Modern PID Control

S.P. Bhattacharyya
Texas A&M University

A Short Course at
University of California, Berkeley

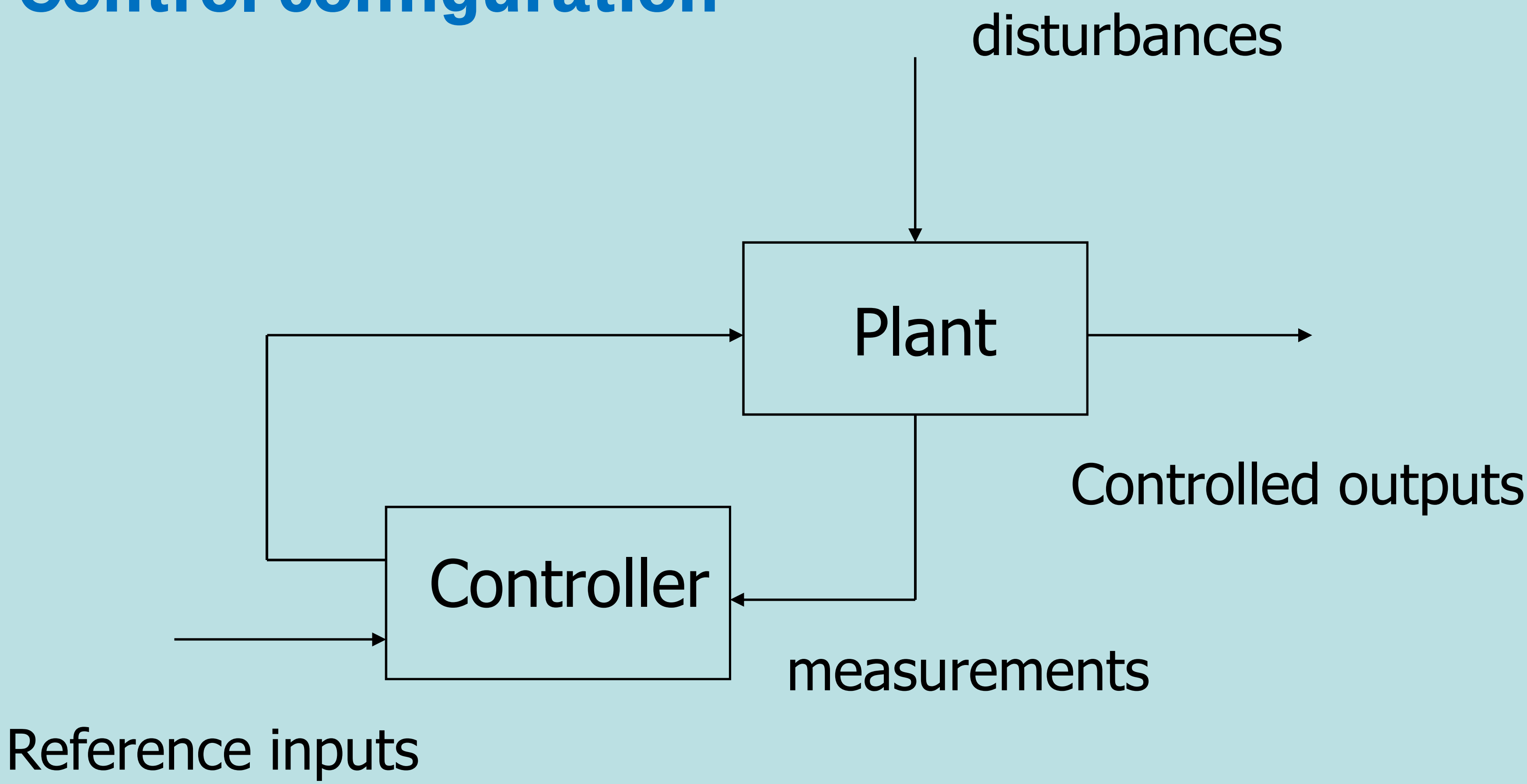
March 2013

PID Controllers:
An Overview of Classical Theory

Elements in Control Problem

- Outputs – dependent variables to be controlled.
- Inputs – independent variables.
- Disturbances – unknown and unpredictable elements.
- Equations describing the plant dynamics – parameters contained in it are not known precisely.

Control configuration



To determine the characteristics of the controller so that the controlled output can be

1. **Set to equal the reference; (tracking)**
2. **Maintained at the reference values despite the unknown disturbances; (disturbance rejection)**
3. **Conditions (1) and (2) are met despite the inherent uncertainties and changes in the plant dynamic characteristics (robustness)**

The Magic of Integral Control

$$y(t) = K \int_0^t u(\tau) d\tau + y(0) \quad \text{or} \quad \frac{dy(t)}{dt} = K u(t)$$

Integrator gain

For a constant y(t),

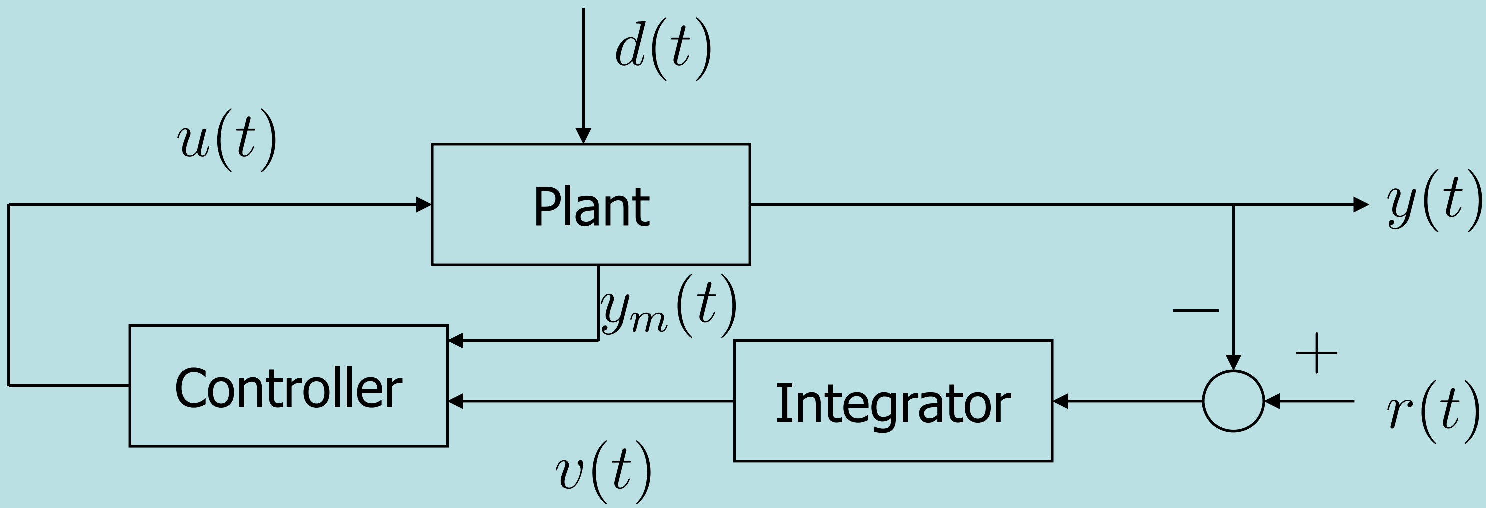
$$\frac{dy(t)}{dt} = 0 = K u(t) \quad \text{for all } t > 0$$

$$\frac{dy(t)}{dt} = 0 = Ku(t) \quad \text{for all } t > 0$$

Facts observed

1. If the output of an integrator is constant over a segment of time, then the input must be identically zero over that same segment.
2. The output of an integrator changes as long as the input is nonzero.

Solution to servomechanism problem



y(t) tracks a constant r(t)

⇔

1. Attach an integrator to the plant and make e(t) the input to the integrator.
2. Ensure asymptotic stability of the closed-loop system.

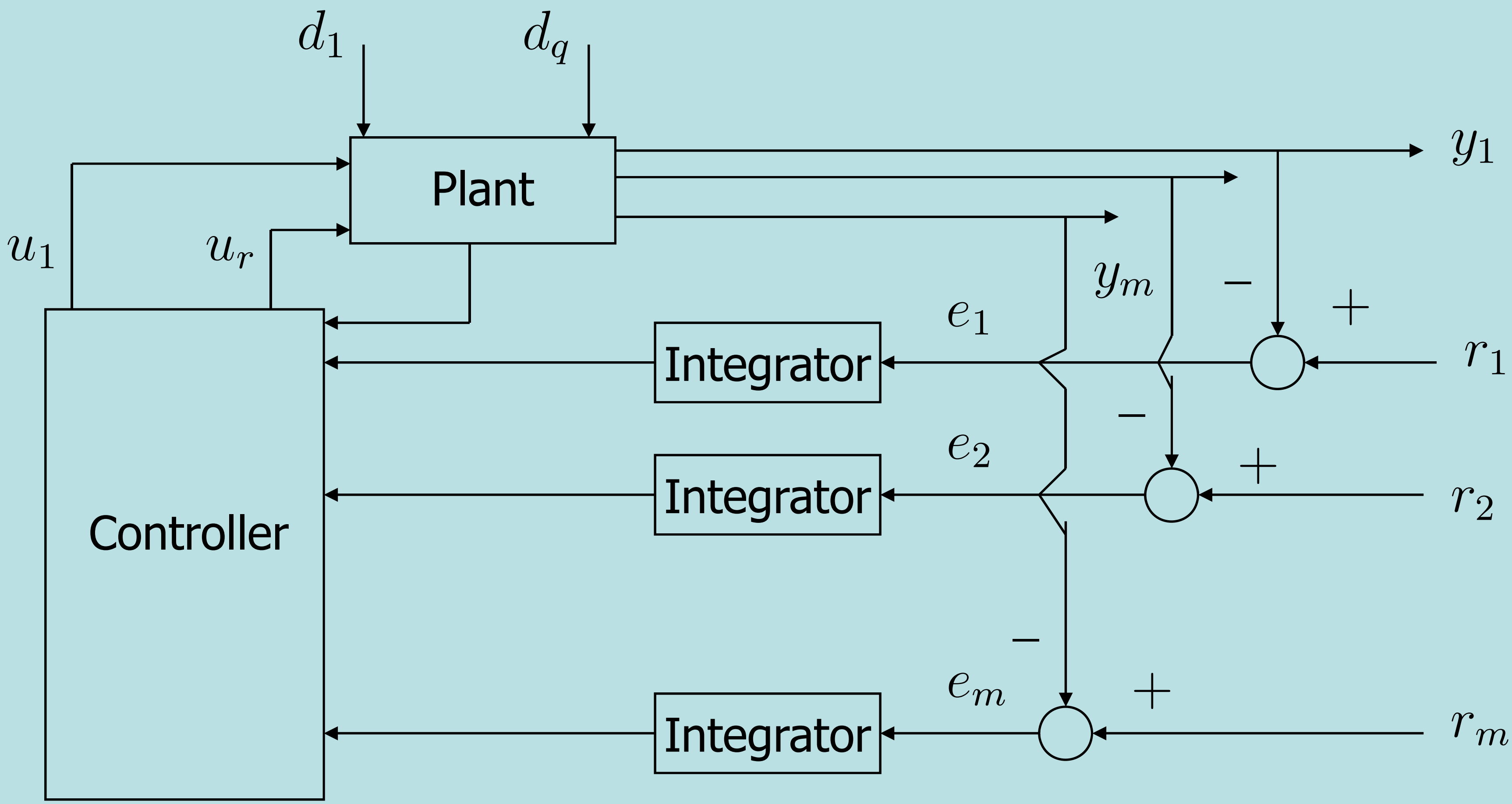
Fundamental fact about operation of an integrator

- It the closed-loop system is asymptotically stable, all signals will tend to constant values including the integrator output v(t).
- It follows that the integrator input tends to zero.

The steady-state tracking property is very robust.

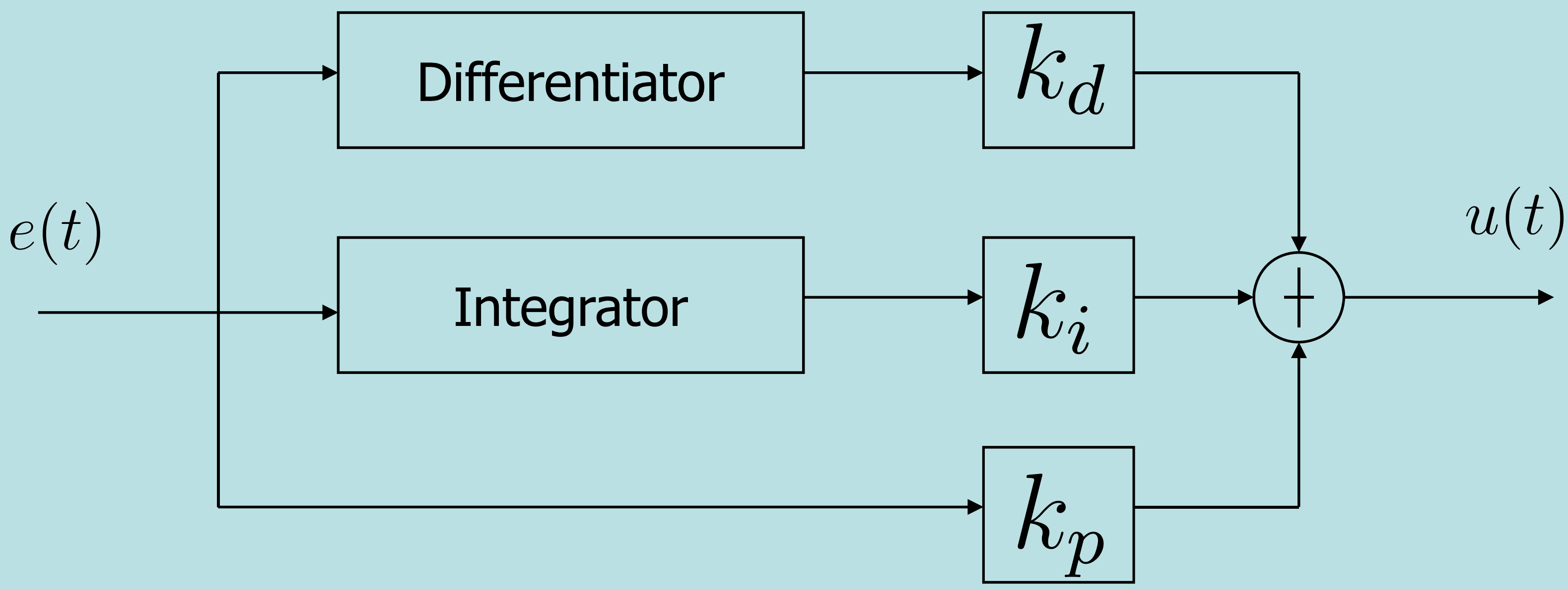
It holds as long as the closed-loop is asymptotically stable and is

1. independent of the particular values of the constant disturbances or references,
2. independent of the initial conditions of the plant and controller, and
3. independent of whether the plant and controller are linear or nonlinear.



Multivariable servomechanism

PID Controllers



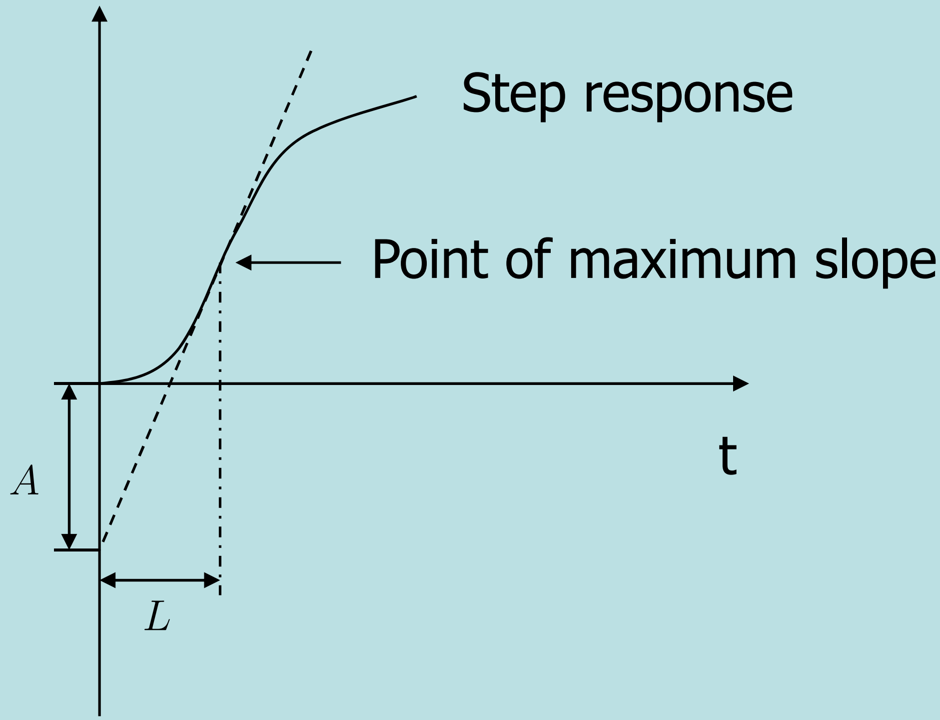
Classical PID Controller Design

The Ziegler-Nichols Step Response Method

$$C(s) = k_p + \frac{k_i}{s} + k_d s$$

$$\frac{k_d s}{1 + T_d s} \quad \text{for a small } T_d$$

The method is an experimental open-loop tuning method and is applicable to open-loop stable plants.



$$k_p = \frac{1.2}{A}$$

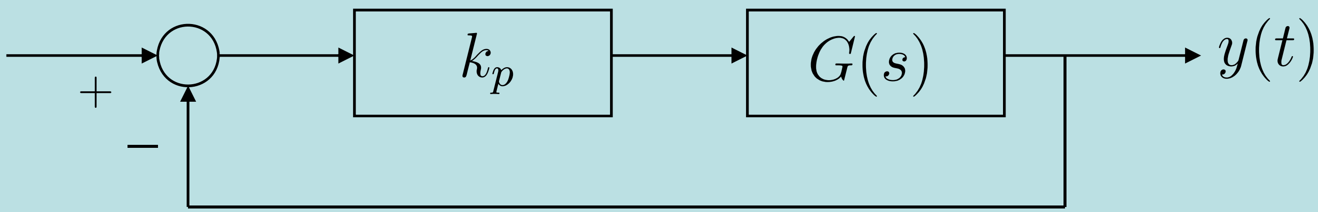
$$k_i = \frac{0.6}{AL}$$

$$k_d = \frac{0.6L}{A}$$

These formulas for the controller parameters were selected to obtain an amplitude decay ratio of 0.25, which means that the first overshoot decays to $1/4^{th}$ of its original value after one oscillation.

The Ziegler-Nichols Frequency Response Method

- The ZNFRM is a closed-loop tuning method.
- It first determines the point where the Nqyuist curve of the plant intersects the negative real axis.
- The closed-loop system must be stable with $k_i = k_d = 0$



- Slowly increase k_p until a periodic oscillation in $y(t)$ is observed.
- $k_u = \max k_p$ (ultimate gain), T_u (ultimate period)
- PID parameters: $k_p = 0.6k_u, \quad k_i = \dfrac{1.2k_u}{T_u}, \quad k_d = 0.075k_uT_u$

Nyquist Interpretation of ZNFRM

Using PID control, it is possible to move a given point on the Nyquist curve to an arbitrary position in the complex plane.

- The point where the Nyquist curve of the plant intersects the negative real axis

$$\left(-\frac{1}{k_u}, 0\right)$$

$= 2\pi$ $= \tan 25^o$

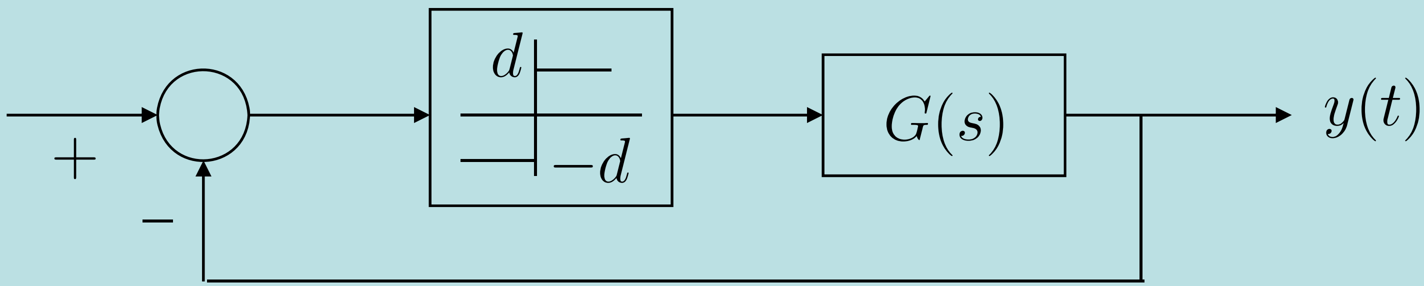
$$C(j\omega_u) = 0.6k_u - j\left(\frac{1.2k_u}{T_u\omega_u}\right) + j\left(0.075k_u T_u\omega_u\right) = 0.6k_u(1 + j0.4671)$$

$$G(j\omega_u)C(j\omega_u) = -0.6(1 + j0.4571) = -0.6 - j0.28$$

- The point $\left(-\frac{1}{k_u}, 0\right)$ is moved to the point (-0.6, -0.28)
- The distance from this point to (-1,0) is almost 0.5
- It means that the frequency response method gives a sensitivity greater than 2.
- The procedure requires the closed-loop system be operated close to instability.

Astrom and Hagglund's Method (An alternative to ZNFRM)

Using a relay to generate a relay oscillation for measuring the ultimate gain and ultimate period.

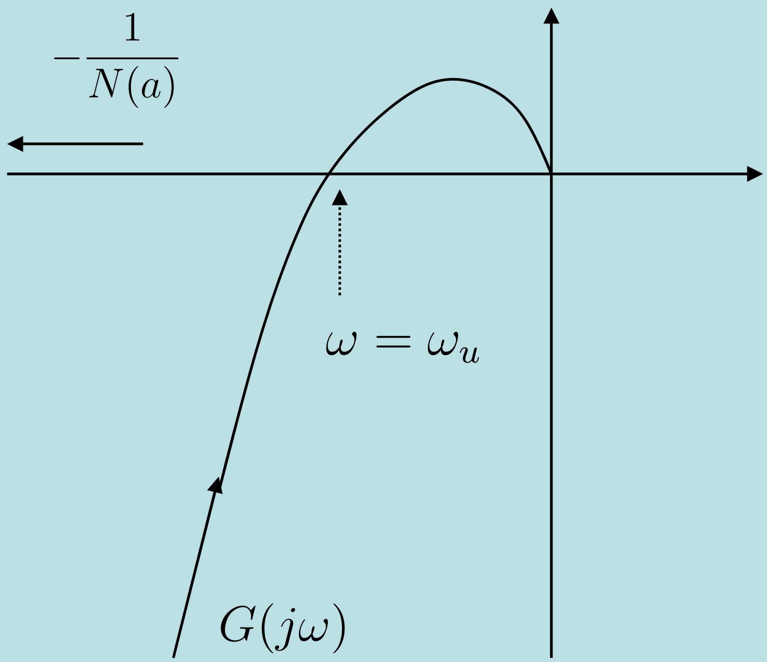


- Describing function for the delay $$N(a) = \frac{4d}{a\pi}$$

  where a is the amplitude of the sinusoidal input given to the relay and d is the relay amplitude.

- Condition for sustained oscillations at ω is $G(j\omega)N(a) = -1.$

Solution of $G(j\omega)N(a) = -1$



Nyquist plot of G(jw) and the describing function $-\frac{1}{N(a)}$

Determining ultimate gain and period

$$|G(j\omega_u)| = \frac{a\pi}{4d} \equiv \frac{1}{k_u}$$

$$\arg\left[G(j\omega_u)\right] = -\pi$$

PID parameters

Remarks on Ziegler-Nichols Based Tuning Methods

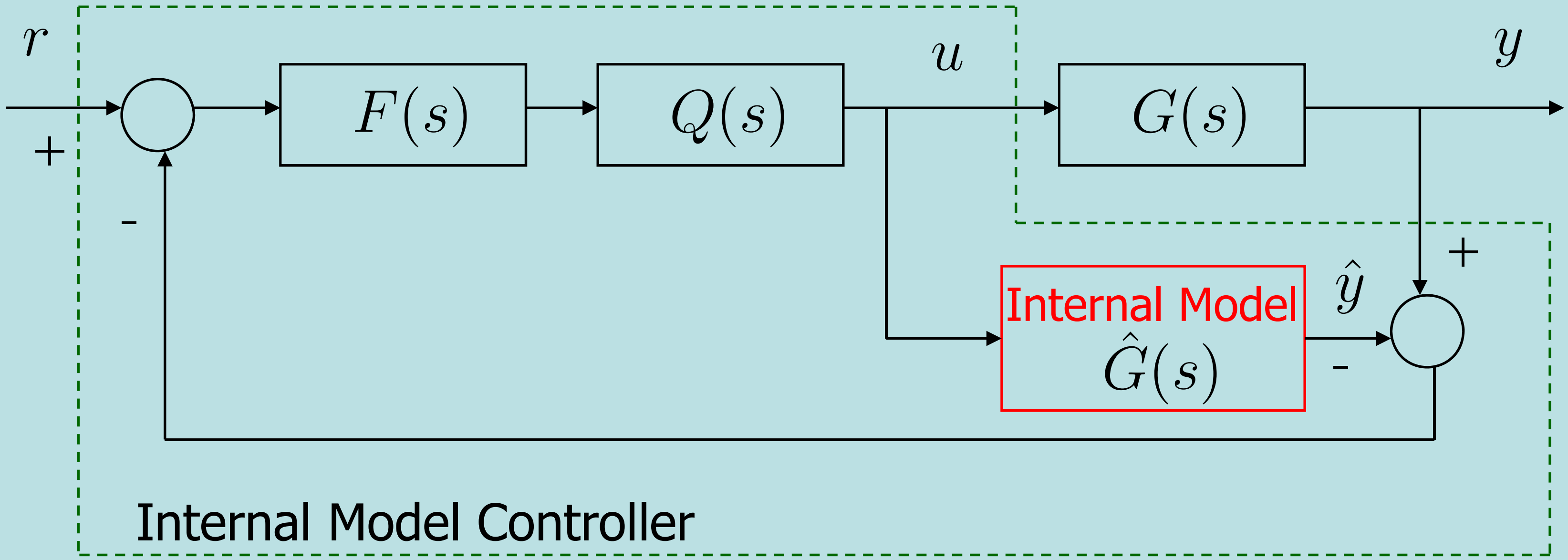
Pros

- Very little knowledge of the plants is required.
- Simple formulas are given for controller parameter settings.
- Formulas are obtained by extensive simulations of many simple stable systems.
- The main design criterion is to obtain a quarter amplitude decay ratio for the load disturbance response.

Cons

- Little emphasis is given to measured noise, sensitivity to process variations, and set-point response.
- The resulting closed-loop system can be poorly damped and sometimes can have poor stability margins.

Technique Using Internal Model Controller



$$C(s) = \frac{F(s)Q(s)}{1 - F(s)Q(s)\hat{G}(s)}$$

Choose Q(s) which minimizes the L_2 norm of the tracking error $r - y$ (achieves an H_2 optimal control design)

NOTE: For first-order plans with dead-time and a step command signal, the IMC H_2-optimal design results in a controller with a PID structure.

- $$G(s) = \left[\frac{k}{1+Ts}\right] e^{-Ls}$$

- H_2-optimal design is achieved by choosing Q(s) for which

$$\|[1 - \hat{G}(s)Q(s)]R(s)\|_2 \quad \text{where } R(s) = \frac{1}{s}$$

- Approximating the dead-time with a first order Pade approximation,

$$e^{-Ls} \approx \frac{1-\frac{L}{2}s}{1+\frac{L}{2}s} \quad \text{and} \quad \hat{G}(s) = \left(\frac{k}{1+Ts}\right)\left(\frac{1-\frac{L}{2}s}{1+\frac{L}{2}s}\right)$$

- Q(s) that minimizes $\|[1-\hat{G}(s)Q(s)]R(s)\|_2$

$$Q(s) = \frac{1+Ts}{k}$$

- Since Q(s) is improper, we choose

$$F(s) = \frac{1}{1+\lambda s} \quad \text{where } \lambda > 0 \text{ is a small number}$$

- The equivalent feedback controller becomes

$$C(s) = \frac{F(s)Q(s)}{1-F(s)Q(s)\hat{G}(s)} = \frac{(1+Ts)(1+\frac{L}{2}s)}{ks(L+\lambda+\frac{L\lambda}{2}s)} \approx \frac{(1+Ts)\left(1+\frac{L}{2}s\right)}{ks(L+\lambda)}$$

- PID parameters

$$k_p = \frac{2T+L}{2k(L+\lambda)}, \quad k_i = \frac{1}{k(L+\lambda)}, \quad k_d = \frac{TL}{2k(L+\lambda)}$$

Remarks

- Since the first-order Pade approximation was used for the time-delay, ensuring the robustness of the design to the modeling errors is important.

⇩ Selecting the design variable λ

- Appropriate compromise between performance and robustness

- A suitable choice proposed by Morari and Zafiriou - $\lambda > 0.2T, \quad \lambda > 0.25L.$

PROs

> Since the PMC PID design procedure minimizes the L2 norm of the tracking error due to set-point changes, the method gives good response to set-point changes.

Cons

> For lag dominant plants, the method gives poor load disturbance response because of the pole-zero cancellation inherent in the design methodology.

Dominant Poles Design: The Cohen-Coon Method

- The method is based on the first order plant model with dead-time

$$G(s) = \left[\frac{k}{1+Ts}\right] e^{-Ls}$$

- Attempt to locate three dominant poles (a pair of complex poles and a real pole) so that

  1. the amplitude decay ratio for load disturbance is 0.25 and
  2. $\int_0^\infty e(t)dt$ is minimized.

- Based on analytical and numerical computation, the following formulas are obtained:

$$k_p = \frac{1.35(1-0.82b)}{a(1-b)}, \quad k_i = \frac{1.35(1-0.82b)(1-0.39b)}{aL(1-b)(2.5-2b)}, \quad k_d = \frac{1.35L(0.37-0.37b)}{a(1-b)}$$

- where $a = \frac{kL}{T}, \quad b = \frac{L}{L+T}$

- For small b, the controller parameters are close to the parameters obtained by the Ziegler-Nichols step response method.

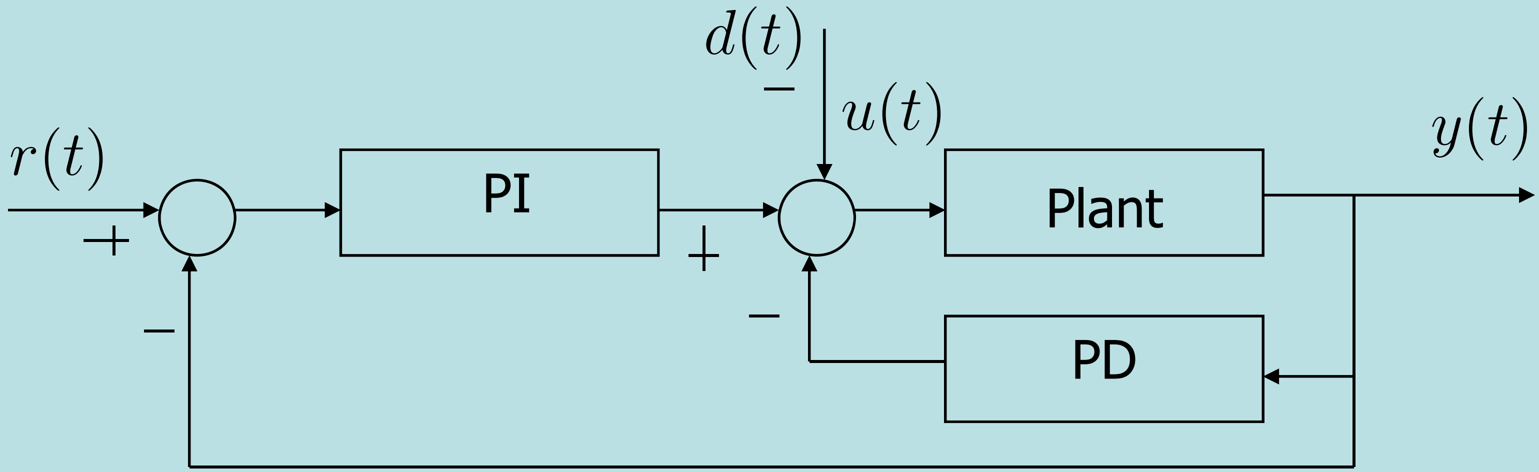
Time Domain Optimization Methods

- To choose the PID parameters to minimize an integral cost functional.
- (Zhang and Atherton) $$J_n(\theta) = \int_0^\infty t^n e^2(\theta, t)dt$$

  where θ is a vector containing the PID parameters and e(t) is error.

- Experimentation showed that for n=1, the controller obtained produced a step response of desired form.
- (Pessen) $$J(\theta) = \int_0^\infty |e(\theta, t)|dt$$

- (Atherton and Majhi) Modified PID controller



- An internal PD feedback to change the poles of the plant to more desirable locations.
- Then a PI controller is used in the forward loop.
- The parameters are obtained by minimization of the ISTE criterion.

PID Controllers: An Overview (Continue)

Frequency Domain Shaping

- Seek a set of controller parameters that gives a desired frequency response.
- (Astrom and Hagglund) Proposed a set of rules to achieve a desired phase margin specification.
- (Ho, Hang, and Zhou) developed a method to obtain the gain and phase margin specifications.
- (Voda and Landau) presented a method to shape the frequency response of the compensated system.

Optimal Control Methods

- Desire to incorporate several control system performance objectives such as reference tracking, disturbance rejection, and measurement noise rejection.
- (Grimble and Johnson) Incorporated specifications into an LQG optimal control problem.
- (Panagopoulos, Astrom, and Hagglund) PID design method that captures demands on load disturbance rejection, set-point response, measurement noise, and model uncertainty
  - Good load disturbance rejection by minimizing integral error
  - Good set-point response by using a structure with 2-degree of freedom
  - Measurement noise was dealt with by filtering
  - Robustness was achieved by requiring a maximum sensitivity

INTEGATOR WINDUP

- For the controller of the PID type, the error will continue to be integrated. This results in the error term becoming very large (windup).
- To return to a normal state, the error signal needs to be an opposite sign for a long time.
- These may lead to large transients when the actuator saturates.

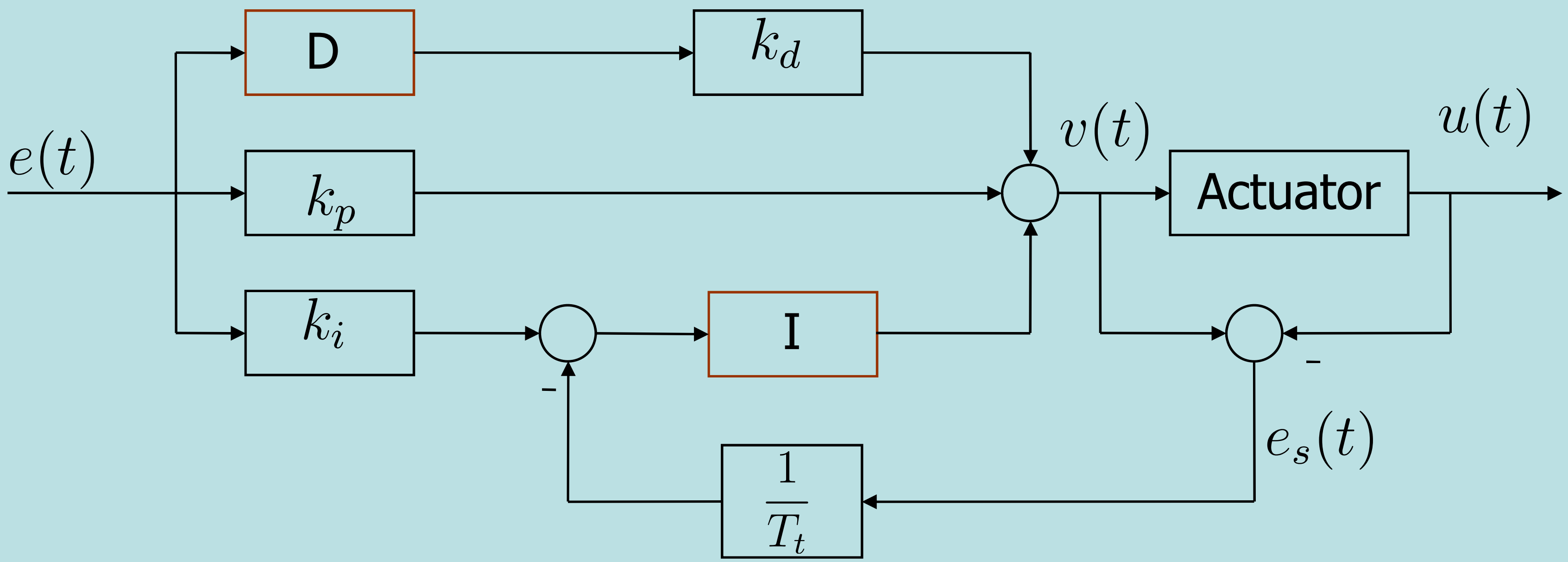
Set-point Limitation

Introduce limiters on the set-point variations so that the controller output will never reach the actuator bounds

drawbacks

1. leads to conservative bounds;
2. imposes limitations on controller performance;
3. does not prevent windup caused by disturbances.

Back-Calculation and Tracking



- When the actuator is within its operating range, $e_s(t)$ is zero. So there is no effect on normal operation.
- When the actuator saturates, $e_s(t) \neq 0$. This results in a new feedback path around the integrator and prevents the integrator from windup.
- The rate at which the controller output is reset is governed by $1/T_t$.
- (Astrom and Hagglund) suggest $k_d/k < T_t < k/k_i$

Conditional Integration

- An alternative to the back-calculation techniques.
- Simply switching off the integral action when the control is far from the steady-state.
- Means that the integral action is only used when certain conditions are fulfilled, otherwise the integral action is kept constant.

Examples of Switching Conditions

- switch off the integral action when control error e(t) is large.
- switch off the integral action when the actuator saturates.

Drawback: The controller may get stuck at a nonzero control error if the integral term has a large value at the time of switch off.

- switch off when the controller is saturated and the integrator update is such that it causes the control signal to become more saturated.