Equations Warm-up: Rules for manipulating equations

Learning objectives:

3.A.3. to be able to rearrange equations using the following rules:

add or **subtract** the same thing **to both sides**	if a = b then a **+ c** = b **+ c**
multiply or **divide both sides** by the same thing	if a = b then a **x c** = b **x c**
replace any term or expression by another equal expression	if a + **b** = c and **b = d x e** then a + **(d x e)** = c
square or **square root both sides**	if a + b = c then $(a + b)^2 = c^2$ also if $a^2 = \frac{b}{c}$ then $a = \sqrt{\frac{b}{c}}$
expand out an equation	**y**(a + x) = 1 becomes **y**a + **y**x = 1
simplify (**factorise**)	**a**b + **a**c = **a**(b + c)

Now use these rules to answer the following questions.

You may want to think about some of these tips.

When rearranging an equation, don't be afraid to use a lot of small steps and write down every step.

Sometimes it isn't at all clear how best to proceed – just start, remembering what it is that you need to make the subject of the equation – and eventually you will get there. There can be a lot of different ways of doing it.

Brackets are useful because you can move the whole term (ie what is inside the brackets) around as if it is a single item.

Questions:

Q1. Consider the equation $v = u + at$. Make a the subject of the equation.

Q2. Rearrange $s = ut + ½ at^2$ to make a the subject of the equation.

Q3. Rearrange $v = \sqrt{\frac{m}{p}}$ to make p the subject

Q4. Rearrange $F = \frac{L}{4\pi d^2}$ to make d the subject.

Q5. Rearrange $y = \frac{1}{1+x}$ to make x the subject.

Q6. If $V = \frac{C}{k}$ and $k = \frac{0.69}{t}$, write an equation for V in terms of C and t.

Q7. Drugs in the blood can be bound to plasma proteins and/or free in solution, in practice there is an equilibrium whereby C_{free} + protein <--> C_{bound}.

The percentage of drug bound is given by $b = 100\frac{C_{bound}}{C_{total}}$ where $C_{total} = C_{bound} + C_{free}$

The fraction of drug in plasma that is free is given by $f = \frac{C_{free}}{C_{total}}$.

Express f as a function of b.

Q8. Rewrite the following so that the brackets are removed. (a – 2)(b – 3) = 0

Q9. Rewrite the following so that the brackets are removed. (x + 3y + 2)(x – 3) = 0

Q10. Factorise $3x^2 – x$

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Q11. Factorise the following expression: $25 - y^2$

Q12. In pharmacology, the proportion of receptors bound with drug D is given by eqn 1.

eqn 1. $proportionbound = \frac{DK}{DK+1}$ (K is the affinity constant.)

when a competing drug B is added, a higher concentration of drug D^1 is given to get the same number of receptors bound with drug D.

eqn 2. $proportionbound = \frac{D^1 K}{D^1 K + BK_B + 1}$ (K_B is the affinity constant for drug B)

since the proportion bound is the same in eqn 1 and eqn 2 we can make the right hand side of eqn 1 equal to the right hand side of eqn 2.

$$\frac{DK}{DK+1} = \frac{D^1 K}{D^1 K + BK_B + 1}$$

Simplify this as much as possible, getting D^1 as a function of B.

Answers:

A1.

start		$v = u + at$
Step 1.	You want to get a on its own on the left. So start by reversing it.	$u + at = v$
Step 2.	You want a to be on its own so start by subtracting u from both sides	$u - u + at = v - u$ $at = v - u$
Step 3.	To get a on its own , you have to divide both sides by t.	$a = \frac{v}{t} - \frac{u}{t} = \frac{(v-u)}{t}$

A2.

Step 1.	You want to get a on its own on the left. So start by reversing it.	$ut + ½at^2 = s$
Step 2.	You want a to be on its own so start by subtracting ut from both sides	$ut - ut + ½at^2 = s - ut$ $½at^2 = s - ut$
Step 3.	To get a on its own , you have to multiply both sides by 2 then divide both sides by t^2	$at^2 = 2(s - ut)$ $\frac{at^2}{t^2} = \frac{2(s-ut))}{t^2}$ $a = \frac{2(s-ut))}{t^2}$

A3.

Step 1.	Start by squaring both sides.	$v^2 = \frac{m}{p}$
Step 2.	You want p to be on the left, so multiply both sides by p.	$pv^2 = \frac{m}{p} \times p$ $pv^2 = m$
Step 3.	Now divide both sides by v^2	$p = \frac{m}{v^2}$

A4.

Step 1.	You need to get d^2 off the bottom. To do this multiply both sides by $4\pi d^2$	$4\pi d^2 F = \frac{L}{4\pi d^2} \times 4\pi d^2$ $4\pi d^2 F = L$
Step 2.	Now to leave d^2 on its own, divide both sides by $4\pi F$	$\frac{4\pi d^2 F}{4\pi F} = \frac{L}{4\pi F}$ $d^2 = \frac{L}{4\pi F}$
Step 3.	Now square-root both sides.	$d = \sqrt{\frac{L}{4\pi F}}$

A5.

Step 1.	You need to get (1+x) off the bottom. To do this multiply both sides by (1+x) Here you are treating what's inside the brackets (1+x) as a single term.	$y(1+x) = \frac{1}{(1+x)} \times (1+x)$ $y(1+x) = 1$
Step 2.	You want to get x on its own, so expand out the brackets.	y+xy = 1
Step 3.	Now subtract y from both sides to leave xy on the left on its own.	xy = 1 – y
Step 4.	Now to get x on its own, divide both sides by y.	$\frac{xy}{y} = \frac{1}{y} - \frac{y}{y}$ $x = \frac{1}{y} - 1$ *or* $x = \frac{1-y}{y}$ these last two expressions are equivalent.

A6. $V = C \times \frac{t}{0.69} = \frac{Ct}{0.69}$

A7. $C_{total} = C_{bound} + C_{free}$
rearranging: $C_{free} = C_{total} - C_{bound}$

$$f = \frac{C_{free}}{C_{total}}$$

$$f = \frac{C_{total}}{C_{total}} - \frac{C_{bound}}{C_{total}}$$

$$f = 1 - \frac{C_{bound}}{C_{total}}$$

$$f = 1 - \frac{b}{100}$$

A8.

Step 1.	This is our starting point.	(a – 2)(b – 3) = 0
Step 2.	You have to multiply each term in the first bracket by each term in the second bracket.	ab – 2b – 3a + 6 = 0

A9.

Step 1.	This is our starting point.	(x + 3y + 2)(x – 3) = 0
Step 2.	You have to multiply each term in the first bracket by each term in the second bracket.	$x^2 + 3xy + 2x - 3x - 9y - 6 = 0$
Step 3.	Then collect similar terms together.	$x^2 + 3xy + 2x - 3x - 9y - 6 = 0$ $x^2 + 3xy - x - 9y - 6 = 0$

A10. The term “factorise” means to find the terms which were multiplied together to give this. In this case you can take x out of both terms

$$3x^2 - x = x(3x - 1)$$

A11. Here you have to remember that the difference of two square numbers is the same as the product of their sum and difference i.e. $25 - y^2 = (5 - y)(5 + y)$

A12.

Step 1.	This is our starting point. Our aim is to get all terms with D^1 on the left and all terms with B in them on the right.	$\frac{DK}{DK+1} = \frac{D^1K}{D^1K + BK_B + 1}$
Step 2.	In order to move things around we need to get them off the bottom (denominator to use the technical). Start by multiplying both sides by (DK+1). (OK so it doesn't look a lot better but just wait...)	$\frac{DK}{(DK+1)} \times (DK+1) = \frac{D^1K}{D^1K + BK_B + 1} \times (DK+1)$ $DK = \frac{(D^1K)(DK) + (D^1K)}{D^1K + BK_B + 1}$
Step 3.	Now multiply both sides by ($D^1K + BK_B+1$) I've used brackets strategically so that I can see it more easily – otherwise it can look like a real mess. Setting things out clearly is really important here.	$DK(D^1K + BK_B + 1) = (D^1K)(DK) + (D^1K)$ $(DK)(D^1K) + (DK)(BK_B) + DK = (D^1K)(DK) + (D^1K)$
Step 4.	Now have a look and see what terms appear on both sides. See that $(DK)(D^1K)$ appears on both sides, so you can subtract $(DK)(D^1K)$ from both sides leaving... Which looks much better.	$(DK)(D^1K) + (DK)(BK_B) + DK = (DK)(D^1K) + (D^1K)$ $(DK)(BK_B) + DK = (D^1K)$
Step 5.	Now you have (DK) appearing in both terms on the left hand side. Try simplifying this...	$DK(BK_B + 1) = D^1K$
Step 6.	Now you can see that you can divide both sides by K which will get rid of the K.	$DK(BK_B + 1) = D^1K$ $D(BK_B + 1) = D^1$
Step 7.	If you want to you can divide both sides by D so you have both D and D^1 on the same side but that is a bit cosmetic.	$BK_B + 1 = D^1/D$

looking back, you started with something not too big, then went through something that looked really quite horrible, then ended up with something quite simple.