CONSTRUCTING PLAYGROUNDS: FIBRED DOUBLE CATEGORIES

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In previous work on concurrent game semantics [7], the second author has described a general construction which takes as input a *pseudo double category* [2, 3, 5, 6, 10, 4] $\mathbb{D}$ with extra structure – there called a *playground*, and outputs a notion of concurrent strategy. When fed with adequate pseudo double categories, $\mathbb{D}_{CCS}$ and $\mathbb{D}_{\pi}$, the construction produces meaningful notions of concurrent strategy for CCS [7] and the π-calculus [1]. The pseudo double categories $\mathbb{D}_{CCS}$ and $\mathbb{D}_{\pi}$ are constructed in very similar ways, which raises the question of how to generalise the process.

Furthermore, from the beginning, the construction was hoped to be expressive enough to recover known results on deterministic, functional languages such as PCF [8], albeit in a new manner, part of a general framework for game semantics. However, one particular point of functional languages cast some doubt on this issue, namely multiple use of variables. It may not be immediately clear why this point would not cause any trouble in CCS or π, in particular since λ-calculus translates into π. This may be due to the fact that λ is much tamer than π, and hence provides less tests to discriminate subtly different strategies. Anyway, issues naturally arise when trying to model, e.g., the simply-typed λ-calculus, $\Lambda_{\rightarrow}$, using the same approach.

In this work, we propose a generalisation of the construction, which encompasses both $\mathbb{D}_{CCS}$ and $\mathbb{D}_{\pi}$, but in fact also works for $\Lambda_{\rightarrow}$. This particular example and its adequacy will be studied in more detail in a follow-up paper, where we will show that Ong and Tsukada's sheaf model [11] naturally arises as an instance; instead we here focus on the most difficult part of the construction.

In a first part, starting with rather basic data, namely what we call a *signature* S, we construct a pseudo double category $\mathbb{D}(\mathsf{S})$, which, when instantiated adequately, produces $\mathbb{D}_{CCS}$ and $\mathbb{D}_{\pi}$. Our main contribution is then, in a second part, to prove that under suitable hypotheses, $\mathbb{D}(\mathsf{S})$ satisfies a certain *fibredness* property. This property is necessary for proving that $\mathbb{D}(\mathsf{S})$ is a playground, so the present work is only a first step in that direction. Fibredness demands that plays in the game generated by S, when viewed as indexed over their initial positions, form a Grothendieck fibration. Intuitively, this says that given any play P with initial position X and any subposition Y of X, there is a canonical way of restricting P to Y.

The technical development mainly rests on a combination of cofibrantly generated (strong) factorisation systems, adhesivity [9], and limits w.r.t. a coreflective subcategory.

References

[1] Clovis Eberhart, Tom Hirschowitz, and Thomas Seiller. An intensionally fully-abstract sheaf model for π. In *CALCO*, Leibniz International Proceedings in Informatics (LIPIcs). Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik, 2015. To appear.

[2] Charles Ehresmann. Catégories structurées. *Annales scientifiques de l'Ecole Normale Supérieure*, 80(4):349–426, 1963.

[3] Charles Ehresmann. *Catégories et structures*. Dunod, 1965.

[4] Richard H. G. Garner. *Polycategories*. PhD thesis, University of Cambridge, 2006.

[5] Marco Grandis and Robert Paré. Limits in double categories. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, 40(3):162–220, 1999.

[6] Marco Grandis and Robert Paré. Adjoints for double categories. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, 45(3):193–240, 2004.

[7] Tom Hirschowitz. Full abstraction for fair testing in CCS (expanded version). *Logical Methods in Computer Science*, 10(4), 2014. doi: 10.2168/LMCS-10(4:2)2014.

[8] J. M. E. Hyland and C.-H. Luke Ong. On full abstraction for PCF: I, II, and III. *Information and Computation*, 163(2):285–408, 2000. doi: 10.1006/inco.2000.2917.

[9] Stephen Lack and Paweł Sobociński. Adhesive categories. In *FoSSaCS*, volume 2987 of *LNCS*, pages 273–288. Springer, 2004. doi: 10.1007/978-3-540-24727-2_20.

[10] Tom Leinster. *Higher Operads, Higher Categories*, volume 298 of *London Mathematical Society Lecture Notes*. Cambridge University Press, Cambridge, 2004.

[11] Takeshi Tsukada and C.-H. Luke Ong. Nondeterminism in game semantics via sheaves. In *LICS*. IEEE Computer Society, 2015.