Fundamentals of High Accuracy Inertial Navigation

Averil B. Chatfield

Volume 174
PROGRESS IN
ASTRONAUTICS AND AERONAUTICS

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Charles Stark Draper Laboratory, Inc.
Cambridge, Massachusetts

Published by the
American Institute of Aeronautics and Astronautics, Inc.
1801 Alexander Bell Drive, Reston, Virginia 20191-4344

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ISBN 1-56347-243-0

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Preface

Inertial navigation involves a blend of inertial instrumentation, mathematics. control system design, and geodesy. The technology is used in many military, civilian, engineering. and scientific activities. Examples include the navigation and attitude control of aircraft, submarines. ballistic and cruise missiles, and spacecraft. Since the 1940s, when the development of inertial navigation systems for aircraft began in earnest, there has been a steady improvement in the reduction in size and the increase in accuracy of the two primary inertial sensors: the gyroscope and the accelerometer. During this same time period, there have been tremendous improvements in digital computer technology. The inertial measurement unit in the Snark cruise missile developed in the 1950s weighed several hundred pounds; today inertial measurement units are king manufactured that weigh only a few pounds. Computations in the Snark missile were handled by a simple digital differential analyzer. Today inertial navigation systems employ high-capacity, high-speed, general-purpose digital computers capable of thousands of operations per second.

The focus of the book is on the physical and mathematical principles forming the basis for inertial navigation. Although specific inertial system designs are not treated, the material in the book is applicable to the inertial navigation of all types of vehicles whether on land, in or on the ocean, in the atmosphere, or in space in the vicinity of the Farth. The reader is assumed to be familiar with the algebra and calculus of matrices, analytical mechanics, elementary control system theory. elementary statistics, and Kalman filtering.

The book is written primarily for the third or fourth year engineering student majoring in inertial technology. It is also intended for the engineer or scientist trained in mathematics, control theory, analytical mechanics, geodesy. or physics, who has a need for understanding the basic principles of inertial navigation technology.

Previous books on inertial navigation have dealt almost exclusively with the inertial instrumentation and mathematical aspects. The essential Darts played by control system theory and geodesy have largely been ignored. Elementary control system theory provides a means for obtaining the impulse response function of a navigation system operating in a central force field, which is essential for understanding the propagation of errors through the system. By itself inertial navigation can only provide the change in state from one time to the next with ever-increasing errors. Geodesy provides the absolute initial conditions so that the absolute state can be calculated. It also provides the physical data used for inertial instrument calibration and for deriving the navigation gravity model used in navigation computations. For the first time, this book comprehensively treats these aspects of the blend of inertial navigation technology and geodesy.

The accuracy of inertial instrumentation decreases with time and with environmental changes. Measurements independent of the inertial equipment are often incorporated into the navigation computations to improve accuracy. To accommodate this situation the term inertial navigation has been broadened to include the use of external equipment to periodically measure position, velocity, or orientation to update the navigation state vector. The physical basis for using external measurements for this purpose is derived from basic principles, and the several types of external measurements that can be employed are described. Of particular interest are the descriptions of the basis for using GPS position and velocity information to improve the accuracy. If external measurements are used, the process is referred to as aided inertial navigation.

As in previous texts on inertial navigation, the subject of error propagation is dealt with at length and the propagation of inertial instrument errors is given extensive treatment. For the first time, this book deals with the effects on accuracy of errors in the external measurements and geodetic data. Also for the first time, the book treats the subject of accuracy criteria and evaluation. Equations are derived and verified for accurately computing the Circular Error Probable (CEP) and the Spherical Error Probable (SEP) that account for biases as well as random errors.

The book is divided into three parts: inertial navigation. inertial navigation with aids. and accuracy analysis. The first two parts are designed to impart to the reader an understanding of the fundamentals of high-accuracy inertial navigation without requiring an understanding of the statistical analysis techniques involved in determining the effects of errors on accuracy. The first two parts provide the reader with enough information to understand how inertial navigation systems function and to some extent how they are designed. The third part defines the criteria for determining the accuracy and then leads the reader through the complex process of determining the accuracy. Accuracy information is required lo establish and verify the system design specifications, to predict operational performance, and to determine the actual performance achieved. A more complete description of the material in Parts I. II, and III is provided at the beginning of each part.

Averil B. Chatfield
September 1997

Chapter 1

Introduction

THE invention of the sextant and mechanical chronometer had a profound effect on navigation before the 20th century. These two instruments made possible navigation over the featureless seas and oceans. The development of inertial navigation sensors has had a similar striking influence on present-day navigation. Inertial instruments have improved navigation over the featureless oceans and have made possible navigation under the oceans, in the atmosphere. and in space. Beginning in the late 1940s. systems were developed for aircraft. ship. submarine, and space vehicle navigation. During the following decades, steady improvements were made in the accuracy and reliability of inertial sensors and navigation computers. Today inertial navigation systems are used in all types of commercial and military aircraft, commercial and military ships, military tanks. submarines. missiles of all sizes, and space vehicle boosters.

The development of the fundamentals of inertial navigation systems begins with an examination of the forces producing motion in an inertial frame. Because these forces must be defined in an inertial frame. we next define the inertial coordinate frame and then derive the fundamental equation of inertial navigation in this frame. With this background. a formal description of inertial navigation is given. and the measurements made by inertial sensors are described. The chapter is concluded with a description of a model of the earth suitable for use as a reference for inertial navigation computations.

I. Forces Producing Motion

If motion is at aconstant speed in a fixed direction in an inertial space containing no attracting matter. we know from Newton's first law of motion that there is no need to consider forces. The position and velocity of the vehicle is predictable for all time. It is when changes in motion occur that the concept of forces comes into play. Two types of forces determine the motion of a vehicle: gravity and inertia.

In the description of these forces, no distinction is made between gravitational mass and inertial mass. Gravitational mass has been described' as being like a charge the object feels in proportion to its gravitational mass; whereas inertial mass describes the resistance of a vehicle to changing the state of motion, Because the equivalence of the two types of masses has been established to approximately one part in 10^{12}, there is no need to distinguish between the two types of masses for navigation in the vicinity of the Earth (Ref. 2).

A. Gravitation

In accordance with Newton's universal law of gravitation. the force due to the gravitational attraction of a mass. such as the Earth. is proportional to the product