CONVEXITY AND OPTIMIZATION IN $\mathbb{R}^n$

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A Wiley-Interscience Publication
JOHN WILEY & SONS, INC.

This book is printed on acid-free paper. ∞

Published simultaneously in Canada.

For ordering and customer service, call 1-800-CALL-WILEY.

Library of Congress Cataloging-in-Publication Data:

Berkovitz, Leonard David, 1924–
Convexity and optimization in R [superscript n] / Leonard D. Berkovitz.
p. cm.--(Pure and applied mathematics)
"A Wiley-Interscience publication."
Includes bibliographical references and index.
ISBN 0-471-35281-0 (cloth : alk. paper)
1. Convex sets. 2. Mathematical optimization. I. Title. II. Pure and applied mathematics (John Wiley & Sons : Unnumbered)

QA640.B46 2001
516′.08--dc21

2001045391

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

To my wife, Anna

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PREFACE

This book presents the mathematics of finite-dimensional optimization, featuring those aspects of convexity that are useful in this context. It provides a basis for the further study of convexity, of more general optimization problems, and of numerical algorithms for the solution of finite-dimensional optimization problems. The intended audience consists of beginning graduate students in engineering, economics, operations research, and mathematics and strong undergraduates in mathematics. This was the audience in a one-semester course at Purdue, MA 521, from which this book evolved.

Ideally, the prerequisites for reading this book are good introductory courses in real analysis and linear algebra. In teaching MA 521, I found that while the mathematics students had the real analysis prerequisites, many of the other students who took the course because of their interest in optimization did not have this prerequisite. Chapter I is for those students and readers who do not have the real analysis prerequisite; in it I present those concepts and results from real analysis that are needed. Except for the Weierstrass theorem on the existence of a minimum, the "heavy" or "deep" theorems are stated without proof. Students without the real variables prerequisite found the material difficult at first, but most managed to assimilate it at a satisfactory level. The advice to readers for whom this is the first encounter with the material in Chapter I is to make a serious effort to master it and to return to it as it is used in the sequel.

To address as wide an audience as possible, I have not always presented the most general result or argument. Thus, in Chapter II I chose the "closest point" approach to separation theorems, rather than more generally valid arguments, because I believe it to be more intuitive and straightforward for the intended audience. Readers who wish to get the best possible separation theorem in finite dimensions should read Sections 6 and 7 of Chapter II. In proving the Fritz John Theorem, I used a penalty function argument due to McShane rather than more technical arguments involving linearizations. I limited the discussion of duality to Lagrangian duality and did not consider Fenchel duality, since the latter would require the development of more mathematical machinery.

The numbering system and reference system for theorems, lemmas, remarks, and corollaries is the following. Within a given chapter, theorems, lemmas, and remarks are numbered consecutively in each section, preceded by the section number. Thus, the first theorem of Section 1 is Theorem 1.1, the second, Theorem 1.2, and so on. The same applies to lemmas and remarks. Corollaries are numbered consecutively within each section without a reference to the section number. Reference to a theorem in the same chapter is given by the theorem number. Reference to a theorem in a chapter different from the current one is given by the theorem number preceded by the chapter number in Roman numerals. Thus, a reference in Chapter IV to Theorem 4.1 in Chapter II would be Theorem II.4.1. References to lemmas and remarks are similar. References to corollaries within the same section are given by the number of the corollary. References to corollaries in a different section of the same chapter are given by prefixing the section number to the corollary number; references in a different chapter are given by prefixing the chapter number in Roman numerals to the preceding.

I thank Rita Saerens and John Gregory for reading the course notes version of this book and for their corrections and suggestions for improvement. I thank Terry Combs for preparing the figures. I also thank Betty Gick for typing seemingly innumerable versions and revisions of the notes for MA 521. Her skill and cooperation contributed greatly to the success of this project.

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