Faraday's Law: An Application of the Derivative

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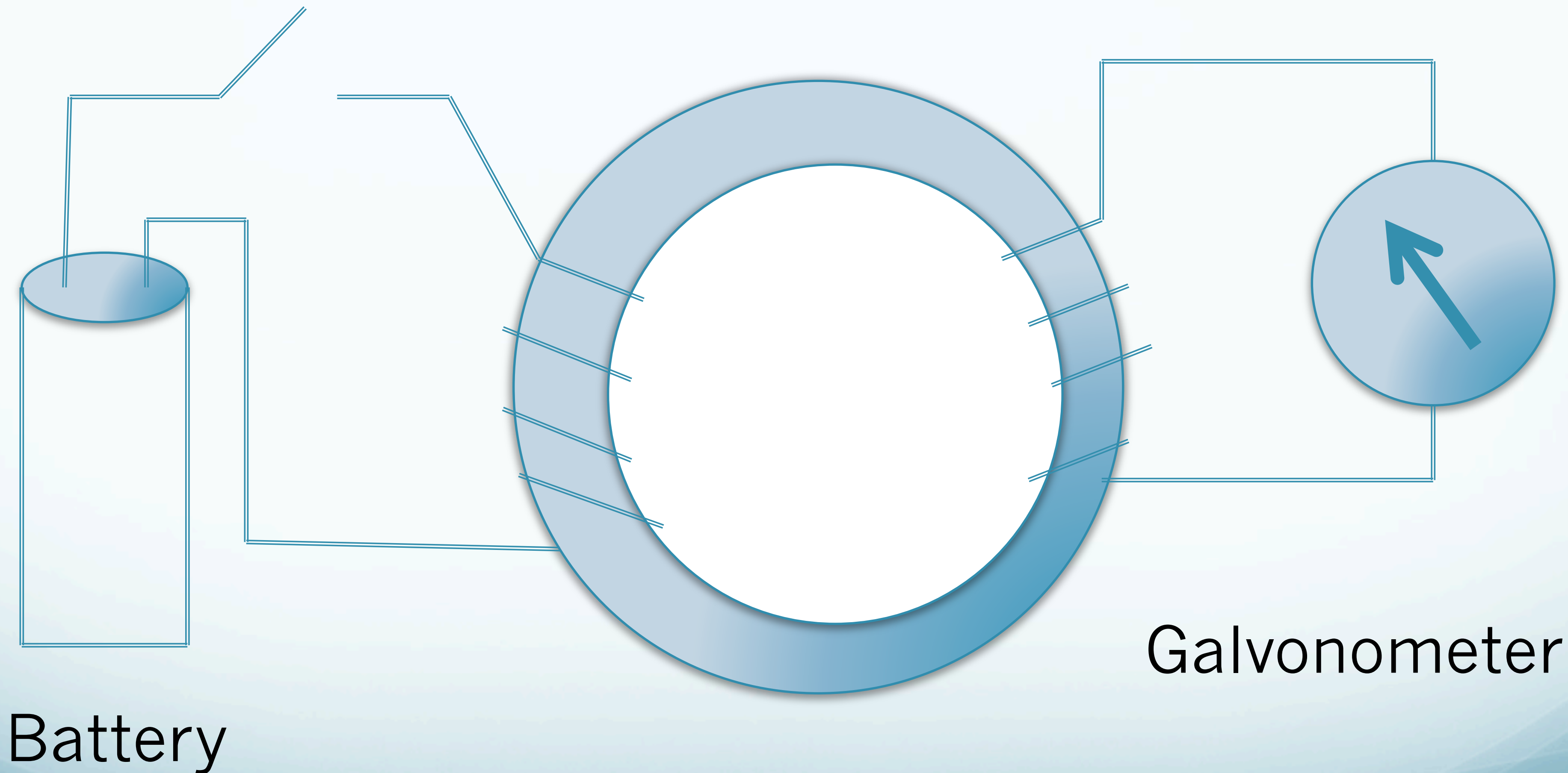
Introduction

- Faraday's law of induction is a basic law of electromagnetism predicting how a magnetic field will interact with an electric current to produce an electromotive force (EMF) – a phenomenon called electromagnetic induction.
- It is a fundamental operating principle of transformers, inductors and many types of electric motors, generators and solenoids.

Faraday's Experiment

- Electromagnetic induction was discovered independently by Michael Faraday and Joseph Henry in 1831; however, Faraday was the first to publish the results of his experiments.[4][5]
- In Faraday's first experimental demonstration of electromagnetic induction (August 29, 1831[6]), he wrapped two wires around opposite sides of an iron ring or "torus" (an arrangement similar to a modern toroidal transformer).
- Based on his assessment of recently discovered properties of electromagnets, he expected that when current started to flow in one wire, a sort of wave would travel through the ring and cause some electrical effect on the opposite side.

Faraday's Iron Ring Apparatus (Simplified)



Faraday’s Observation

- He plugged one wire into a galvanometer, and watched it as he connected the other wire to a battery.
- Indeed, he saw a transient current (which he called a "wave of electricity") when he connected the wire to the battery, and another when he disconnected it.[7]
- This induction was due to the change in magnetic flux that occurred when the battery was connected and disconnected.[3]

Mathematical Relationship between Flux and EMF

- The Electromagnetic Force (E) is equal to the negative of the rate of change of the magnetic flux (Φ) with respect to time
- $E = - d\Phi / dt$

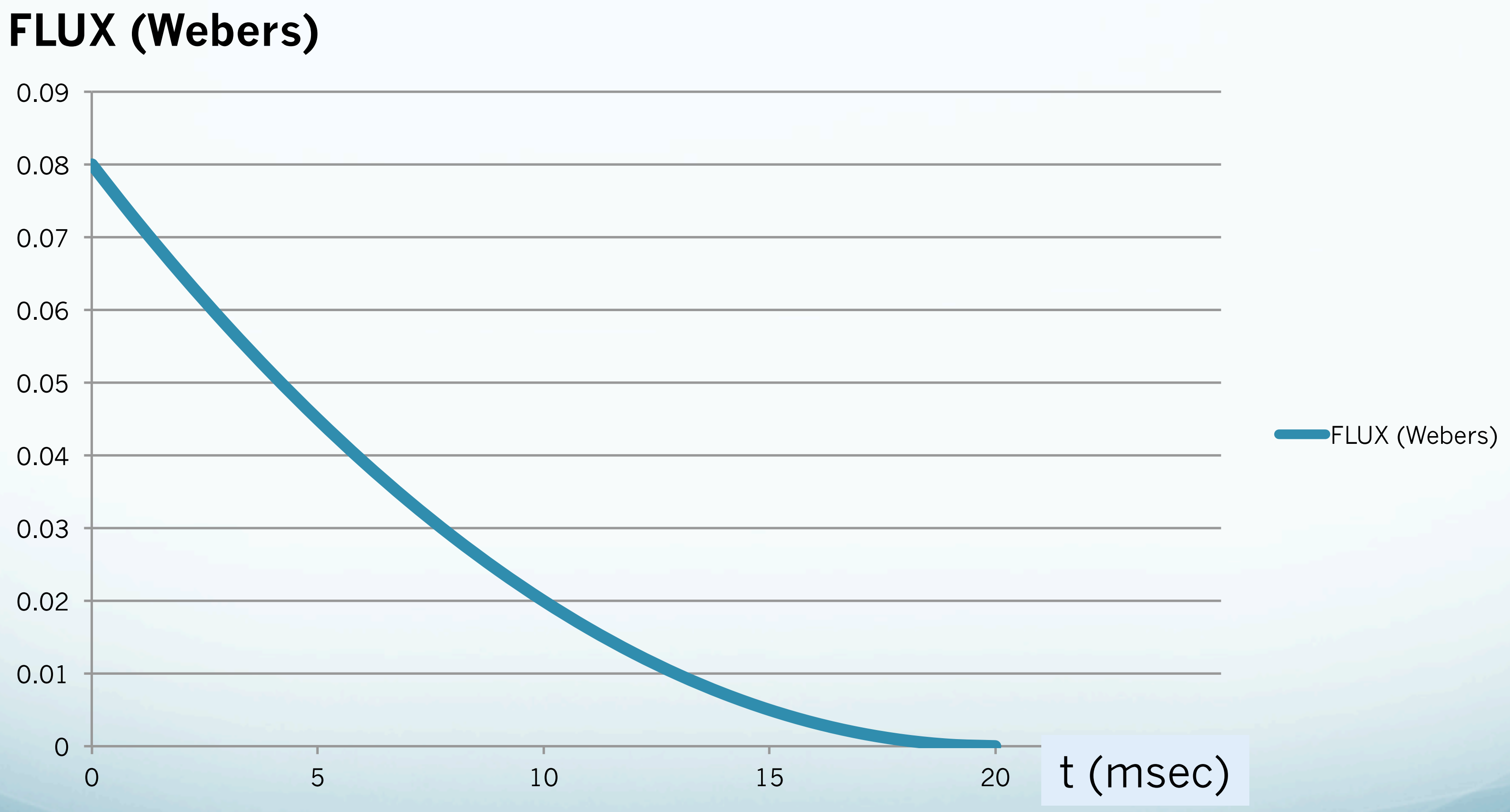
Example

- Suppose that we take measurements of the Electromotive Force and the Magnetic Flux and store the values in a table.

Table of Values

t (msec)	FLUX (Webers)
0	0.08
1	0.0722
2	0.0648
3	0.0578
4	0.0512
5	0.045
6	0.0392
7	0.0338
8	0.0288
9	0.0242
10	0.02
11	0.0162
12	0.0128
13	0.0098
14	0.0072
15	0.005
16	0.0032
17	0.0018
18	0.0008
19	0.0002
20	0

Plot of the Data



Calculate the Electromotive Force

- We can use the derivative to calculate the electromotive force.
- A formula relates the Electromotive Force to the derivative of the Magnetic Flux

$$E = - d\Phi / dt$$

Numerical Approximation

- We can determine an approximation for the Electromotive Force by using an approximation for the derivative.

Calculation Results

t (msec)	FLUX (Webers)	EMF(Volts)
0	0.08	-
1	0.0722	7.8
2	0.0648	7.4
3	0.0578	7
4	0.0512	6.6
5	0.045	6.2
6	0.0392	5.8
7	0.0338	5.4
8	0.0288	5
9	0.0242	4.6
10	0.02	4.2
11	0.0162	3.8
12	0.0128	3.4
13	0.0098	3
14	0.0072	2.6
15	0.005	2.2
16	0.0032	1.8
17	0.0018	1.4
18	0.0008	1
19	0.0002	0.6
20	0	0.2

Plot of Electromotive Force