Double and Half Angle Formulas

** More trigonometric formulas - based on the addition formulas from previous section (Gotta Love Them).

Recall: $\sin(x+y) =$

$\cos(x+y) =$

$\tan(x+y) =$

$\sin(2\theta) =$

$\cos(2\theta) =$

$\tan(2\theta) =$

Double Formulas

Example: If $\sin(\theta) = \frac{3}{5}, \ \frac{\pi}{2} < \theta < \pi$, find the exact value of $\sin(2\theta)$ and $\cos(2\theta)$

Example: Find an expression for the triple angle formula $\cos(3x)$ in terms of cosine only.

Example: Find the exact value of the following expression: $\sin\left[2\cos^{-1}\left(\frac{5}{13}\right)\right]$

Double Angle Formulas

As a consequence of the double angle formulas from $\cos(2\theta)$, we can construct the following useful identities.

$$\sin^2(\theta) =$$

$$\cos^2(\theta) =$$

$$\tan^2(\theta) =$$

Example: Simplify the following using the double angle formulas until the powers of sine and cosine are not greater than 1:

$$\sin^2(2x)\cos^2(2x)$$

Half Angle Formulas

As a consequence of the double angle formulas on the previous page, we can construct the ***half-angle formulas:***

$$\sin\left(\frac{1}{2}u\right)=\pm\sqrt{\frac{1}{2}\left(1-\cos(u)\right)} \qquad \cos\left(\frac{1}{2}u\right)=\pm\sqrt{\frac{1}{2}\left(1+\cos(u)\right)}$$

$$\tan\left(\frac{1}{2}u\right)=\frac{1-\cos(u)}{\sin(u)}=\frac{\sin(u)}{1+\cos(u)}$$

The +/- depends on the *quadrant* in which $\frac{1}{2}u$ lies.

Example: Find the exact value of

(a) $\cos\left(15^\circ\right)$

(b) $\csc\left(\frac{7\pi}{8}\right)$

Example: Find the exact value of the expression: $\cos^2\left[\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)\right]$

Double and Half Angle Formulas

Establish the identity: $\dfrac{\cot(\theta)-\tan(\theta)}{\cot(\theta)+\tan(\theta)}=\cos(2\theta)$

Example: If $x=8\tan(\theta)$, express $\sin(2\theta)$ as a function of *x*.