College Algebra 6.5 Test Review Name ______________________________

Period _______

1. Use a sum or difference formula to find the exact value of the expression.

a. $\cos 110^\circ \cos 20^\circ + \sin 110^\circ \sin 20^\circ$ $\cos(110-20) = \cos 90^\circ$

b. $\sin 110^\circ \cos 20^\circ + \cos 110^\circ \sin 20^\circ$ $\sin(110+20) = \sin 130^\circ$

c. $\dfrac{\tan 110^\circ + \tan 20^\circ}{1 - \tan 110^\circ \tan 20^\circ}$ $\tan(110+20) = \tan 130^\circ$

2. Find the exact value ~~of $\cos 2u$, $\sin 2u$, $\tan 2u$~~ using a double angle formula and given the following:

a. $\cos u = -\dfrac{2}{3}$, $\dfrac{\pi}{2} < u < \pi$ Q2 $\sin 2u = \dfrac{-4\sqrt{5}}{9}$

$\sin 2u = 2\sin u \cos u$

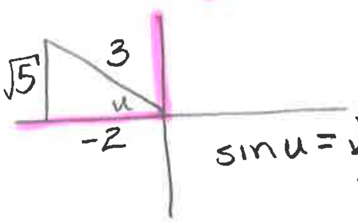
$= 2\left(\frac{\sqrt{5}}{3}\right)\left(-\frac{2}{3}\right)$

$= -\frac{4\sqrt{5}}{9}$

$x^2+2^2=3^2$

$x^2+4=9$

$x^2=5$



$\sin u = \frac{\sqrt{5}}{3}$

b. $\cot u = -6$, $\dfrac{3\pi}{2} < u < 2\pi$ Q4 $\cos 2u = \dfrac{35}{37}$

$\tan u = -\frac{1}{6}$

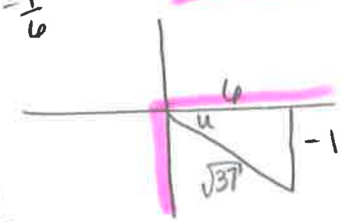
$\cos 2u = 2\cos^2 u - 1$

$= 2\left(\frac{6}{\sqrt{37}}\right)^2 - 1$

$= 2\left(\frac{36}{37}\right) - 1$

$= \frac{72}{37} - \frac{37}{37}$

$= \frac{35}{37}$



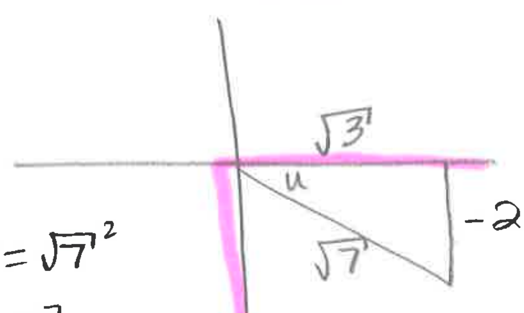
$6^2+(-1)^2=c^2$

$36+1=c^2$

$c^2=37$

$\cos u = \frac{6}{\sqrt{37}}$

c. $\sin u = -\dfrac{2}{\sqrt{7}}$, $\dfrac{3\pi}{2} < u < 2\pi$ $\tan 2u = \dfrac{12}{\sqrt{3}}$ OR $4\sqrt{3}$



$\tan 2u = \dfrac{2\tan u}{1-\tan^2 u}$

$x^2+2^2=\sqrt{7}^2$

$x^2+4=7$

$x^2=3$

$\tan u = \frac{-2}{\sqrt{3}}$

$= \dfrac{2\left(\frac{-2}{\sqrt{3}}\right)}{1-\left(\frac{-2}{\sqrt{3}}\right)^2} = \dfrac{\frac{-4}{\sqrt{3}}}{\frac{3}{3}-\frac{4}{3}}$

$\dfrac{\frac{-4}{\sqrt{3}}}{-\frac{1}{3}}$ $\frac{-4}{\sqrt{3}} \cdot \frac{3}{-1} = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$

3. Use a sum or difference formula to find the solution(s) of the equation on the interval $[0, 2\pi)$.

No $+2\pi n$ or $+\pi n$

$$\cos\left(\overset{\alpha}{x}+\overset{\beta}{\frac{\pi}{2}}\right)-\cos\left(x-\frac{\pi}{2}\right)=\sqrt{3}$$

$x=\frac{4\pi}{3}, \frac{5\pi}{3}$

$\cos x\cos\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}-\left(\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}\right)=\sqrt{3}$

$\cancel{\cos x\cos\frac{\pi}{2}}-\sin x\sin\frac{\pi}{2}-\cancel{\cos x\cos\frac{\pi}{2}}-\sin x\sin\frac{\pi}{2}=\sqrt{3}$

$-2\sin x\sin\frac{\pi}{2}=\sqrt{3} \rightarrow -2\sin x(1)=\sqrt{3}$

$\sin x=\frac{-\sqrt{3}}{2}$

ref. angle $=\frac{\pi}{3}$

$\frac{4\pi}{3}$ $\frac{5\pi}{3}$

4. Use a double angle formula to rewrite the equation and find the exact solutions in the interval $[0, 2\pi)$.

a. $\sin 2x+\cos x=0$

$2\sin x\cos x+\cos x=0$

$\cos x(2\sin x+1)=0$

$\cos x=0$ $2\sin x+1=0$

$\sin x=-\frac{1}{2}$

Ref angle $\pi/6$

$x=\frac{\pi}{2}, \frac{3\pi}{2}$ $x=\frac{7\pi}{6}$ $x=\frac{11\pi}{6}$

b. $\sin 2x\sin x=\cos x$

$\sin 2x\sin x-\cos x=0$

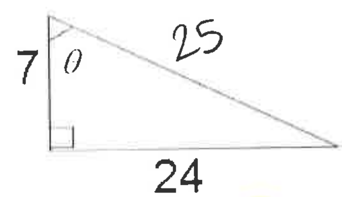
$2\sin x\cos x\sin x-\cos x=0$

$\cos x(2\sin x-1)=0$

$\cos x=0$ $\sin x=\frac{1}{2}$

$x=\frac{\pi}{2}, \frac{3\pi}{2}$ $\frac{5\pi}{6}$ $\frac{\pi}{6}$

5. Use the figure to find the exact value of each trigonometric function.



a. $\sin\frac{\theta}{2}=\sqrt{\frac{1-\cos\theta}{2}}=\sqrt{\frac{1-\frac{7}{25}}{2}}=\sqrt{\frac{\frac{25}{25}-\frac{7}{25}}{2}}=\sqrt{\frac{\frac{18}{25}}{2}}=\sqrt{\frac{\cancel{18}^{9}}{25}\cdot\frac{1}{\cancel{2}}}=\frac{3}{5}$

b. $\cos\frac{\theta}{2}=\sqrt{\frac{1+\frac{7}{25}}{2}}=\sqrt{\frac{\frac{25}{25}+\frac{7}{25}}{2}}=\sqrt{\frac{\cancel{32}^{16}}{25}\cdot\frac{1}{\cancel{2}}}=\frac{4}{5}$

c. $\tan\frac{\theta}{2}=\frac{1-\cos\theta}{\sin\theta}=\frac{1-\frac{7}{25}}{\frac{24}{25}}=\frac{\frac{25}{25}-\frac{7}{25}}{\frac{24}{25}}=\frac{18}{\cancel{25}}\cdot\frac{\cancel{25}}{24}=\frac{3}{4}$

$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$  $\sin\frac{5\pi}{6} = \frac{1}{2}$

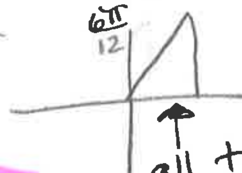
$2 \cdot \frac{\alpha}{2} = \frac{5\pi}{12} \cdot 2$

$\alpha = \frac{5\pi}{6}$

6. Use half-angle formulas to determine the exact values of the following if $\theta = \frac{5\pi}{12}$.

a. $\sin\theta$ $\quad \boxed{\sin\frac{5\pi}{12}} = \sqrt{\frac{1-\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{1-\frac{-\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$



b. $\tan\theta$ $\quad \boxed{\tan\frac{5\pi}{12}} = \frac{1-\cos\frac{5\pi}{6}}{\sin\frac{5\pi}{6}} = \frac{1-\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{\frac{1}{2}} \quad \frac{2+\sqrt{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{1} = 2+\sqrt{3}$

all trig functions are +

c. $\cos\theta$ $\quad \boxed{\cos\frac{5\pi}{12}} = \sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}} = \sqrt{\frac{2+\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

7. Use double-angle formulas to rewrite the expression 14sinxcosx

$7(2\sin x\cos x) = 7(\sin 2x)$

8. Rewrite the expression in terms of the first power of the cosine.

a. $\sin^2 2x$ $\quad * \sin^2\theta = \frac{1-\cos 2\theta}{2} \rightarrow \frac{1-\cos 2(2x)}{2}$

$\theta = 2x$ $\qquad = \frac{1-\cos 4x}{2}$

b. $\cos^2\frac{x}{2}$ $\quad * \cos^2\theta = \frac{1+\cos 2\theta}{2} \rightarrow \frac{1+\cos 2\left(\frac{x}{2}\right)}{2}$

$\theta = \frac{x}{2}$ $\qquad = \frac{1+\cos x}{2}$