Newton's Law of Universal Gravitation and the Scale Principle

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Earlier this year I wrote a paper entitled Scale Factors and the Scale Principle. In that paper I formulated a new law which describes a number of fundamental quantum mechanical laws and part of Einstein's theory of relativity. The purpose of this article is to show that this theory also predicts Newton's law of universal gravitation. Thus this new formulation can be extended to classical mechanics.

Keywords*: Newton's law of universal gravitation, Coulomb's law, Planck charge, Planck length, Planck time, Planck mass, Planck acceleration, Planck force.*

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1. Introduction

In a previous article [1] I introduced the scale principle or scale law through the following mathematical relationship

(1)

Scale principle or scale law
$\left(\frac{Q_1}{Q_2}\right)^n \left[\le \; \middle| = \middle| \; \ge \right] S \left(\frac{Q_3}{Q_4}\right)^m$

Where

a) Q_1, Q_2, Q_3 and Q_4 are physical quantities of identical dimension (such as Length, Time, Mass, Temperature, etc), or

b) Q_1 and Q_2 are physical quantities of dimension 1 or dimensionless constants while Q_3 and Q_4 are physical quantities of dimension 2 or dimensionless constants. However, if Q_1 and Q_2 are dimensionless constants then Q_3 and Q_4 must have dimensions and viceversa.
The physical quantities can be variables, constants, dimensionless constants, differentials, derivatives (including Laplacians), integrals, vectors, any mathematical operation between the previous quantities, etc.

(e.g.: Q_1 and Q_2 could be quantities of Mass while Q_3 and Q_4 could be quantities of Length).

c) The relationship is one of three possibilities: **less than or equal to** inequation ($\leq$), or **equal to** - equation (=), or a **greater than or equal to** inequation ($\geq$).

d) *S* is a dimensionless scale factor. This factor could be a real number, a complex number, a real function or a complex function (strictly speaking real numbers are a particular case of complex numbers). The scale factor could have more than one value for the same relationship. In other words a scale factor can be a quantum number.

e) *n* and *m* are integers 0, 1, 2, 3,… (In general these two numbers are different. e.g. 1: *n* =1 and *m* = 1. e.g. 2: *n* = 1 and *m* = 2. *n* and *m* cannot be both zero in the same relationship). It is worthy to remark that so far these integers are not greater than 2, however this could change in the future.

2. The Planck Force

In this section I shall derive the Planck force from two known laws. In subsection 2.1 I start the analysis from the Coulomb's law while in subsection 2.1 the starting point is Newton's second law of motion. The results of these two independent derivations are identical as it should be. The Planck force is used in section 3 to derive the law of universal gravitation.

2.1 Derivation of the Planck Force based on Coulomb's Law

We shall derive the Planck force from Coulomb's law

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \text{(Coulomb's law)} \qquad (2)$$

We shall substitute both charges q_1 *and* q_2 with the Planck charge Q_P and the distance r with the Planck length L_P, this yields

$$F_P = \frac{Q_P^2}{L_P^2} \qquad (3)$$

Where

$$Q_P \equiv \sqrt{2\varepsilon_0 h c} \qquad \text{(Planck charge)} \qquad (4)$$

$$L_P \equiv \sqrt{\frac{hG}{2\pi\, c^3}} \qquad \text{(Planck length)} \qquad (5)$$

$$F_P = \frac{1}{4\pi\,\varepsilon_0}\left(\frac{2\varepsilon_0 h c}{\frac{hG}{2\pi\, c^3}}\right) \qquad (6)$$

Finally the Planck force is

$$F_P = \frac{c^4}{G} \qquad \text{(Planck Force)} \qquad (7)$$

2.2 Derivation of The Planck Force based on Newton's Second Law of Motion

We shall derive the Planck force from Newton's second law of motion

$$F = ma \qquad (8)$$

The Planck force can also be defined as

$$F_P \equiv M_P a_P \qquad (9)$$

Where

F_P = Planck force
M_P = Planck mass
a_P = Planck acceleration

The Planck mass is defined as

$$M_P \equiv \sqrt{\frac{hc}{2\pi\, G}} \qquad (10)$$

The Planck acceleration can be defined as

$$a_P \equiv \frac{c}{T_P} \qquad (11)$$

Where

c = speed of light in vacuum
T_P = Planck time

$$T_P \equiv \sqrt{\frac{hG}{2\pi\, c^5}} \qquad (12)$$

Substituting T_P in equation (11) by the second side of equation (12) yields

$$a_P = c\sqrt{\frac{2\pi\, c^5}{hG}} \qquad (13)$$

$$a_P = \sqrt{\frac{2\pi\, c^7}{hG}} \qquad (14)$$

Substituting M_P and a_P in equation (9) with the second side of equations (10) and (14) respectively yields

$$F_P = \sqrt{\frac{h\,c}{2\pi\, G}\frac{2\pi\, c^7}{hG}} \qquad (15)$$

Finally the expression for the Planck force is

$$F_P = \frac{c^4}{G} \qquad \text{(Planck Force)} \qquad (16)$$

3. Derivation of the Universal Law of Gravitation

In this section I shall derive Newton's law of universal gravitation from the scale principle. In 1687 Isaac Newton published his Principia where he stated his universal law of gravitation as follows

$$F_G = G\frac{m_1 m_2}{r^2} \qquad \text{(Newton's law of universal gravitation)} \qquad (17)$$

Where

F_G = Gravitational force between two any bodies of masses m_1 and m_2 (this force is also known as force of universal gravitation, gravity, gravity force, force of gravitational attraction, force of gravity, Newtonian force of gravity, force of universal mutual gravitation, etc.)
G = Gravitational constant (also known as constant of gravitation, constant of gravity,

gravitational force constant, universal constant of gravity, universal gravitational constant, Newtonian gravitational constant, etc.)

m_1 = mass of body 1

m_2 = mass of body 2

r = distance between the centers of body 1 and body 2

We start the analysis by observing that the numerator of equation (16) [or equation (7)] is c^4. This suggests that the equation we are looking for should originate from the product of two relativistic energies such as $E_1 \equiv m_1 c^2$ and $E_2 \equiv m_2 c^2$. We also notice that the c^4 factor of the product $m_1 m_2 c^4$ should cancel out with the numerator of equation (16). Thus, taking into consideration these facts, we draw the following scale table

Work	Work	Energy (relativistic)	Energy (relativistic)
W_G	W	E_1	E_2

TABLE 1: *This scale table is used to derive Newton's law of universal gravitation.*

In summary, the quantities shown in Table 1 must be defined as follows

$W_G \equiv F_G r$ (Work done by F_G) (18)

$W \equiv F_P r$ (Work done by F_P) (19)

$E_1 \equiv m_1 c^2$ (Relativistic energy of body 1) (20)

$E_2 \equiv m_2 c^2$ (Relativistic energy of body 2) (21)

From the table we establish the following relationship

$$W_G W = S E_1 E_2 \qquad (22)$$

Replacing the variables W_G, W, E_1 *and* E_2 by equations (18), (19), (20) and (21) respectively we get

$$F_G r F_P r = S m_1 c^2 m_2 c^2 \qquad (23)$$

$$F_G = S \frac{G}{c^4} \frac{1}{r^2} m_1 c^2 m_2 c^2 \qquad (24)$$

$$F_G = S\,G\frac{m_1 m_2}{r^2} \qquad (25)$$

If $S = 1$ we obtain the Newton's law of universal gravitation (see equation 17).

Thus we have proved that Newton's law of universal gravitation can be derived from the scale principle. It is worthy to remark that the gravitational constant G is not the scale factor as one might be tempted to think. Scale factors are dimensionless while the gravitational constant is not.

Now let us express equation (23) in the form of the scale principle

$$\frac{F_G\, r}{m_1 c^2} = S\,\frac{m_2 c^2}{F_P\, r} \qquad (26)$$

Comparing equation (26) with equation (1) we find that equation (26) has the following form

$$\frac{Q_1}{Q_2} = S\frac{Q_3}{Q_4} \qquad (27)$$

Where

$n = m = 1$
$Q_1 = F_G r$
$Q_2 = m_1 c^2$
$Q_3 = m_2 c^2$
$Q_4 = F_P r$

Thus we have proved that Newton's law of universal gravitation obeys the scale principle.

4. Conclusions

In this paper we have formulated a relativistic quantum mechanical derivation of the Newton's law of universal gravitation. The derivation is relativistic because we used the total relativistic energy $m_1 c^2$ and $m_2 c^2$ of the attracting bodies. At the same time the derivation is quantum mechanical because we introduced the Planck force (c^4/G). However the law we obtained is a classical law.

Taking into account that the scale law describes several known laws of physics as I have shown both on previous papers [1, 2, 3, 4, 5] and on this paper, we can consider the scale law as a more universal law than the specific laws it describes.

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