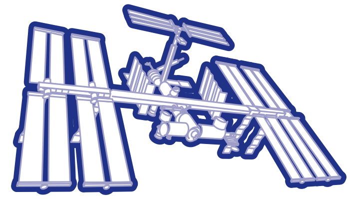
Educational Product	
Educators	Grades 5-12

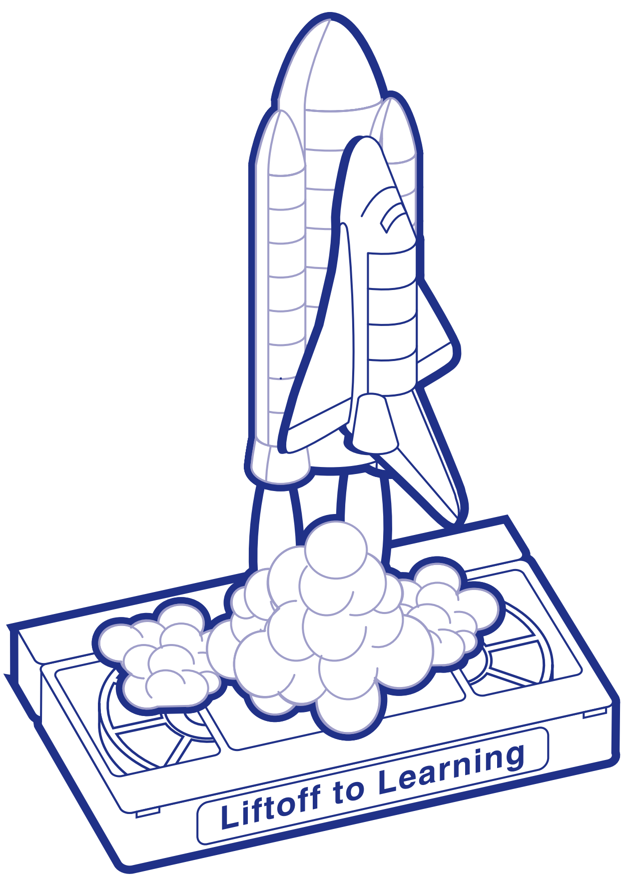
National Aeronautics and
Space Administration

Liftoff to Learning



Mathematics of Space - Rendezvous

A Videotape for Mathematics



Video Resource Guide

EV-1998-02-014-HQ

Video Synopsis

Title: The Mathematics of Space - Rendezvous

Length: 17 minutes

Subjects: The mathematics of spacecraft orbital rendezvous.

Description:
This program addresses the basic mathematical operations of spacecraft rendezvous in Earth orbit. Middle school students in a mathematics class work to solve some problems that permit the Space Shuttle to rendezvous and dock with the Russian Space Station Mir. The video has stopping points to permit viewers to work the problems as well.

Mathematics Standards:

Mathematics as Problem Solving
Mathematics as Communication
Mathematics as Reasoning
Computation and Estimation
Algebra
Geometry
Measurement

Background

In a few years, the International Space Station (ISS) will be ready for full-time occupancy by crews of astronauts. From their vantage point in space, astronauts will study Earth's environment and conduct a variety of scientific and technological experiments that will ultimately help to improve life on Earth.

One of the critical tasks to the construction and use of a space station is the ability to rendezvous in space. Consequently, the first phase of the International Space Station program is a series of Space Shuttle mission rendezvous and docking activities with the Russian Space Station Mir. By bringing crew and equipment to Mir, Shuttle astronauts have gained valuable practice in the maneuvers they will need to work with the ISS. These maneuvers are complex and docking is a delicate operation. Each maneuver employs extensive use of mathematics to achieve the objective.

The objective of this video is to invite students to work out some of the fundamental equations Shuttle crews use for rendezvous. The equations presented in the video are listed in a following section of this guide.

Some students may become confused with some of the mathematical operations shown. The video is designed with stopping places for you to work the mathematics with your students.

One rendezvous concept presented in the video may need some additional explanation. By firing its rocket engines to increase velocity, the Space Shuttle actually slows down. Conversely, firing engines to slow down causes the vehicle to speed up. These paradoxical statements are easier to understand when you remember that movement in space is a three-dimensional problem. Changes in velocity leads to changes in altitude.

On Earth, rendezvous between two automobiles is a relatively simple operation. Both drive at specific speeds to arrive at a specific location. When the automobiles arrive at the right place, they stop. In space, the rendezvous location is over a specific place on Earth but it is also at a specific altitude. When the two spacecraft arrive, they cannot stop. Doing so will cause them to fall back to Earth. Instead, their rendezvous is at a specific location and altitude (three dimensions), and at a specific time. Time is important when you consider that the spacecraft will be traveling at 5 or more kilometers per second. A mere 5-second error will cause the spacecraft to miss each other by 25 kilometers.

The nature of space rendezvous is also complicated by some basic physical laws. Altitude and velocity of a spacecraft are related. Spacecraft in low orbits travel very fast because the gravitational pull is strong. In higher orbits, spacecraft travel slower because the force of gravity is less. The force of gravity between two objects (Earth and the Shuttle) is determined by the following mathematical relationship that was first formulated by Isaac Newton and later modified by Henry Cavendish.

$$f = G \frac{m_1 m_2}{r^2}$$

G in the equation is the gravitational constant. The r in the equation is the distance between the center of Earth and the center of the Shuttle (not the altitude of the Shuttle over Earth's surface). As you can see, r has an inverse square relationship in the equation. That means that the closer the centers of the two bodies are to each other, the greater the force of attraction. It also means that increasing the distance between the centers decreases the attraction by an inverse square.

The difference in gravitational attraction with change in distance (orbital altitude) is where the speed-up/slow-down paradox comes in. You must travel faster in a lower orbit than a higher one to stay in orbit. If you want to go to a higher orbit, you must fire your rocket engines to accelerate. The acceleration causes your spacecraft to climb higher above Earth. As you climb higher, your velocity diminishes until you are traveling at the right velocity for the higher orbit. It is a slower velocity than you were traveling before the firing of the engines. In other words, you sped up so that you could slow down in a higher orbit.

The reverse is true if you want to go to a lower orbit. To descend, you fire rocket engines in the opposite direction you are traveling. This causes your spacecraft to slow. Earth's gravity pulls your spacecraft downward, and as you fall, your speed increases until you are at the right speed for the new altitude. Your speed is greater in the lower orbit. Thus, you slowed down to speed up.

Although somewhat complicated, this paradox helps to accomplish rendezvous. For example, when the two spacecraft are on opposite sides of Earth from each other, having one spacecraft in a lower orbit will enable it to close the distance. In the lower orbit, the spacecraft will not only travel faster than the higher spacecraft, but the orbit has a smaller circumference as well. After closing the distance, the lower spacecraft can begin maneuvers to adjust to the right altitude for the rendezvous. How long to fire rocket engines, when to do it, and in what directions is determined with mathematics.

Equations Used In the Program

Degrees longitude orbital ground track shifts eastward with each orbit.

$$\frac{x}{92 \text{ min}} = \frac{360^o}{24 \text{ hrs } (60 \text{ min/hr})} \qquad x = 23^o$$

Number of orbits so that Mir flies over Moscow.

Mir - Moscow = Longitude Distance

$$\begin{array}{r} 105^o \\ -\ 37 \\ \hline 68^o \end{array}$$

$$\frac{68^o}{23^o\text{/orbit}} = 2.9565 \text{ orbits}$$

or 3 orbits

Time for Mir's orbit to cross Moscow.

$$\begin{array}{r} 92 \text{ min/orbit} \\ \times\ 3 \text{ orbits} \\ \hline 276 \text{ min} \end{array} \quad \text{or} \quad \frac{276 \text{ min}}{60 \text{ min/hr}} = 4.6 \text{ hr}$$

Mir's orbital speed.

$$\frac{\text{Distance}}{\text{Time}} = \text{Speed}$$

Distance (circumference) Mir travels during one orbit. (The altitude is the distance from Earth's center to Mir.)

$$6{,}778 \text{ km} \times 2\pi = C_{\text{Mir}}$$

$$42.587 \text{ km} = C_{\text{Mir}}$$

Mir's orbital speed.

$$\frac{42{,}587 \text{ km}}{92 \text{ min}} = \text{speed}$$

$$463 \text{ km/min} = \text{speed}$$

or

$$\begin{array}{r} 63 \text{ km/min} \\ \times\ 60 \text{ min/hr} \\ \hline 27{,}780 \text{ km/hr} \end{array}$$

Shuttle speed change needed to raise orbit 7 kilometers. (It is stated in the video that a change in velocity of 0.4 meters per second raises the Shuttle 1 kilometer.)

$$\begin{array}{rr} \text{target altitude} & 305 \text{ km} \\ \text{present altitude} & -\ 298 \text{ km} \\ \hline & 7\text{km}/1\text{km} \end{array}$$

$$\frac{7 \text{ km}}{1 \text{ km}} \times 0.4 \text{ m/sec} = 2.8 \text{ m/sec}$$

Classroom Activities

How High?

Materials: Earth globe
Metric ruler

Objective: To learn why it is necessary to exaggerate altitudes when orbits are shown in model form.

Procedure:
Measure the diameter of the globe you are using for the model. Determine its scale. To do this, you will need to know the actual diameter of Earth (12,756 km). Using the same scale, determine how high above the globe's surface the Space Shuttle and Mir would be (400 km).

Discussion:
Diagrams of planets and spacecraft orbiting them are difficult to portray accurately. The diagrams of Earth, the Space Shuttle, and Mir Space Station used in the video greatly exaggerate the distance the orbiting spacecraft are above Earth. Without doing this, the orbits would lie so closely to the surface of Earth that the lines would be indistinguishable.

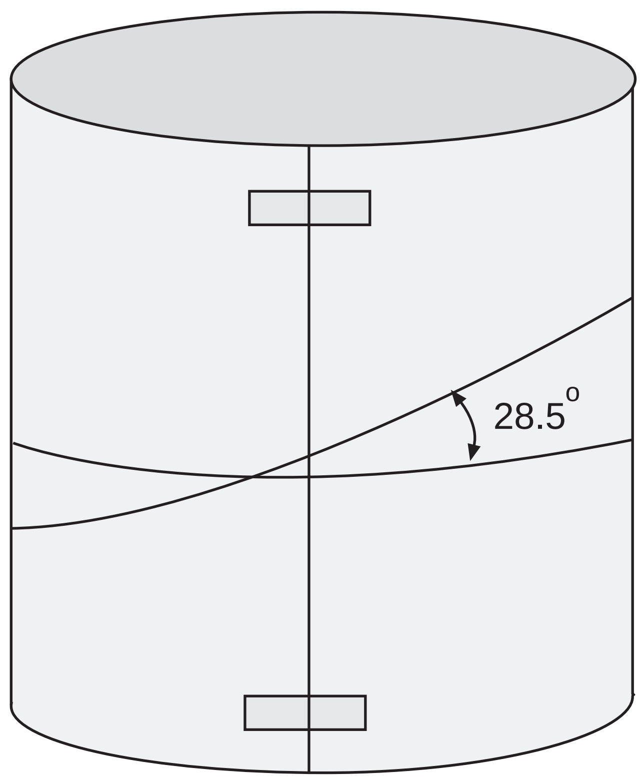
Extension:
Using the same scale, determine how far the Moon would be from Earth.

Sine Curve Orbits?

Materials: Earth globe
Paper
Tape
Marker
Scissors

Objective: To show why a sine curve orbital plot is created when an orbit is portrayed on a flat map.

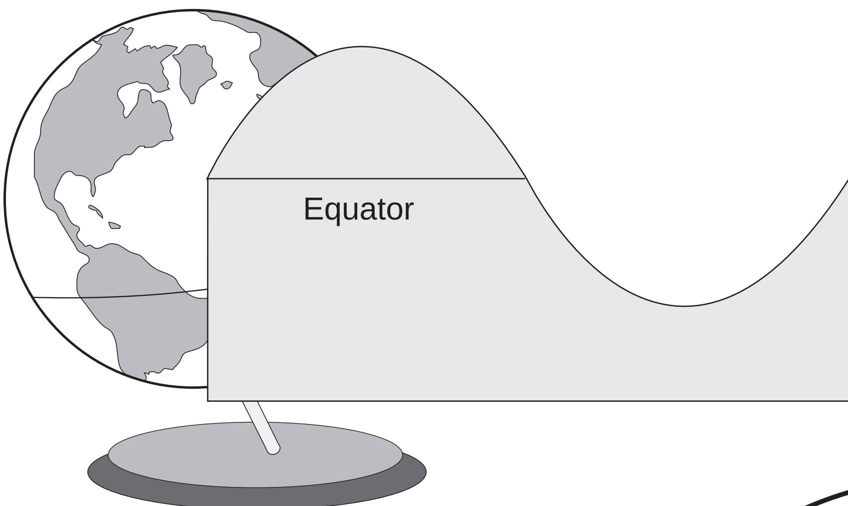
Procedure:
When shown on a flat map, Space Shuttle orbits resemble sine curves. To show why this happens, wrap and tape a cylinder of paper around an Earth globe. Use a marker pen to draw an orbit around the cylinder. Start with an orbit inclined 28 degrees. Draw the line around the cylinder so that it falls on a plane inclined to the globe's equator by 28 degrees. Remove the cylinder and cut the

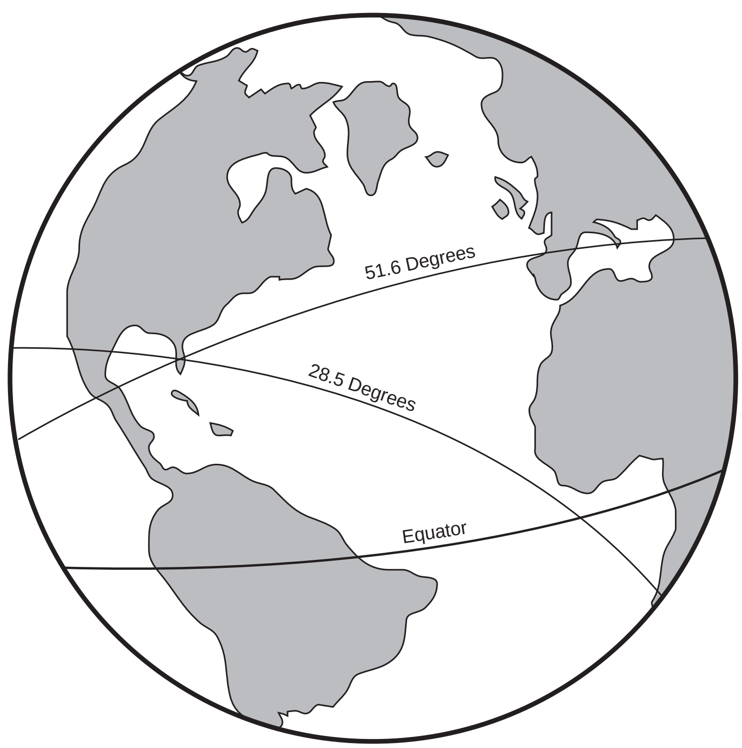


paper along the line you drew. If you drew the line carefully, the edge of the cut will fall on a plane. Unwrap the cylinder and look at the shape of the orbit.

Discussion:
Orbital maps displayed in Mission Control at the NASA Johnson Space Center show three Space Shuttle orbits at a time. A small Space Shuttle orbiter is displayed on one of the orbits over the geographic position the actual orbiter is flying. The curve of the orbits resembles a sine curve. The steepness of the curve is determined by the angle in which the Space Shuttle was launched in respect to Earth's equator. Many Shuttle orbits are inclined at 28.5 degrees. This is the geographic latitude of the Kennedy Space Center. When a Shuttle is launched due east, its orbit is inclined 28.5 degrees. This happens because an orbit must be concentric with the center of Earth. In geographic terms, the orbit must be a great circle.

Extension:
Create other cylinders for different orbits such as 35 degrees and 51.6 degrees (orbit of the International Space Station). Compare the steepness of the curves when the cylinders are flattened.





References

NASA ON-LINE RESOURCES FOR EDUCATORS

NASA On-line Resources for Educators provide current educational information and instructional resource materials to teachers, faculty, and students. A wide range of information is available, including science, mathematics, engineering, and technology education lesson plans, historical information related to the aeronautics and space program, current status reports on NASA projects, news releases, information on NASA educational programs, useful software and graphics files. Educators and students can also use NASA resources as learning tools to explore the Internet, access information about educational grants, interact with other schools, and participate in on-line interactive projects with NASA scientists, engineers, and other team members to experience the excitement of real NASA projects.

Access these resources through the NASA Education Home Page:
http://education.nasa.gov

Other web sites of interest:
http://www.jsc.nasa.gov
http://station.nasa.gov/core.html
http://www.osf.hq.nasa.gov/heds

STS-84 Crew Biographies

Commander: Charles J. Precourt (Col., USAF)
Charles Precourt was born in Waltham, Massachusetts, and grew up in Hudson, Massachusetts. He received a B.S. degree in aeronautical engineering from the USAF Academy, an M.S. degree in engineering management from Golden Gate University, and an M.A. degree in national security affairs and strategic studies from the U.S. Naval War College. He also studied as an exchange student at the French Air Force Academy. Precourt flew the F-15 while based at Bitburg Air Base in Germany. As a test pilot at Edwards Air Force Base, California, Precourt flew the F-T5E, F-4, A-7, and A-37 aircraft. His flight experience includes more than 5,500 hours in over 50 types of civil and military aircraft. Precourt was selected as an astronaut in 1990 and flew as a mission specialist on STS-55 in 1993 and as a pilot aboard STS-71 (the first Space Shuttle mission to dock with the Russian Mir Space Station and exchange crews) in 1995. He has spent nearly 20 days in space. As commander of STS-84, Precourt was in charge of the sixth Shuttle mission scheduled to rendezvous and dock with Mir.

Pilot: Eileen M, Collins (Lt. Col., USAF)
Eileen Collins was born in Elmira, New York. She received an A.A. degree in mathematics science from Corning Community College, a B.S. degree in mathematics economics from Syracuse University, an M.S. degree in operations research from Stanford University, and an M.A. degree in space systems management from Webster University. Collins was one of the first women to graduate from pilot training at Vance Air Force Base (AFB), Oklahoma, and later became the first woman T-38 instructor pilot at that base. She served as a C-141 pilot, aircraft commander, and instructor pilot at Travis AFB, California. In 1983, she flew in Operation Urgent Fury in Grenada. She was later assigned to the USAF Academy where she was an assistant professor in mathematics and a T-41 instructor pilot. In 1990, she graduated from the AF Test Pilot School at Edwards AFB, California, where she was the class leader. Collins has logged over 4,000 hours in 30 different types of aircraft and was selected as an astronaut in 1990. Aboard STS-63, she spent more than 8 days in space, and became the first woman pilot of a Space Shuttle. STS-84 was her second Shuttle mission.

Payload Commander, Mission Specialist: Jean-Francois Clervoy (Ingenieur en Chef de l Armement, ESA Astronaut)
Jean-Francois Clervoy was born in Longeville-les Metz, France. He graduated from Ecole Polytechnique, Paris, in 1981; from Ecole Supérieure de l' Aéonautiqe et de l' Espace, Toulouse, in 1983; and from Ecole du Personnel Navigant d Essais et de Reception,lstres, as a flight test engineer, in 1987. Seconded from the Délégation Générale pour L' Armement to the French Space Agency, Clervoy worked on various satellite projects when he was selected in the second group of French astronauts. In 1985, Clervoy was the Chief Test Director of the Parabolic Flight Program and provided support to the European Manned Space Programs Hermes and Columbus until he was selected as a European Space Agency (ESA) astronaut and reported to NASA Johnson Space Center for astronaut training in 1992. Clervoy has also received training on the Russian Space Systems Soyuz and Mir at Star City, Moscow. He served as a mission specialist aboard STS-66 during which he logged more than 10 days in space. STS-84 was Clervoy's second Space Shuttle mission.

Mission Specialist: Edward T. Lu (Ph.D)
Edward Lu was born in Springfield, Massachusetts, raised in Webster, New York and most recently, resided in Honolulu, Hawaii. He received a B.S. degree in electrical engineering from Cornell University and a Ph.D. in applied physics from Stanford University.

Since receiving his Ph D., Dr. Lu has been a research physicist working in the fields of solar physics and astrophysics. He was a visiting scientist at the High Altitude Observatory in Boulder, Colorado, and, at one point, simultaneously worked with the Joint Institute for Laboratory Astrophysics at the University of Colorado. He was a postdoctoral fellow at the Institute for Astronomy in Honolulu and has developed a number of new theoretical advances which have provided, for the first time a basic understanding of the underlying physics of solar flares. Dr. Lu has published articles on a wide range of topics including solar flares, cosmology, solar oscillations, statistical mechanics, and plasma physics. He has given numerous invited lectures at various universities and international conferences. He also holds a commercial pilot certificate with instrument and multi-engine ratings. Dr. Lu was selected as a NASA astronaut in 1994 and has worked in the Computer Support Branch of the Astronaut Office. STS-84 was Dr. Lu's first Space Shuttle mission.

Mission Specialist: Carlos I. Noriega (Major, USMC) Carlos Noriega was born in Lima, Peru, and raised in Santa Clara, California. He received a B.S. degree in computer science from the University of Southern California. After flight school he flew CH-46 Sea Knight helicopters. In addition to two 6-month shipboard deployments which included operations in support of the Multi-National Peacekeeping Force in Beirut, Lebanon, he served as an aviation safety officer and instructor pilot. He has logged approximately 2,000 flight hours in various fixed wing and rotary wing aircraft. From the Naval Postgraduate School, he received two masters degrees, one in computer science and the other in space systems operations. Upon graduation, he was assigned to the United States Space Command in Colorado Springs. In addition to serving as a Space Surveillance Center Commander, he was command representative for the development and integration of the major space and missile warning computer system upgrades for Cheyenne Mountain Air Force Base. Noriega was selected as an astronaut in 1994 and has worked technical issues concerning extravehicular activity (spacewalks). He served as flight engineer (mission specialist) on STS-84, his first Space Shuttle flight.

Mission Specialist: Elena V. Kondakova
Elena Kondakova was born in Mitischi, Moscow Region. She graduated from the Moscow Bauman Technical Institute. Upon graduation, Kondakova started work in RSC-Energia, completing science projects, experiments, and research work. In 1989, she was selected as a cosmonaut candidate by the RSC-Energia Main Design Bureau and sent to the Gagarin Cosmonaut Training Center to start the course of general space training. Upon course completion, Kondakova was qualified as a flight engineer. During 1994 she was in training for the seventeenth main Mir mission known as the EuroMir-94 flight. She completed her first flight on board the spacecraft Soyuz TM-17 and the orbital complex Mir as a flight engineer in March 1995. Kondakova has logged approximately 169 days in orbit. She served as a mission specialist on exchange from the Russian Space Agency during STS-84, her first Space Shuttle mission.

Mission Specialist, NASA-Mir 4: Jerry M. Linenger (Capt., USN)
Jerry Linenger was born and raised near Eastpointe, Michigan. He earned a B.S. degree from the U S. Naval Academy, an M.D. from Wayne State University, an M.S. in systems management from the University of Southern California, and both an M.P.H. in health administration and a Ph.D in epidemiology from the University of North Carolina. He completed surgical internship, aerospace medicine, and preventive medicine programs. Linenger served as a naval flight surgeon at Cuba Point, Republic of the Philippines. He was then assigned as medical advisor to the Commander, Naval Air Forces, U.S. Pacific Fleet, San Diego. He later became a research principal investigator at the Naval Health Research Center and a faculty member at the University of California-San Diego School of Medicine in the Division of Sports Medicine. Dr. Linenger was selected to be an astronaut in 1992 and flew aboard STS-64, during which he spent 11 days in space. Following training in Star City, he traveled to the Mir Space Station aboard STS-81. Linenger returned with the STS-84 crew aboard the Space Shuttle after spending approximately 130 days aboard the Mir Space Station.

Mission Specialist, NASA-Mir 5: C. Michael Foale (Ph.D.)
Michael Foale was born in Louth and raised in Cambridge, England. He attended the University of Cambridge Queens' College, where he earned a B.A. degree in physics, National Sciences Tripos, with first class honors. While at Queens' College he completed a Ph.D. in laboratory astrophysics. As a postgraduate at Cambridge University, Dr. Foale participated in the organization and execution of scientific scuba diving projects including surveying underwater antiquities in Greece. Dr. Foale joined NASA Johnson Space Center in 1983 in the payload operations areas of the Mission Operations Directorate. He was selected as an astronaut in 1987 and flew as a mission specialist on STS-45, the first of the ATLAS series of missions to address the atmosphere and its interaction with the sun, and again as a mission specialist on STS-56, carrying ATLAS-2. He served as a mission specialist

on STS-63 (February 2-11, 1995), the first rendezvous with the Russian Space Station Mir. During the flight he made a spacewalk (extravehicular activity) for 4 hours, 39 minutes, evaluating the effects of extremely cold conditions on his spacesuit as well as moving the 2,800-pound Spartan satellite as part of a mass-handling experiment. He launched to Mir on STS-84. After spending 4 months aboard Mir as a part of the Mir-24 crew, he returned to Earth with the crew of STS-86.

NASA Liftoff to Learning
Mathematics of Space - Rendezvous

EDUCATOR REPLY CARD
Video Resource Guide

To achieve America's goals in Educational Excellence, it is NASA's mission to develop supplementary instructional materials and curricula in science, mathematics, geography, and technology. NASA seeks to involve the educational community in the development and improvement of these materials. Your evaluation and suggestions are vital to continually improving NASA educational materials.

Please take a moment to respond to the statements and questions below. You can submit your response through the Internet or by mail. Send your reply to the following Internet address:

http://ehb2.gsfc.nasa.gov/edcats/educational_videotape

You will then be asked to enter your data at the appropriate prompt.

Otherwise, please return the reply card by mail. Thank you.

1. With what grades did you use the video and video resource guide?
   Number of Teachers/Faculty:
   ______ K-4 ______ 5-8 ______ 9-12 ______ Community College
   College/University - ______ Undergraduate ______ Graduate

   Number of Students:
   ______ K-4 ______ 5-8 ______ 9-12 ______ Community College
   College/University - ______ Undergraduate ______ Graduate

   Number of Others:
   ______ Administrators/Staff ______ Parents ______ Professional Groups
   ______ General Public ______ Civic Groups ______ Other

2. What is your home 5- or 9-digit zip code? __ __ __ __ __ — __ __ __ __

3. This is a valuable video and video resource guide.
   ❏ Strongly Agree ❏ Agree ❏ Neutral ❏ Disagree ❏ Strongly Disagree

4. I expect to apply what I learned in this video and video resource guide.
   ❏ Strongly Agree ❏ Agree ❏ Neutral ❏ Disagree ❏ Strongly Disagree

Fold along line and tape closed.

5. What kind of recommendation would you make to someone who asks about this video and video resource guide?
   ❏ Excellent ❏ Good ❏ Average ❏ Poor ❏ Very Poor

6. How did you use this video and video resource guide?
   ❏ Background Information ❏ Critical Thinking Tasks
   ❏ Demonstrate NASA Materials ❏ Demonstration
   ❏ Group Discussions ❏ Hands-On Activities
   ❏ Integration Into Existing Curricula ❏ Interdisciplinary Activity
   ❏ Lecture ❏ Science and Mathematics Standards Integration
   ❏ Team Activities
   ❏ Other: Please specify: ____________________

7. Where did you learn about this video and video resource guide?
   ❏ NASA Educator Resource Center
   ❏ NASA Central Operation of Resources for Educators (CORE)
   ❏ Institution/School System
   ❏ Fellow Educator
   ❏ Workshop/Conference
   ❏ Other: Please specify: ____________________

8. What features of this video and video resource guide did you find particularly helpful?
   ____
   ____

9. How can we make this video and video resource guide more effective for you?
   ____
   ____

10. Additional comments:
   ____
   ____

Today's Date: ____________ **EV-1998-02-014-HQ**

Please Place
Stamp Here
Post Office
Will Not Deliver
Without Proper
Postage

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
EDUCATION DIVISION
MAIL CODE FE
WASHINGTON DC 20546–0001

Fold along line and tape closed.