Section 1.1, selected answers
Math 114 Discrete Mathematics

D Joyce, Spring 2018

2. Which of the following are propositions? What are the truth values of those that are propositions?

a. Do not pass go. Not a proposition; it's a command.

b. What time is it? Not a proposition; it's a question.

c. There are no black flies in Maine. It's a proposition. Right now, in January, it's probably false. You might say it's not a proposition if you don't consider the time to be fixed.

d. $4+x = 5$. Not a proposition since there's a free variable x in it.

e. The moon is made of green cheeese. A false proposition

f. $2^n \geq 100$. Not a proposition since there's a free varibile n in it.

4. Let p and q be the propositions "I bought a lottery ticket this week," and "I won the million dollar jackpot on Friday," respectively. Express each of the following propositions as an English sentence.

There are many ways each one can be expressed. Your answers may differ from the ones here.

a. $\neg p$. I didn't buy a lottery ticket this week.

b. $p \vee q$. I bought a lottery ticket this week, or I won the million dollar jackpot on Friday.

c. $p \to q$. If I bought a lottery ticket this week, then I won the million dollar jackpot on Friday.

d. $p \wedge q$. I bought a lottery ticket this week, and I won the million dollar jackpot on Friday.

e. $p \leftrightarrow q$. I bought a lottery ticket this week if and only if I won the million dollar jackpot on Friday.

f. $\neg p \to \neg q$. If I didn't buy a lottery ticket this week, then I didn't win the million dollar jackpot on Friday.

g. $\neg p \wedge \neg q$. I didn't buy a lottery ticket this week, and I didn't win the million dollar jackpot on Friday.

h. $\neg p \vee (p \wedge q)$. Either I didn't buy a lottery ticket this week, or I did and I won the million dollar jackpot on Friday.

10. Let p be the proposition "you get an A on the final exam," let q be the proposition "you do every exercise in this book," and let r be the proposition "you get an A in this class." Write these propositions using p, q, and r and logical connectives.

a. You get an A in this class, but you do not do every exercise in this book. $r \wedge \neg q$.

b. You get an A on the final, you do every exercise in this book, and you get an A in this class. $p \wedge q \wedge r$.

c. To get an A in this class, it is necessary for you to get an A on the final. $r \to p$.

d. You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class. $p \wedge \neg q \wedge r$.

e. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class. $p \wedge q \to r$.

f. You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final. $r \longleftrightarrow q \vee p$.

16. Which of these are inclusive ORs and which are exclusive?

a. Experience with C++ or Java is required. Inclusive, since if you know both, that's okay.

b. Lunch includes soup or salid. Are you going to get both? Hardly. This is an exclusive OR.

c. To enter the country you need a passport or a voter registration card. Inclusive, since it's okay to have both.

d. Publish or perish. This means if you don't publish research, you'll lose your job. But it's possible to publish and still perish. Inclusive.

32. Construct a truth table for the compound propositions.

a. $(p \vee q) \vee r$.

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

c. $(p \wedge q) \vee r$.

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

e. $(p \vee q) \wedge \neg r$.

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \wedge \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

54. What Boolean search would you use to look for web pages about hiking in West Virginia? What if you wanted

to find web pages about hiking in Virginia, but not in West Virginia.

First, hiking AND West AND Virginia. Second, hiking AND Virginia AND NOT West. With most search engines, you would simply write

hiking West Virginia
hiking West -Virginia

Furthermore, most search engines allow strings to be specified, like "West Virginia" which require the word West to immediately precede Virginia. With those, the first answer could be: hiking AND "West Virginia", and the second: hiking AND Virginia AND NOT "West Virginia". Note that BUT NOT is the same as AND NOT.

62. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known?

1. Either Kevin or Heather, or both, are chatting.
2. Either Randy or Vijay, but not both, are chatting.
3. If Abbey is chatting, so is Randy.
4. Vijay and Keven are either both chatting or neither is.
5. If Heather is chatting, then so are Abby and Kevin.

Explain your reasoning.

It's easiest to use symbolic notation. Then we have the following five propostions.

1. $K \vee H$.
2. $R \oplus V$.
3. $A \to R$.
4. $V \leftrightarrow K$.
5. $H \to A \wedge K$.

There are many ways to proceed. Here's one. Note that 1 and 5 both involve K and H. 1 says either K or H, but 5 says that if H, then also $A \wedge K$, so K in that case, too. Thus, H occurs whenever K occurs, so in all cases K. Now 1 is satisfied.

So we now know K. By 4, therefore also V, and 4 is now satisfied. By 2, therefore $\neg R$, and 2 is satisfied. Then by 3, therefore $\neg A$, and 3 is satisfied. But by 5, since $\neg A$, therefore $\neg H$, and 5 is satisfied.

By now, we've satisfied all five statements in the only way possible, namely, with K, V, but not R, not A, and not H.

Thus, the answer to the question is: we can conclude that Kevin and Vijay are chatting, but the other three aren't.

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