8} = \frac{8}{12} \begin{array}{c} L1 \\ L2 \end{array}$

$64 \neq 48$

corr. legs of small & med. Δ are not in proportion so ΔABC cannot be a rt. Δ.



Aim #4: What is the area and circumference of a circle and how do we calculate percent error of area approximations?

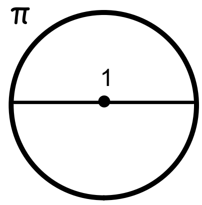
Do Now: The circumference of circle $C_1 = 9$ cm, and the circumference of $C_2 = 2\pi$ cm. What is the value of the ratio of the areas of C_1 to C_2 ?

$r = \frac{2\pi}{9}$ $r^2 = \left(\frac{2\pi}{9}\right)^2 = \frac{4\pi^2}{81}$ $\boxed{81 : 4\pi^2}$

Let us define π to be the circumference of a circle whose diameter is 1.

$C = 2\pi r$ or $C = \pi d$

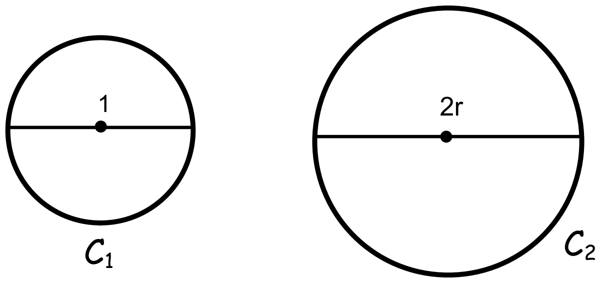
circumference of a circle.



$C = \pi(1)$

$C = \pi$ u

We are going to show why the circumference of a circle has the formula $2\pi r$.
Circle C_1 below has a diameter of $d = 1$, and circle C_2 has a diameter of $d = 2r$.



a. All circles are similar. What scale factor of the similarity transformation takes C_1 to C_2?

$r = \frac{\text{new}}{\text{old}}$ $r = \frac{2r}{1}$ $r = 2r$

b. Since the circumference of a circle is a one-dimensional measurement, the value of the ratio of two circumferences is equal to the value of the ratio of their respective diameters. 1-D! Rewrite the following equation by filling in the appropriate values for the diameters of C_1 and C_2:

$$\frac{\text{Circumference } C_2}{\text{Circumference } C_1} = \frac{\text{diameter } C_2}{\text{diameter } C_1}$$

$$\frac{\text{Circumference } C_2}{\pi} = \frac{2r}{1}$$

c. Rewrite the equation to show a formula for the circumference C_2.

$C_2 = 2\pi r$

Since C_2 is a general circle, we have shown the circumference of any circle is $2\pi r$.

Exercises

1. What is the radius of a circle whose circumference is π ?

$C = 2\pi r$

$\dfrac{\pi = 2\pi r}{2\pi}$

$r = \dfrac{\pi}{2\pi} = \boxed{r = \frac{1}{2}\text{ u.}}$

2. The circumference of a circle and the perimeter of a square are each 50 cm. Which figure has the greater area?

Circle

$C = 2\pi r$

$\dfrac{50 = 2\pi r}{2\pi}$

$\left(\dfrac{50}{2\pi}\right) = r$

$r \approx 7.958$

$A = \pi r^2$

$A = \pi (7.958)^2$

$A \approx 199\text{ cm}^2$

$\boxed{\text{circle}}$

Square

sides = 12.5cm

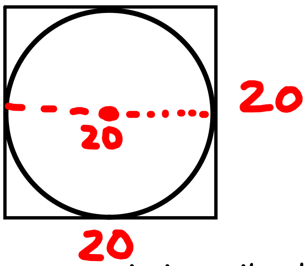
$A = s^2$

$A = (12.5)^2$

$A = 156.25\text{ cm}^2$

3. The side of a square is 20 cm long. What is the circumference of the circle when

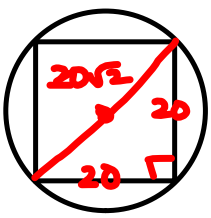
a) the circle is inscribed within the square



$C = \pi d$

$C = 20\pi \approx 62.8\text{ cm}$

b) the square is inscribed within the circle



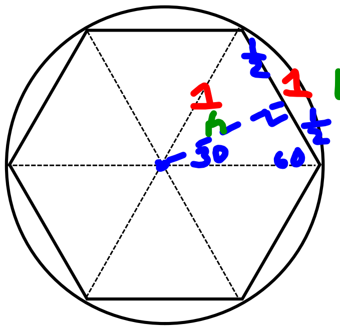
$C = \pi d$

$C = \pi (20\sqrt{2})$

$C = 20\sqrt{2}\,\pi\text{ cm}$

$\approx 88.9\text{cm}$

4. a. To the nearest hundredth, approximate the area of a circle of radius 1 using an inscribed regular hexagon.



$h = \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2}$

$a = \frac{1}{2}ph$

$a = \frac{1}{2}(6)\left(\frac{\sqrt{3}}{2}\right)$

$a = 3\left(\frac{\sqrt{3}}{2}\right)$

$a = \frac{3\sqrt{3}}{2} = 1.5\sqrt{3}$

$\boxed{\approx 2.60\,u^2}$

b. Find the exact area of a circle that has a radius of 1.

$A = \pi r^2 \quad A = \pi(1)^2 = \boxed{\pi\,u^2}$

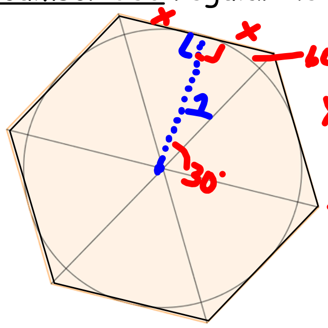
c. What is the percent of error, to the nearest tenth, of the approximation of the area in part a?

$$\text{Percent error} = \frac{|\text{measured} - \text{actual}|}{\text{actual}} \times 100$$

$$\%\ \text{error} = \frac{|1.5\sqrt{3} - \pi|}{\pi} \times 100$$

$$= 17.3007 = \boxed{17.3\%}$$

5. a. To the nearest hundredth, approximate the area of a circle of radius 1 using a circumscribed regular hexagon.



$x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$s = \frac{2\sqrt{3}}{3} \cdot 6$

$\frac{12\sqrt{3}}{3} = 4\sqrt{3}$

$a = \frac{1}{2}ph$

$a = \frac{1}{2}(4\sqrt{3})(1)$

$a = 2\sqrt{3}\,u^2$

$\approx 3.46\,u^2$

b. What is the percent of error, to the nearest tenth, of the approximation of the area in part a?

$$\%\ \text{error} = \frac{|2\sqrt{3} - \pi|}{\pi} \times 100 \approx 10.3\%$$

6. a. Find the average of the approximations for the area of a circle of radius 1 using the inscribed and circumscribed regular hexagon.

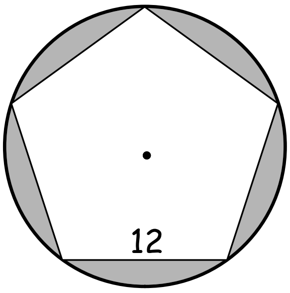
$$\frac{1.5\sqrt{3} + 2\sqrt{3}}{2} = \frac{3.5\sqrt{3}}{2} \approx 3.03\,u^2$$

b. What is the percent of error, to the nearest tenth, of the average approximation of the area in part a?

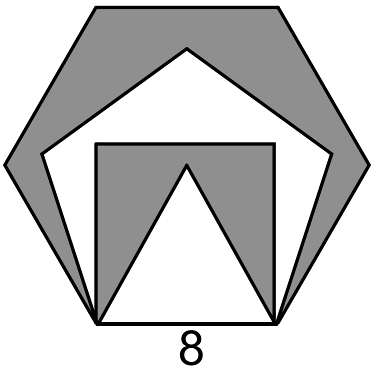
$$\%\ \text{error} = \frac{\left|\frac{3.5\sqrt{3}}{2} - \pi\right|}{\pi} \times 100 \approx 3.5\%$$

Mixed Practice:

1. Find the area of the shaded region (nearest tenth). The pentagon is regular.



2. An equilateral triangle lies inside a square inside a regular pentagon inside a regular hexagon. Find the approximate area of the entire shaded region to the nearest whole number.



3. The areas of two similar triangles are in the ratio of 4:1. The length of a side of the smaller triangle is 5. Find the length of the corresponding side in the larger triangle.

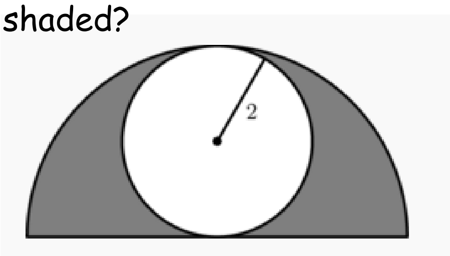
4. Find the ratio of the areas of two similar triangles in which the lengths of two corresponding sides are:

a) s = 1 and s' = 5 b) s = 10 and s' = 15 c) s = 9 and s' = 3

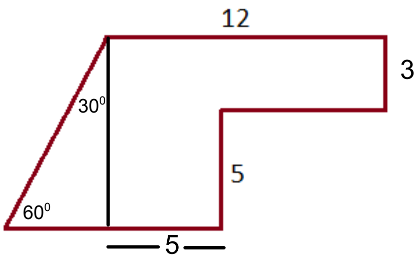
5. Find the ratio of the perimeters of two similar polygons if the ratio of their areas is:

a) 16:1 b) 1:4 c) 9:25 d) 3:1

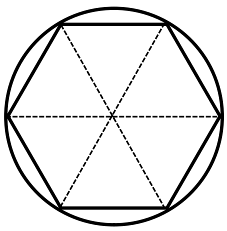
6. A circle of radius 2 is inscribed in a semicircle as shown. The area inside the semicircle, but outside the circle is shaded. What fraction of the semicircle's area is shaded?



7. Find the exact area:



8. A regular hexagon is inscribed in a circle. If the length of the radius of the circle is $8\sqrt{3}$ in., find the perimeter and area of the hexagon.



9. If a regular pentagon circumscribes a circle and the length of its height is 7 in., what is the length of the diameter, and the circumference and area of the circle?