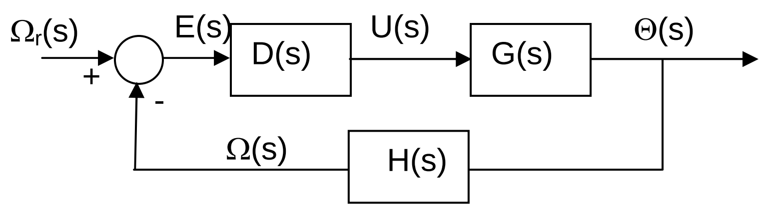
Motor Velocity Control Experiment

Part A: Speed Control

In-Class Component

Goal: Investigate P, PI, and PID speed control for a DC motor. Determine effect on relative stability and steady-state accuracy as the gains as varied.

Background:
The analog version of the block diagram that should be implemented is shown below:



where

- G(s) is the DC motor
- D(s) is the controller
- H(s) is the sensor
- Θ(s) is the angle, θ(t) is in degrees
- Ω(s) is the speed, ω(t) is in deg/sec
- $\Omega_r(s)$ is the reference speed, $\omega_r(t)$ is in deg/sec
  - -the reference is chosen to be a square wave between 100 and 200 deg/sec
- $E(s) = \Omega_r(s) - \Omega(s)$ is the error signal

There is no tachometer to measure speed, so the angle sensor is used to obtain the motor shaft angle θ(t). This must be differentiated to obtain ω(t). Since digital approximation to differentiation creates a very noisy signal, an ideal differentiator H(s) = s is replaced with a filtered differentiator $H(s) = \frac{as}{s+a}$. In this case, a is chosen as 30.

The control can be chosen as a P, PI, or PID controller. The PID controller has the form:

$$D(s) = K_P + \frac{K_I}{s} + K_D s$$

The PID controller is implemented digitally as:

$$D(z) = \frac{U(z)}{E(z)} = K_P + \frac{K_I T z}{z-1} + \frac{K_D (z-1)}{Tz}$$

where T is the sampling period. This expression is easiest to implement using parallel blocks where

$$U(z) = K_P E(z) + \frac{K_I T z}{z-1} E(z) + \frac{K_D (z-1)}{Tz} E(z)$$

In the time domain, this corresponds to the following sum of terms:

$$u[n] = u_p[n] + u_i[n] + u_d[n]$$

The digital version of H(s) is obtained using a bilinear transformation.

Ziegler-Nichols Tuning Rules:

Slowly increase the gain K_P until there are sustained oscillations. This value of gain is termed K_u and the period of oscillations is termed P_u. The tuning rules are:

	K_P	**T_I**	**T_D**
P	$0.5K_u$		
PI	$0.45K_u$	$P_u/1.2$	
PID	$0.6K_u$	$P_u/2$	$P_u/8$

$K_I = K_P/T_I$ and $K_D = K_P T_D$

Experiment:

1) You will implement the controllers using LabVIEW. Start with a Proportional controller, $D(z) = K_P$. Set the gains as $K_P = 0.1$, and $K_I = K_D = 0$.

2) Examine the speed response of the motor to the reference input with $K_P = 0.1$.

   Observations: average steady-state error___________

3) Increase the gain K_P until you reach sustained oscillations. Note the signal is naturally noisy due to the errors in the differentiation approximation, but the sustained oscillations are at a lower frequency. They first appear as an overshoot, then the system's damping ratio becomes smaller as the gain is increased. Record the average steady-state error when the oscillations appear to be sustained K_P.

   K_u = _______________, P_u = ___________________

   Observations: average steady-state error___________

   Summarize what has happened to the response as K_P was increased, in terms of steady-state accuracy and relative stability.

4) Determine the gains using the Ziegler-Nichols Tuning Rules:

	K_P	T_I	T_D	K_I	K_D
P					
PI					
PID					

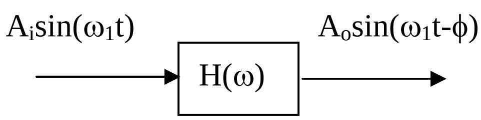
5) Summarize your observations for each control, P, PI, and PID, with the Ziegler-Nichols gains. Compare them in terms of steady-state error, rise time, and noise as well as maximum control effort required.

6) Tune the gains further by modifying them in an effort to improve the performance. What seemed to work the best and why?

Out-of-Class Component:

This portion of the lab will ask you to verify the tracking accuracy of the closed loop PID control that you designed and also to plot the frequency responses of the open loop and closed loop systems.

Background:
The steady-state output of a linear system, H(s), to a sinusoidal input is a sinusoid with the same frequency:



Where $H(\omega) = H(s)\big|_{s=j\omega}$

The output amplitude, A_o, and the output phase, ϕ, are found from $H(\omega)$ as follows:

$$A_o = A_i\left|H(\omega_1)\right| \qquad \text{and} \qquad \phi = \angle H(\omega_1)$$

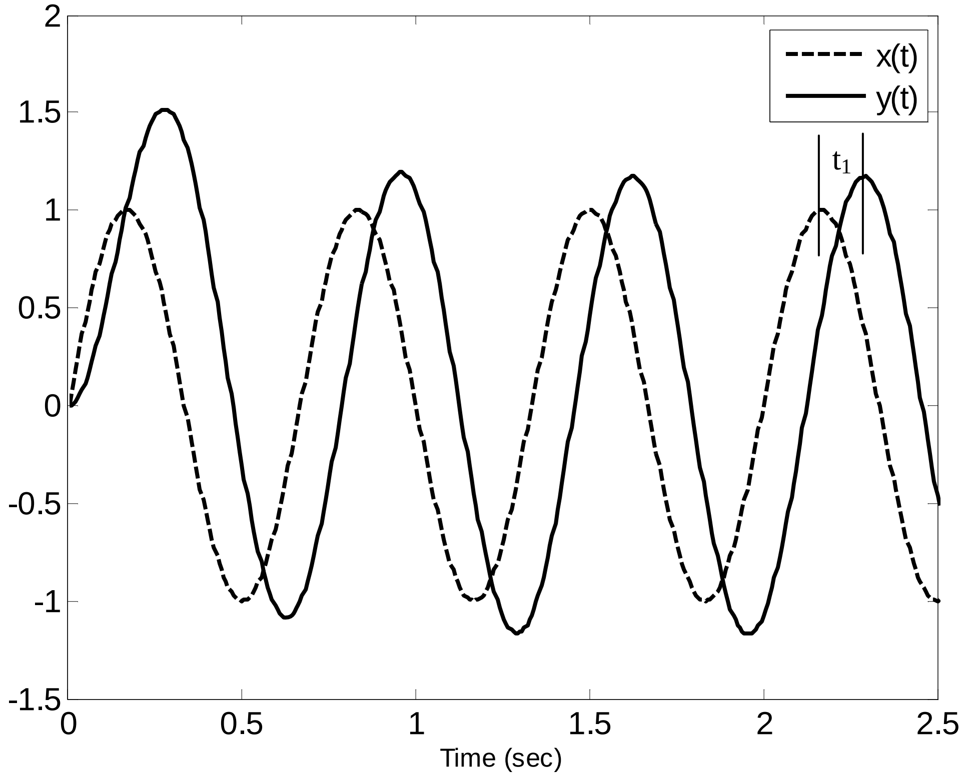
To determine $H(\omega)$, find the sinusoidal response to a wide range of input frequencies. From each sinusoidal response, measure A_o and ϕ. Fill in a table such as the one below for each input frequency.

ω_1	$\phi=\angle H(\omega_1)$	$\lvert H(\omega_1)\rvert=A_o/A_i$	$20\log\lvert H(\omega_1)\rvert$

Consider, for example, the response, y(t), of a system with input $x(t)=\sin(3\pi t)$ shown below. For this input, $\omega_1=3\pi$ and $A_i = 1$. Measure A_o and t_1, the time lag, in steady-state. The phase is calculated as

$$\phi = -\frac{t_1}{T} * 360^o$$

where T is the period of the signal.



From the plot, $t_1 = 0.12s$ and $A_o = 1.17$. $|H(\omega_1\}| = 1.17$ and $\phi = -0.12(360)/0.67 = -64.7^o$.

ω_1	$\phi=\angle H(\omega_1)$	$\|H(\omega_1)\|=A_o/A_i$	$20\log\|H(\omega_1)\|$
3π	-64.7^o	1.17	1.36dB

Select a wide range of values of input frequencies that fully characterize the frequency response. If sampling is being used in the system being measured, make sure that the input frequency selected does not violate the sampling theorem.

Once the data is collected, draw the frequency response of the system. You can determine the system transfer function from the Bode plot.

Experiment:

1) Experimentally obtain the data to plot the frequency response for the open loop system, $G(\omega)H(\omega) = \dfrac{\Omega(\omega)}{U(\omega)}$, and the closed loop systems with the final PI and PID controllers,

$G_{cl}(\omega) = \dfrac{\Omega(\omega)}{\Omega_r(\omega)}$. Give the sinusoidal input an offset so that the velocity never changes direction. Your response will also have an offset in it; *do not include this offset* in your measurements. Only include the amplitude of the sinusoidal part of the response. Note that the output signal is noisy due to measurement errors. Use the average values of the output signal to determine the amplitude and phase.

> Note: The offset in the input signal is used because the motor has a nonlinearity, called a dead zone, when the velocity goes through 0. The dead zone arises due to friction in the motor assembly. The motor will not turn when the expected response is too low in magnitude, typically at high frequency inputs. The goal in this experiment is to model the linear behavior of the motor, so it is necessary to avoid this nonlinear deadzone region.

2) Plot the frequency response of both the open loop system, $G(\omega)H(\omega)=\frac{\Omega(\omega)}{U(\omega)}$, and the closed loop systems with the final PI and PID controllers, $G_{cl}(\omega)=\frac{\Omega(\omega)}{\Omega_r(\omega)}$ using linear scales, for example, $|G(\omega)H(\omega)|$ vs ω and $\angle G(\omega)H(\omega)$ vs ω. Find the bandwidth of the open and closed loop systems. The bandwidth is defined as the frequency for which the response is 0.707 of the DC value (on a Bode plot, this is the same as being 3dB down from the DC value).

3) For each of the PI and PID controllers, record the closed loop responses for a sinusoid input reference velocity that is at a frequency that is half of the bandwidth. Plot the corresponding reference velocity and actual velocity. Repeat this procedure for an input reference signal that is twice the bandwidth.

4) Plot the Bode plot of the open loop system and identify the open loop transfer function $G(s)H(s)$.