KENDERDINE MATHS TUTORING

BLOOD ALCOHOL CONCENTRATION - HSC GENERAL MATHEMATICS

Prepared by Dr Richard Kenderdine [1]

Abstract
Widmark's Formula is used to estimate Blood Alcohol Concentration (BAC). The recently revised HSC General Mathematics 2 Preliminary Course includes a formula for BAC that differs from the format used elsewhere. This note examines the differences.

General Mathematics formulae
The formulae for Blood Alcohol Concentration (BAC) given in the HSC General Mathematics 2 course is

$$BAC = \begin{cases} \frac{10N-7.5H}{6.8M} & \text{males} \\ \\ \frac{10N-7.5H}{5.5M} & \text{females} \end{cases} \tag{1}$$

where N= number of standard drinks consumed, H = hours since commencement of drinking and M= weight in kg.

Textbooks then state that the time taken for the BAC to reduce to 0 is calculated by dividing BAC by 0.015 ie

$$Time\ to\ zero = \frac{BAC}{0.015} \tag{2}$$

I have not been successful in locating formulae (1) except in material prepared by the Board of Studies.

Examples from Internet search

The form of Widmark's equation in Gullberg is:

$$N = \frac{Wr[C_t - \beta t]}{dZ} \tag{3}$$

where:
N = *number of drinks*
W = *body weight (ounces)*
r = *volume of distribution (L/Kg)*
C_t = *blood alcohol concentration at time t (Kg/L)*
β = *alcohol elimination rate (Kg/L/hr)*
t = *time since drinking began (hours)*
d = *density of alcohol (0.82 oz./fl.oz.)*
Z = *the fluid ounces of ethyl alcohol per drink*

Rearranging (3) to solve for BAC as a function of the number of drinks yields:

$$C_t = \frac{NdZ}{Wr} - \beta t \tag{4}$$

Gullberg states *'formulae (3) and (4) represent the most common forms applied in forensic toxicology today.'*

Keates states the basic Widmark formula as :

$$\%BAC = \frac{5.14A}{Wr} - 0.015t \tag{5}$$

[1] 6 Braeside Drive, BOWRAL NSW 2576
www.kenderdinemathstutoring.com.au email: rdkdine@acenet.com.au

where A is the total number of ounces of alcohol ingested by the individual starting at the first drink, W stands for the weight, in pounds, of the individual, H is the period of time, in hours, that alcohol was eliminated from the body and r is the distribution ratio for alcohol through the body. For men, r stands for 0.73, while for woman the value is 0.66.
The value 5.14 is a mathematical conversion factor (.823x100/16) used to convert liquid ounces to regular weighted ounces, with the 100 used to convert the final value to a percentage, and lastly, 16 is used to convert lbs to ounces.

Note: the above two extracts originate in the USA and use ounces as the measure.

MATH1050 uses the form

$$BAC = \frac{A}{rW} \times 100 - Vt \tag{6}$$

where BAC is a percent at any time t in hours since commencing drinking, A is the amount of alcohol consumed in grams (a standard drink contains 10 grams of alcohol), V is the rate at which the body eliminates alcohol measured in % per hour (let $V = 0.015\%$ per hour), W is the body mass in grams and r is the Widmark factor. The Widmark factor estimates the proportion of body mass that is water. The precise value of r depends on factors such as gender, age and percentage body fat, but reasonable estimates are $r \approx 0.7$ for males and $r \approx 0.6$ for females.

Comparing Formulae (1) and (6)

The variable A in (6) represents the quantity of alcohol consumed which equals $10N$ in (1) when each of N drinks contains 10g of alcohol (a 'standard drink')

The value of $\frac{100}{rW}$ where $r = 0.7$ and W is the weight in grams is almost equivalent to $\frac{1}{6.8M}$ where M is in kg.

Thus far we have agreement. To achieve complete agreement requires

$$\frac{7.5}{6.8M} = 0.015 \tag{7}$$

This occurs when $M = 73.53kg$, presumably an average weight for a male.

There are a couple of points:

1. Formula (1) is not in the same format that appears to be adopted elsewhere. It would be preferred that formats are consistent; formulae (4), (5) and (6) are consistent apart from differences arising from units of measurement or whether the number of standard drinks or amount of alcohol is included.

2. Formulae (5) and (6) are consistent with (2) as they both use the elimination rate 0.015. Formula (4) is consistent with (2) in the sense that it explicitly shows the elimination rate (β). The use of 0.015 in (2) does not appear to have any connection to (1)

3. Formula (1) seems to indicate that BAC is accurately determined from the number of drinks, hours since drinking commenced and the person's weight. In reality, BAC is affected by other factors in addition to variability in r and the alcohol elimination rate. It should be clearly stated that (1) is only an estimate of BAC - one textbook discusses the uncertainty in the calculation only after presenting the formula and examples.

Conclusion
Formula (6) appears to be the best way of presenting Widmark's formula and is consistent with the calculation required to determine the time to achieve zero BAC after drinking alcohol.

References
GULLBERG, Rod G, *Estimating the Uncertainty Associated With Widmarks Equation As Commonly Applied in Forensic Toxicology.* Accessed 20 May 2014 from http://www.icadts2007.org/print/108widmarksequation.pdf
KEATES, Robert L, *Explaining Blood Alcohol Levels (BAC) (What does the* .08% *BAC mean? How many drinks is that?).*Accessed 20 May 2014 from http://www.avvo.com/legal-guides/ugc/explaining-blood-alcohol-levels-bac-what-does-the-08-bac-mean-how-many-drinks-is-that-1
MATH1050, Accessed 20 May 2014 from http://www.maths.uq.edu.au/courses/MATH1050/2011/investigation2011.pdf