Fundamentals of the Hydrodynamic Mechanism of Splitting in Dispersion Processes

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The splitting of globules is an important phenomenon during the final stages of disintegration processes. Three basic types of deformation of globules and six types of flow patterns causing them are distinguished.

The forces controlling deformation and breakup comprise two dimensionless groups: a Weber group N_{We} and a viscosity group N_{Vi}. Breakup occurs when N_{We} exceeds a critical value $(N_{We})_{crit}$. Three cases are studied in greater detail: (a) Taylor's experiments on the breakup of a drop in simple types of viscous flow, (b) breakup of a drop in an air stream, (c) emulsification in a turbulent flow.

It is shown that $(N_{We})_{crit}$ depends on the type of deformation and on the flow pattern around the globule. For case (a) $(N_{We})_{crit}$ shows a minimum value ~ 0.5 at a certain value of N_{Vi} and seems to increase indefinitely with either decreasing or increasing ratio between the viscosities of the two phases. For case (b) $(N_{We})_{crit}$ varies between 13 and ∞, depending on N_{Vi} and on the way in which the relative air velocity varies with time, the lowest value refers to the true shock case and $N_{Vi} \rightarrow 0$. For case (c) $(N_{We})_{crit}$, which determines the maximum drop size in the emulsion, amounts to ~ 1, and the corresponding values of N_{Vi} appear to be small. A formula is derived for the maximum drop size.

Efficient dispersion with a view to obtaining large interfacial areas is of great importance for heat and mass transfer processes in gas-liquid systems (atomization, froth-formation) and liquid-liquid systems (emulsification).

Most of the ordinary diminution processes function by injection of one phase into the other or by inducement of turbulence in the continuous phase. In the case of injection the potential energy of the phase to be dispersed is converted into kinetic energy, and at the same time usually interaction takes place with another medium, which may be gaseous, liquid, or solid. This interaction generates forces in the phase to be dispersed which may result in its breaking up. In the case of turbulence it is the kinetic energy of the turbulent motion in the continuous phase that brings about the breakup of the other phase.

In general the disintegration process takes place in stages. When one fluid is just squirted into another or when stirring processes have just started, the fluid to be dispersed is at first present in bulk. The deformation of this bulk of fluid and its initial breakup into chunks of fluid, which may break up further into smaller parts, has been much studied (*8, 6, 4*). There seems to be a mechanism that is common to such disintegration of liquids, to emulsification, and to atomization, namely the penetration of lamellae and ligaments of one fluid into the other. These ligaments then break up into globules, which may further split up into smaller parts.

When parts of a drop are separated or when a long ligament breaks up into droplets, in most cases secondary small droplets are also formed. Moreover, the ligaments at the moment of breakup will in general not all be equally thick. Hence during these disintegration processes drops of different sizes are formed. As the sizes seem to depend on many uncontrollable conditions, at present it is practically impossible to predict theoretically the final size distribution of a dispersion. Of course this does not exclude the possibility of making predictions from empirical correlations.

Concerning the various stages of disintegration processes, important contributions have already been made by theoretical and experimental studies of the instability of liquid films and cylindrical liquid ligaments and of the breakup of drops (*6, 15*). These theories can be applied to some simple types of atomization, but for the reasons stated above they still fail to describe quantitatively "chaotic" disintegration processes. More knowledge is required concerning the detailed mechanisms of breaking-up processes, as for instance the splitting up of small fluid lumps of the dispersed phase during the final stages of disintegration. For this case an attempt is made in the present paper to systematize the various ways in which single globules can break up. These considerations are applied to the breakup of globules in a viscous flow, to drops exposed to an air flow, and to emulsification in turbulent flow. In these few cases it appears possible to obtain an expression for the average value of the maximum globule size that can withstand the forces of a known hydrodynamic flow field.

BASIC TYPES OF GLOBULE DEFORMATION AND FLOW PATTERN

Globules can split up owing to hydrodynamic forces in a number of different ways that depend on the flow pattern around them. In general the following three basic types of deformation (illustrated in Figure 1) can be recognized.

Type 1. The globule is flattened, forming in the initial stages an oblate ellipsoid (lenticular deformation). How deformation proceeds during the subsequent stages leading to breakup seems to depend on the magnitude of the external forces causing the deformation. One possibility is that the drop deforms into a torus, which, after being more or less stretched, breaks into many small droplets.

Type 2. Here the globule becomes more and more elongated, forming in the initial stages a prolate ellipsoid, until ultimately a long cylindrical thread is formed, which breaks up into droplets (cigar-shaped deformation).

TABLE 1.—POSSIBLE CONDITIONS FOR BREAKUP OF A GLOBULE

Type of deformation	*A*. Dynamic pressures			*B*. Viscous stresses		
	Flow pattern around the globule					
1. Lenticular ..	(*a*) Parallel	(*c*) Axisymm. hyperbol.	(*e*) Rotat.	(*a*) Parallel	(*c*) Axisymm. hyperbol.	(*e*) Rotat.
2. Cigar-shaped	(*b*) Plane hyperbol.	(*d*) Couette		(*b*) Plane hyperbol.	(*d*) Couette	
3. Bulgy	(*f*) Irregular					

Type 3. As the surface of the globule is deformed locally, bulges and protuberances occur, and thus parts of the globule become bodily separated (bulgy deformation).

Of the various flow patterns that may cause a globule to deform in one of these basic ways, those considered here are given in Figure 2. In addition an irregular flow field *f* as occuring in turbulent flow is considered. For the rotating flow pattern *e* it is assumed that the fluid rotates bodily. It is further assumed that the flow patterns considered are large compared with the globule size but are still local flow patterns in the entire flow field. Finally one may distinguish between deformation by external dynamic pressures and that by viscous stresses. The possible combinations of flow patterns and types of deformation, or more precisely the most important of the possible ways in which a globule can be broken up by hydrodynamic forces, are given in Table 1.

Thus it is expected that deformation of type 1 will occur if the globule is subjected to the dynamic pressures or viscous stresses produced by the parallel flow *a*, the axisymmetric hyperbolic flow *c*, and the rotating flow *e*. Type 2 deformation (cigar-shaped deformation) can be caused by the plane hyperbolic flow *b* and the Couette flow *d*, and type 3 (bulgy deformation) is brought about only by the dynamic pressures occurring in the irregular flow pattern *f*.

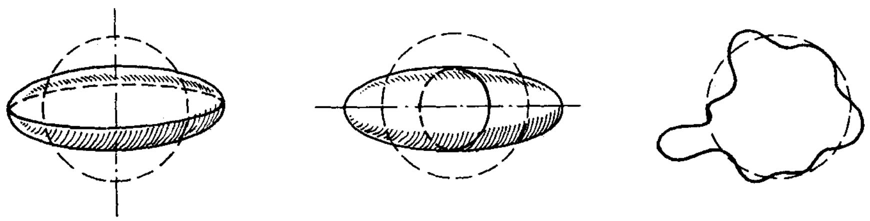


Fig. 1. Basic types of globule deformation.

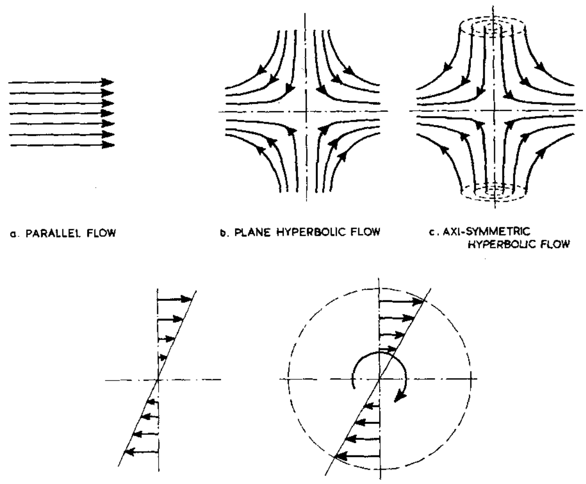


Fig. 2. Flow patterns that can cause one of the basic types of globule deformation.

Breakup will occur if there is a sufficient degree of deformation, that is, if the region of the flow pattern causing a specific deformation is sufficiently large to contain the deformed globule and if the flow pattern persists long enough.

Various possibilities may be considered somewhat more closely.

Lenticular Deformation in Parallel Flow. Here the globule has a translatory motion relative to the ambient fluid. This motion will persist longer if the density of the globule is much greater than that of the ambient fluid. An example of the dynamic case *A1a* (*see* Table 1) is the bursting of a drop in an air flow. If the flow is viscous (case *B1a*), the tangential viscous forces on the interface cause a torodial internal circulation, shown theoretically by Hadamard(*7*) and experimentally by Garner(*5*), which produces at the interface normal forces that in conjunction with the external normal component of the viscous stresses tend to deform the globule.

Lenticular Deformation in Rotating Flow (Cases *A1e* and *B1e*). The globule is assumed to be in a region where the ambient fluid rotates about an axis coinciding with that of the globule. Also here it will in general be necessary to have an appreciable difference in density in order to make the centrifugal forces sufficiently great to bring about breakup of the globule.

Cigar-shaped Deformation in Plane Hyperbolic Flow (Cases *A2b* and *B2b*). The elongation occurs along that axis in the plane of the flow in which the direction of the velocity is away from the origin.

Cigar-shaped Deformation in Couette Flow. In the dynamic case *A2d* the pressure distribution on the globule causes initially an elongation along an axis at 45° to the direction of flow. After this, pressure distribution on the elongated globule will cause further elongation and also a slight rotation, until the elongated globule has reached a position in which only continued elongation with eventual breakup can take place. In the viscous case $B2d$ the tangential forces at the interface cause internal circulation. The normal components of the viscous forces cause the globule to deform into an ellipsoid. The axes of the ellipsoid fall in the principal planes of the stress distribution of the shear flow (that is, the planes with no tangential stress). Also here, after sufficient elongation, breakup may occur.

The third type of deformation takes place if local presure differences at the interface (flow pattern f) cause initial bulges which, helped perhaps by a general parallel flow past the globule, may develop into protuberances, which may then possibly split up.

We thus see that a preference for a given type of deformation and breakup depends not only on the local flow pattern around the drop, but also on the physical properties of the two phases, namely their densities, viscosities, and interfacial tension.

In atomization the possibilities $A1a$ and $A2d$ will be the ones most frequently encountered; sometimes also $A3f$ will be possible.

In the case of gas dispersion in liquids (froth-formation), more particularly if the liquid is in violent turbulent motion, the following conditions will often occur: $A1c$, $A2b$, $A2d$, $A1e$, and $A3f$.

In emulsification processes it is reasonable to expect the conditions $B2b$ and $B2d$, with the odds on $B2d$, to occur in viscous flows; whereas in turbulent flows the conditions $A1c$, $A2b$, $A2d$, and $A3f$ may occur, with the odds perhaps on $A2d$.

FORCES CONTROLLING THE SPLITTING UP OF A GLOBULE

An isolated globule will be considered on the surface of which external forces act in such a way as to cause a deformation of the globule. τ is this force per unit surface area; in general it will vary along the surface and will be a function of time. It may be a viscous stress or a dynamic pressure set up in the surrounding continuous phase.

Owing to the deformation of the globule, internal flows are set up that cause viscous stresses as well as dynamic pressures. Furthermore the interfacial tension σ will give rise to a surface force that will in general counteract the deformation. If D is the diameter of the globule, the surface-tension force will be of the order of magnitude σ/D. In the first instance the dynamic pressure will be of the same order of magnitude as τ, causing flow velocities of the order of magnitude $(\tau/\rho_d)^{\frac{1}{2}}$. Thus the viscous stresses are of the order of magnitude

$$\frac{\mu_d}{D}\sqrt{\frac{\tau}{\rho_d}}$$

where μ_d and ρ_d are the viscosity and density of the globule.

These three forces per unit area

$$\tau,\ \sigma/D,\ \text{and}\ \frac{\mu_d}{D}\sqrt{\frac{\tau}{\rho_d}}$$

control the deformation and breakup of the globule. The ratio between each two of these forces is a dimensionless magnitude. Three dimensionless groups may be formed in this way, only two of which are independent.

For one of the dimensionless groups the combination $N_{We} = \tau D/\sigma$, a generalized Weber group, was chosen. The qualification "generalized" was added because, strictly speaking, the Weber group usually refers to $\rho U^2 D/\sigma$ where ρU^2 is the dynamic pressure of a fluid flow.

For the other dimensionless group either

$$\frac{\mu_d}{\sigma}\sqrt{\frac{\tau}{\rho_d}}\ \text{or}\ \frac{\tau D}{\mu_d}\sqrt{\frac{\rho_d}{\tau}} = \frac{\rho_d\,D}{\mu_d}\sqrt{\frac{\tau}{\rho_d}}$$

may be chosen, the latter being the Reynolds group. However, since the two groups contain the external force τ, the ratio between these two groups was preferred. It contains only the properties of the globule

$$N_{Vi} = \frac{\mu_d}{\sqrt{\rho_d \sigma D}},\ \text{a viscosity group.}$$

This group is called a viscosity group because it accounts for the effect of the viscosity of the fluid in the globule.

Of course the two groups N_{We} and N_{Vi} will immediately result from applying formal dimensional analysis and taking into account the magnitudes τ, D, σ, μ_d, and ρ_d.

The greater the value of N_{We}, that is, the greater the external force τ compared with the counteracting interfacial-tension force σ/D, the greater the deformation. At a critical value $(N_{We})_{crit}$, breakup occurs.

Experiments have shown that the mechanism of breakup is quite simple when N_{We} is equal to or slightly higher than $(N_{We})_{crit}$. The more N_{We} exceeds its critical value, the more complicated this mechanism becomes. For $N_{We} >> (N_{We})_{crit}$ this mechanism is very complex, and the disintegration process is more or less chaotic. This subject will be considered when the bursting of a drop in an air stream is described.

If the breakup of a globule is one of the final stages of a general disintegration process, then usually the N_{We} for breakup does not greatly exceed the critical value, and so the mechanism of breakup remains the simple type.

Since the deformation process can be described in terms of the two dimensionless groups, in general, $(N_{We})_{crit}$ will be a function of N_{Vi}. For this relation the following form is chosen

$$(N_{We})_{crit} = C\,[1 + \varphi\,(N_{Vi})] \quad (1)$$

where the function φ decreases to zero when $N_{Vi} \to 0$. In this form C is the value of We_{crit} for vanishing viscosity effect of the globule liquid. In this relation C and φ are still dependent on the external conditions, since the force τ is determined by the flow conditions in the continuous phase around the globule, which can also be described in terms of other dimensionless groups. The $(N_{We})_{crit}$ will be different for the three basic types of deformation and will also be dependent on the local flow pattern around the globule.

If the flow pattern is the same throughout the entire flow region, all globules larger than their critical size will break up according to the same mechanism. For all these globules the same value of $(N_{We})_{crit}$ is valid.

In more complicated flow fields where locally and temporarily different flow patterns of varying flow velocities occur, as for instance when the continuous phase is in turbulent motion, the $(N_{We})_{crit}$ will not be the same for all the globules present in the flow field. Some statistical mean value of $(N_{We})_{crit}$ will then determine the average size of the largest globules that can still withstand the breakup forces of the flow field. To arrive at this statistical mean value a higher weight must be assigned to those flow patterns and types of deformation which produce a lower value of $(N_{We})_{crit}$.

DEFORMATION OF GLOBULES IN VISCOUS FLOW

Fundamental work on the splitting up of drops in viscous flow was done by Taylor (*16*) and Tomotika (*18*). Taylor developed a theory based upon a few obvious assumptions, such as small deformations, no slip at the interface, and con-

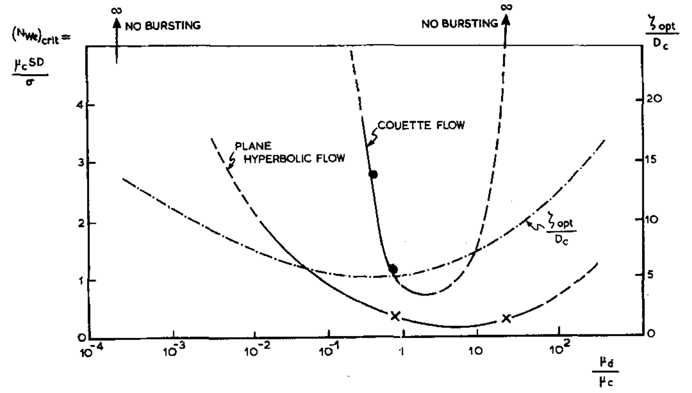


Fig. 3. $(N_{We})_{crit}$ and S_{opt}/D_c as functions of μ_d/μ_c.

tinuous shear stress across the interface. He considered two types of flow, namely the Couette flow (type *d* of Figure 2) and the plane hyperbolic flow (type *b* of Figure 2).

Because of the shear flow of the continuous phase the originally spherical drop is deformed into an ellipsoid (basic type 2), the cases *B2b* and *B2d* of Table 1.

According to Taylor's theory the deformation of the globule is predominantly determined by the generalized Weber group.

$$N_{We} = \frac{\mu_c \, S \, D}{\sigma},$$

where S is the maximum velocity gradient in the flow field of the continuous phase.

In two apparatuses which simulate the Couette flow and the plane hyperbolic flow respectively, Taylor has studied the deformation of a single drop as a function of S; he determined the value of S at which the breakup of the drop occurs, that is, $(N_{We})_{crit}$. This has been done with liquids that show the following ratios between the viscosities:

$$\frac{\mu_d}{\mu_c} = 0.0003, 0.5 \; 0.9, \text{and} \; 20.$$

Throughout the experiments the viscosity of the continuous phase was essentially the same, namely about 100 c.g.s. units.

The few data of Taylor's experiments are given in Figure 3. They show that in all cases a drop bursts more easily in the plane hyperbolic flow than in the Couette flow. If [$(N_{We})_{crit}$ for hyperbolic flow]/[$(N_{We})_{crit}$ for Couette flow] is determined, the curves in Figure 3 seem to indicate a maximum value of about 0.4 at $\mu_d/\mu_c \sim 1$. For both types of flow a minimum value of $(N_{We})_{crit}$ of less than 1 is obtained at a value of $\mu_d/\mu_c > 1$ (roughly at 5).

In the case of the Couette flow Taylor observed the remarkable fact that at $\mu_d/\mu_c = 20$ there was no breakup of the drop; the drop was only slightly deformed and showed a slow rotation in the direction of flow. Such a rotation, however, cannot occur in the plane hyperbolic flow; indeed, in that flow breakup did ultimately occur.

In neither type of flow was there any breakup at all during the experiments at the viscosity ratio $\mu_d/\mu_c = 0.0003$, even at the highest rates of flow applied. This result can be explained by the results of Tomotika's theoretical investigations(*17*). He considered the stability of an infinitely long cylindrical column of a viscous liquid surrounded by another viscous liquid when subjected to disturbances of rotational symmetry. He found that the optimum wave length ζ_{opt}, at which the rate of increase of the disturbance is at its maximum, increases to infinity if the viscosity ratio μ_d/μ_c becomes either infinite or zero. This means that the chance of the breakup of a drop deformed into a cylindrical thread is very slight at those extreme viscosity ratios.

The values of ζ_{opt}/D_c, where D_c is the diameter of the cylindrical thread, are also given in Figure 3. Apparently a minimum value of ζ_{opt} occurs at $\mu_d/\mu_c = 0.28$, that is, at $\mu_d/\mu_c < 1$. Hence, for the same diameter of the column the smallest drops caused by the breakup of the column are obtained at this viscosity ratio.

If the energy necessary to deform the drop is neglected, the power input must be equal to the dissipation in the flow of the continuous phase. This dissipation per unit mass ε is equal to $\mu_c S^2/\rho_c$ for the Couette flow, and to 4 $\mu_c S^2/\rho_c$ for the plane hyperbolic flow.

From Figure 3 it may be concluded that the energy input required to break a given drop reaches its minimum at $\mu_d/\mu_c > 1$. But in view of the results obtained by Tomotika(*17, 18*) it is possible that smaller drops are formed at a $\mu_d/\mu_c < 1$. One can estimate from Figure 3 that the energy input required for breaking the same drop is smaller for the hyperbolic flow than for the Couette flow.

Although it would be very desirable to have more data available from experiments such as those carried out by Taylor, yet it is possible to draw conclusions, albeit with some reserve, from his few experimental results:

1. It will be very difficult to disperse fluids that show a high viscosity ratio. This becomes obvious if one considers the extreme cases: dispersion of gas bubbles in viscous liquid flow, and dispersion (atomization) of liquid drops in viscous gas flow. This explains why in practice dispersion by pure viscous flow is restricted to emulsification processes.
2. For a given energy input per unit time and mass of the continuous phase, the drop that can resist the shear flow without breaking will be smaller if the liquid with the greater viscosity is the dispersed phase ($\mu_d/\mu_c > 1$).
3. A dispersion in which $\mu_d/\mu_c < 1$ can contain a larger fraction of small drops than one in which $\mu_d/\mu_c > 1$, although for the same energy input the largest drops may be larger; that is, the dispersion may be less uniform.

If relation (1) is applied to the results given in Figure 3, the assumption that at least C must be a function of μ_d/μ_c, becoming infinite for $\mu_d/\mu_c \to 0$ and probably also for $\mu_d/\mu_c \to \infty$, is obvious. The function φ may also increase indefinitely with increasing N_{Vi}. If $(N_{We})_{crit}$ is plotted against N_{Vi}, the minimum value for $(N_{We})_{crit}$ is obtained at rather high values of N_{Vi}, namely 100 and more. From the relation between $(N_{We})_{crit}$ and μ_d/μ_c one may conclude that the minimum value of $(N_{We})_{crit}$ will occur at lower values of N_{Vi} as the viscosity of the continuous phase decreases.

BREAKUP OF A DROP IN AN AIR STREAM

Investigation into the bursting of drops in an air stream has a long history, dating to before 1904, when Lenard published his experiments(*14*). Shortly after World War I Hochschwender made similar experiments(*11*), and in recent years drop deformation and breakup have been studied both experimentally and theoretically(*2, 9, 10, 12, 19, 20*) in various countries.

If a drop exposed more or less suddenly to a parallel air flow (flow pattern *a* of Figure 2) is considered for not too small drops and for a relative air velocity sufficiently great to bring about breakup of the drop, the Reynolds number for the flow around the drop will in general be sufficiently great for the external dynamic pressure forces to overshadow the external viscous forces. The external forces can then be expressed in terms of $\rho_c U_o^2$, where ρ_c is the density of the continuous phase and U_o the maximum value of the air velocity relative to the drop. In this case the Weber group has its original notion:

$$N_{We} = \frac{\rho_c U_o^2 D}{\sigma}.$$

Hinze(*9, 10*) has shown theoretically that $(N_{We})_{crit}$ depends not only on N_{Vi} but also on the way in which the relative velocity varies with time. Two cases have been studied by the various investigators, namely that of the true shock exposure and that of the falling drop. $(N_{We})_{crit}$ turns out to be much smaller for the first case.

For $N_{Vi} = 0$ and true shock exposure $(N_{We})_{crit} \simeq 13$, whereas for a falling drop $(N_{We})_{crit} \simeq 22$(*10*). $(N_{We})_{crit}$ increases with increasing N_{Vi}. Figure 4 shows results of laboratory experiments studying this effect of the viscosity of the drop on its breakup. Figure 4 also shows theoretical curves for the case of true shock exposure. These two theoretical curves refer to the cases of slight viscosity effect and great viscosity effect respectively. For great viscosity effects, which occur when $N_{Vi} > 0.5$, the rate of deformation is so low that the relative air velocity decreases substantially during the process of deformation and breakup, and so consequently do the breakup forces. Theoretical considerations show (*see* Figure 4) that $(N_{We})_{crit} \to \infty$ for indefinitely increasing N_{Vi}. The experimental results seem to indicate that already at $N_{Vi} > 2$ breakup will cease.

All investigators of the mechanism of breakup observed that during the breakup process the drop passes in succession through the stages of extreme flattening, formation of a torus with attached hollow-bag—shaped film of increasing size, bursting of the film, and breakup of the torus. Experiments at Delft showed that this mechanism still remains even for large effects of the drop viscosity.

Lane(*12*) was the first to draw attention to the fact that at

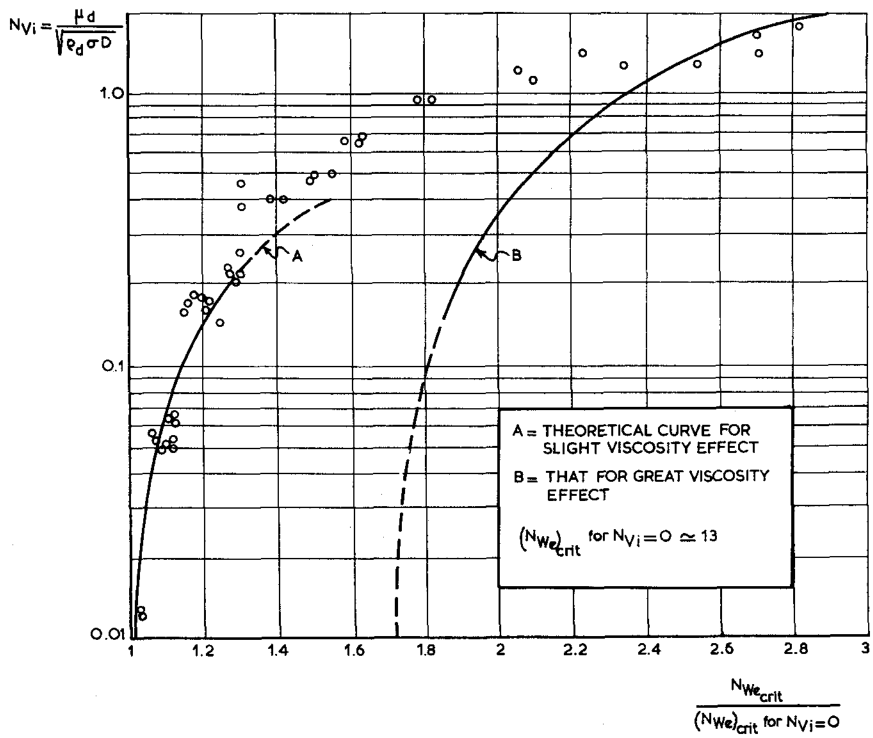
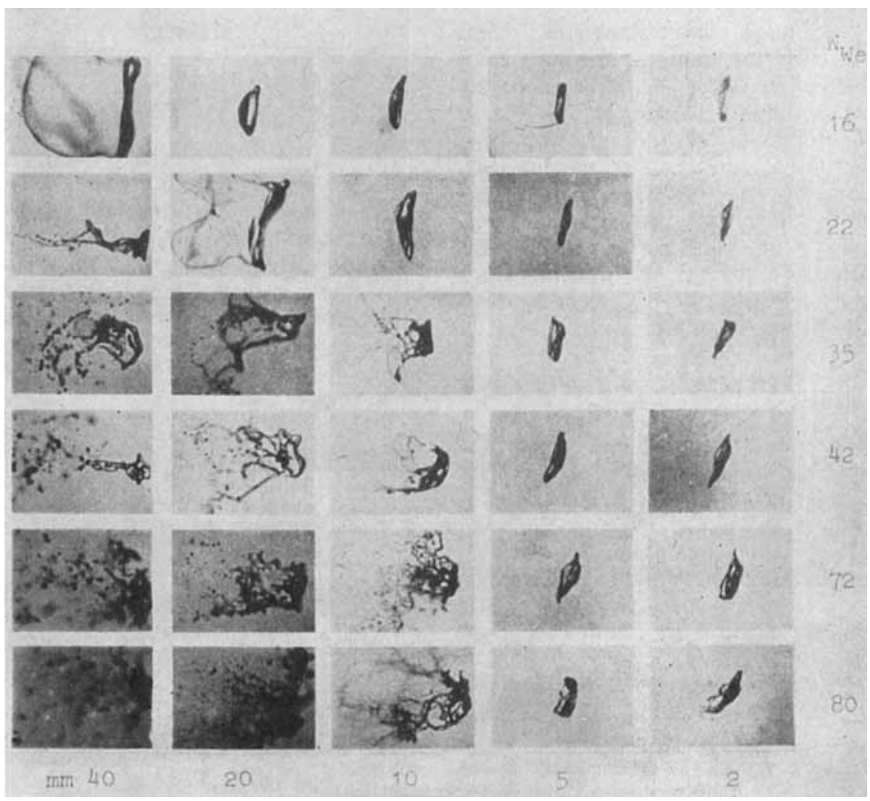


Fig. 4. Effect of N_{Vi} on $(N_{We})_{crit}$ in the case of a drop suddenly exposed to an air stream.



Distance from Pipette

Fig. 5. Short-flash photographs showing the breakup of a drop at increasing values of N_{We}. Gas oil; $D = 3.9$ mm.; $(N_{We})_{crit} \simeq 13$.

$N_{We} >> (N_{We})_{crit}$ the mechanism of breakup is different from the one just described. Figure 5 shows a series of short-flash photographs taken during the Delft experiments of drops launched in a horizontal air flow. With increasing N_{We} the nature of breakup changes in the direction of a more chaotic disintegration process. One may notice that the shape of the bulk of the drop during the initial stages of deformation is different for the two mechanisms. In the case of the first mechanism, that is at low N_{We}, the drop during deformation has the convex part directed downstream, whereas in that of the second mechanism, at large N_{We}, the the convex part is directed upstream. The nonuniform pressure distribution along it causes an acceleration of the drop. The acceleration forces acting on the fluid particles of the drop produce a deformation in addition to that caused along the drop by the pressure distribution, which is assumed to be steady. The first kind of deformation mentioned above is more *comformable to the* pressure distribution along the drop alone, whereas the second kind is more comformable to what might be expected of the acceleration effects alone. Moreover for $N_{We} >> (N_{We})_{crit}$ a kind of stripping off of the drop surface takes place owing to waves and ripples generated there.

Although the transition from the first mechanism to the second with increasing N_{We} is more or less gradual, it appears possible to introduce a second critical N_{We} pertaining to this transition. An analysis of the experimental results concerning this second critical N_{We} shows it to be a quite complicated function of the dimensionless groups N_{Vi}, N_{Re} pertinent to the flow around the drop, and ρ_d/ρ_c.

EMULSIFICATION IN TURBULENT FLOW

The dispersion of one liquid into another as the result of keeping the latter in violent turbulent motion is restricted here to the case of emulsification under noncoalescing conditions, which can be realized, for instance, by taking low concentration of the dispersed phase so that the chance of coalescing, in so far as it is determined by statistical mechanical conditions, is very slight.

More or less unconsciously guided by the mechanism observed by Taylor in viscous shear flow, investigators have commonly believed hitherto that also in turbulent flow splitting up of drops is the result of viscous shearing action(*4, 16*). However, a first requisite for such breakup of drops is that not only the undeformed drop, but also the elongated drop, must be small compared with the local regions of viscous flow. For not too small values of the Reynolds number the spatial dimensions of such local regions are very small, compared at least with the largest drops observed in the emulsion. Hence it is more logical to assume that the dynamic pressure forces of the turbulent motions are the factor determining the size of the largest drops. These dynamic pressure forces are caused by changes in velocity over distances at the most equal to the diameter of the drop. In regard to the area just around the drop, the plane hyperbolic, the axisymmetric and Couette flow patterns are the ones most likely to be responsible for breaking up the largest drops (compare the possibilities *A2b*, *A1c*, and *A2d* of Table 1). The kinetic energy of a turbulent fluctuation increases with increasing wave length. Thus velocity differences due to fluctuations with a wave length equal to $2d$ will produce a higher dynamic pressure than those due to fluctuations with a shorter wave length. If these fluctuations are assumed to be responsible for the breakup of drops, for $(N_{We})_{crit}$ in the relation (1) may be placed

$$(N_{We})_{crit} = \frac{\rho_c \overline{v^2} D_{max}}{\sigma},$$

where $\overline{v^2}$ is the average value across the whole flow field of the squares of velocity differences over a distance equal to D_{max}. To relate this average kinetic energy to this distance, one considers the simplest case, namely an isotropic homogeneous turbulence. For this case of turbulence the main contribution to the kinetic energy is made by the fluctuations in the region of wave lengths where the Kolmogoroff energy distribution law is valid. In this region the turbulence pattern is solely determined by the energy input ϵ per unit mass and unit time. It can be shown that for this region

$$\overline{v^2} = C_1 (\epsilon D)^{2/3}$$

where $C_1 \simeq 2.0$ according to Batchelor(*1*).

If for the moment one assumes that $N_{Vi} << 1$, then one obtains from 1:

$$\frac{\rho_c D_{max}}{\sigma} C_1 (\epsilon D_{max})^{2/3} = \text{const}$$

or

$$D_{max} \left(\frac{\rho_c}{\sigma}\right)^{3/5} \epsilon^{2/5} = C \qquad (2)$$

This simple result also follows directly from dimensional reasoning once it has become evident that only the quantities ρ_c, σ, and ϵ determine the size of the largest drops.

If this result is to be applied to fields of nonisotropic turbulent flow, it must be assumed that the turbulence pattern is practically isotropic in the region of wave lengths comparable to the size of the largest drops. Indeed, many actual, nonisotropic, turbulent flows do show an energy spectrum the high-energy part of which can often be approximated by the Kolmogoroff spectrum(*13*).

Furthermore, nonisotropic turbulent flows are not usually homogeneous, and so the energy input and dissipation are not constant across the flow field; moreover there is no equilibrium between energy input and dissipation at

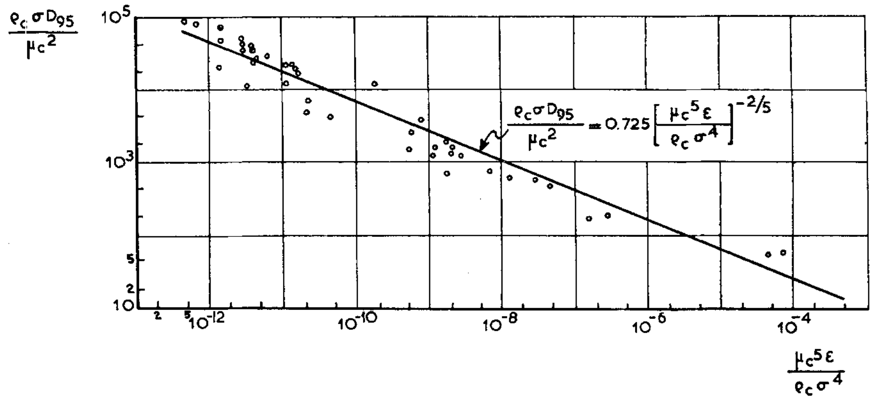


Fig. 6. Maximum drop size as a function of the energy input according to experimental data by Clay.

each point. In this case, even if local isotropy is present, the drop size as determined by (2) is still a function of the space coordinates.

If in the emulsifying apparatus the flow field is not too inhomogeneous, the powerful diffusive action of turbulence causes the average size of the largest drop in the whole field to correspond to the average energy input across this field.

With these assumptions formula (2) has been applied to the results obtained by Clay (*3*) with his model arrangement consisting of two coaxial cylinders, the inner one of which rotated. Clay determined drop-size distributions and from these calculated the value of D_{95}; this is the value for which 95% by volume is contained in the drops with $D \leq D_{95}$. If one takes $D_{max} \simeq D_{95}$ and applies the theory to this D_{95}, one obtains for (2):

$$D_{max}\left(\frac{\rho_c}{\sigma}\right)^{3/5}\epsilon^{2/5} = 0.725$$

with a standard deviation of 0.315.

Figure 6 shows that notwithstanding the wide scatter the data follow pretty well the relation (2). In this figure the relation is written:

$$\frac{\rho_c \sigma D_{95}}{\mu^2} = 0.725\left[\frac{\mu_c^5 \epsilon}{\rho_c \sigma^4}\right]^{-2/5} \qquad (3)$$

An appreciable scatter of test data is quite usual in phenomena with processes governed by statistical laws, but the very large scatter obtained here is most probably due to the rather crude way in which the drop-size distributions were obtained, which made the determination of D_{95} very inaccurate.

It would be interesting to know how far the relation (3) is applicable to other emulsifying apparatus. If the flow field is not too inhomogeneous (for instance, if there is emulsification by turbulence in pipe flow) the relation might well be applied; but in stirring apparatus, with paddles and the like, the intensity of turbulence will vary appreciably, being highest close to the paddles. Although perhaps the maximum drop size may still follow roughly the –2/5 law of the energy input, it is reasonable to expect a much smaller value at least for the numerical constant.

From the relation (3) it is possible to calculate the value of $(N_{We})_{crit}$. This, of course, will be a statistical mean value. If one substitutes $\overline{v^2} = 2.0(\epsilon D_{max})^{2/3}$ in the expression $(N_{We})_{crit} = \rho_c \overline{v^2} D_{max}/\sigma$, one obtains $(N_{We})_{crit} = 2.0\rho_c \epsilon^{2/3} D_{max}^{5/3}/\sigma$; thus with the empirical relation (3): $(N_{We})_{crit} = 2.0 (0.725)^{5/3} \simeq 1.18$. This value of roughly 1.2 is nearer to the value pertinent to the breakup of a drop in viscous shear flow, that is, according to type 2, than to that found for the breakup in a parallel air flow, that is, according to type 1.

Calculation of the values of N_{Vi} pertinent to D_{95} showed that in all cases N_{Vi} was smaller than 0.3 and in most cases even smaller than 0.1. Obviously the duration of the fluctuating turbulence forces that were responsible for the breakup of drops larger than D_{max} was too short to allow of an appreciable viscosity effect.

CONCLUDING REMARKS

In three cases of the breakup of drops, namely that in a viscous shear flow, that in an air flow, and that in a turbulent flow, it has been shown that there may be various mechanisms of breakup in dispersion processes and that $(N_{We})_{crit}$ will then be different too. Apparently $(N_{We})_{crit}$ is appreciably smaller in breakup according to deformation type 2 than in breakup according to the other types of deformation. The difference in density between the dispersed and the continuous phase has an important effect on the way in which breakup occurs.

Thus if one wants to apply values for $(N_{We})_{crit}$ obtained in one way to other dispersion processes, one has first to make sure that there is not a basic difference between the two dispersion processes; if there is, incorrect results might be obtained. This has too often not been realized.

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NOTATION

D = globule diameter, cm.
D_{max} = largest drop size, cm.
D_{95} = value of D for which 95% by volume is contained in the drops with $D < D_{95}$, cm.
D_c = diameter of cylindrical thread, cm.
S = maximum velocity gradient in external flow field, sec.$^{-1}$
$\overline{v^2}$ = average value of square of difference in turbulence velocity, sq. cm./(sec.)^2
U = relative velocity with respect to globule, cm./sec.
U_o = maximum value of U, cm./sec.
τ = surface force per unit area, dyne/sq.cm.
σ = interfacial tension, dyne/cm.
μ = absolute viscosity, g./(cm.)(sec.)
ρ = density, g./cm.^{3}
ζ_{opt} = optimum wave length (cm.)
ϵ = energy input per unit mass and time, or dissipation per unit mass, sq.cm./sec.^{3}
N_{Vi} = viscosity group $= \mu_d/\sqrt{\rho_d \sigma D}$
N_{We} = Weber group $= \tau D/\sigma$, or $\rho_c U_o^2 D/\sigma$, or $\rho_c \overline{v^2} D/\sigma$
$(N_{We})_{crit}$ = critical value of N_{We} for breakup

Subscripts

c = continuous phase
d = dispersed phase
c.g.s. units are used.

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