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Platform for Manipulating Polarization Modes Realized with Jones Vectors in *MATHEMATICA*

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The fundamental conception in physics of the propagation of the electromagnetic wave polarization in matter is newly understood as the cardinal keyword in free-space quantum communication technology and cosmology in astrophysics. Interactive visualization of the propagation mechanism of polarized electromagnetism in a medium with its helicity has accordingly received attention from scientists exploiting the protocol of quantum key distribution (QKD) to guarantee unconditional security in cryptography communication. We have provided a dynamic polarization platform for presenting the polarization modes of a transverse electromagnetic wave, converting the state of polarization through the arrangement of optical elements, using Jones vectors calculations in *Methematica*. The platform graphically simulates the mechanism of production and propagation of the polarized waves in a medium while satisfying Maxwell's equations.

Keywords: polarization modes, cosmology, Jones vector, wave plate, helicity

1. INTRODUCTION

Polarization is a coherent characteristic of the electromagnetic (EM) wave in a medium. In a plane wave, both the electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ of the electromagnetic radiation always oscillates parallel to a fixed direction in space. Light of such character is said to be linearly polarized, and maintains a constant direction of oscillation, and does vary spatially in a regular manner, producing either elliptically polarized or circularly polarized light (Fowles 1975; Jackson 1975; Pedrotti & Pedrotti 1987). Polarization characteristic have been used in radio transmission to reduce interference between channels, particularly at VHF frequencies and beyond (Masayoshi 2007; Yao et al. 2007). Free-space communication has forced the use of circular polarization, which has the fundamental advantage of precluding disturbance from the reflections of signals (Leitch et al. 2002; Elser et al. 2009). Free-space optical (FSO) communication utilizes a spatial diversity receiver to receive the binary signals, which are modulated by two circular polarizations. FSO communication employing a binary polarization shift keying coherent modulation scheme are utilized in atmospheric turbulence channel (Tang et al. 2010) and free-space quantum key distribution (QKD) by rotation-invariant twisted photons are used to guarantee unconditional security in cryptographic communication (Vallone 2014). On 17 March 2014, astronomers from the Harvard Smithsonian Center for Astrophysics announced their detection of signature patterns of polarized light in the Cosmic Microwave Background (CMB) (Ade 2014; Calvin 2014; CfA 2014). The team hunted for a special type of polarization called 'B-mode' which represents a twisting or 'curl' pattern in the polarized orientations of the ancient light. This is the strongest confirmation yet of the cosmic inflation theory (Boyle 2006). Gravitational waves squeeze space as they travel, and the squeezing produces a distinct pattern in the CMB. Gravitational waves have a "handedness" much like

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light wave and can have left- and right-handed polarization.

In the Standard Model, the weak bosons (W±, Z) mediate the weak interactions between different flavors (all quarks and leptons). Experimental results have shown that all produced and observed neutrinos have left-handed helicity, and all antineutrinos have right-handed helicity (Aad et al. 2012; CfA 2014). The helicity of the elementary particle could be a keyword to determine the Standard Model.

The scope of application of polarization has expanded explosively this decade with the development of communication technology. Accordingly, it is fundamentally important for application of a manipulation scheme to understand the physics of polarization's conception and the process of producing polarization modes. Presently the definition of polarization has been modified, and its nomenclature upgraded, which can be confusing to students of physics and researchers. For instance, there are similar customary nomenclature for the right circularly polarization: right circularly (Fowles 1975; Jackson 1975), right-circularly (Pedrotti & Pedrotti 1987), right-hand circularly (Reitz et al. 1993), and right-handed (Born & Wolf 1999; Georgi 1982) circularly polarization. Right circularly polarization and right-handed polarization are different types of polarization, although with similar names. Recently, the visualization of polarization propagating in matter has drawn physicist's attention for potential applications in modern physics and information technology (Tamm 1997; Mooleskamp & Stokes 2015; Yun & Choi 2013).

We have provided a dynamic polarization modes platform for simulating polarization modes with Jones matrices calculations, corresponding to the physical arrangement of optical elements, in a *Mathematica* computing environment (Mathematica 2015).

2. *MATHEMATICA* SIMULATION FOR THE CONVERTING POLARIZATION MODES

2.1 Electromagnetic waves in solids

The propagating process of electromagnetic waves in solids is different from that in the vacuum, since $\vec{E}$ and $\vec{B}$ of the electromagnetic waves interact with the electrons in a solid (Pedrotti & Pedrotti 1987; Fowles 1975). In particular, although the electromagnetic waves are an harmonic plane wave, the fields may pull or push the electrons in the orbital of a solid, which is responsible for inducing dipole moment $\vec{P}$ and magnetization $\vec{M}$ in the solid. If we assume that the medium is not a magnetic material and $\vec{J}$=0, the Helmholtz wave equation in a solid (Fowles 1975; Jackson 1975) is

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) + \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = -u_0 \frac{\partial^2 \vec{P}}{\partial t^2} \quad (1)$$

where $\vec{P}=\chi\varepsilon_0\vec{E}$. If we suppose the EM wave will be a form of solution such as $\hat{E}_0 e^{i(\vec{\mathcal{K}}.\vec{r}-\omega t)}$, we can rewrite Eq. (1) using a wave vector $\vec{k}$

$$\vec{k} \times (\vec{k} \times \vec{E}) + \frac{\omega^2}{c^2}\vec{E} = -\frac{\omega^2}{c^2}\chi\vec{E} \quad (2)$$

It then becomes a vector equation for $\vec{E}$, which indicates that the propagation process varies with the component of electric susceptibility χ. Therefore, we will write the $\vec{E}$ solution of Eq. (2) in a solid as an inhomogeneous plane wave solution below (Pedrotti & Pedrotti 1987; Fowles 1975; Jackson 1975)

$$\begin{aligned}\hat{E}(\vec{r},t) &= (\vec{\varepsilon}_1 E_1 + \vec{\varepsilon}_2 E_2)\ e^{i(\vec{\mathcal{K}}.\vec{r}-\omega t)} \\ &= E_0(\vec{\varepsilon}_1 + \vec{\varepsilon}_2 e^{i\Delta\varepsilon})\ e^{i(\vec{k}.\vec{r}-\omega t)} \quad (3) \\ &= \begin{pmatrix} A \\ B + iC \end{pmatrix} e^{i(\vec{k}\cdot\vec{r}-\omega t)}\end{aligned}$$

As shown above, $\hat{E}$ field vector may be presented as the Jones vector (Jones 1941) with a complex vector amplitude {A, B ± iC} oscillating in an inhomogeneous plane. The wave vector $\vec{k} = k\hat{n}$ and form a real mutually orthogonal unit vectors $(\vec{\varepsilon}_1, \vec{\varepsilon}_2, \hat{n})$. Here $\hat{K} = \vec{k}+i\vec{\alpha}$ is a complex wave vector and N = n+iκ is the complex refraction index. For Eq. (3) to be a solution of Eq. (2) as a homogeneous plane harmonic wave, it should be, $\mathcal{K}=\frac{\omega}{c}$ N. Then we get the relations: $\alpha=\frac{\omega}{c}\kappa$, and $k=\frac{\omega}{c}n$, which result in propagation speeds that are different along the direction in the medium. Therefore, there will be a cumulative phase difference $\Delta\varepsilon$ between the two components of the $\vec{E}$ field vector as they emerge in uniaxial crystals (Quartz, Calcite, etc.). After the wave has traveled a distance d, the phase difference is $\Delta\varepsilon = \frac{\omega}{c}d(n_2\text{-}n_1)$ between E_x wave and E_y wave when the radiation is propagating along the $\vec{k}$ direction. If the $\Delta\varepsilon$=0 while the amplitude of $\vec{E}$ is real, the vector is responsible for the linearly polarized such as Jones vector {A, B}, otherwise the amplitude is the complex vector responsible for the elliptically polarized Jones vector as {A, B±iC}. Specifically, if B=0 and A=C then the wave is a circularly polarized Jones vector such as {1, i}.

A quarter wave plate is a thin birefringent crystal the thickness of which has been adjusted to produce a ± $\pi/4$ phase difference between the ordinary and extraordinary rays at the operating wavelength. We desire a matrix that will transform the element $E_{0x}\, e^{i\varphi_x}$ into $E_{0x}\, e^{i(\varepsilon_x+\varphi_x)}$ and $E_{0y}\, e^{i\varphi_y}$ into $E_{0y}\, e^{i(\varepsilon_y+\varphi_y)}$. The general form of a matrix representing a phase retarder will transform the elements by the matrix operation as follows

$$\begin{pmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{pmatrix} \begin{pmatrix} E_{0x} & e^{i\varphi_x} \\ E_{0y} & e^{i\varphi_y} \end{pmatrix} = \begin{pmatrix} E_{0x} & e^{i(\varepsilon_x+\varphi_x)} \\ E_{0y} & e^{i(\varepsilon_y+\varphi_y)} \end{pmatrix} \quad (4)$$

where ε_x and ε_y represent the advance in phase of E_x - and E_y- component of the incident light. As an example, consider a quarter-wave plate (QWP) which makes $\Delta\varepsilon=\pi/2$. We may write the Jones matrix, M transforming the Jones vector which makes $\Delta\varepsilon=\pi/2$ to produce right-handed circular polarizing light as

$$M= \begin{pmatrix} e^{-i\frac{1}{4}\pi} & 0 \\ 0 & e^{i\frac{1}{4}\pi} \end{pmatrix} = e^{-i\pi/4} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \text{QWP, FA vertical} \quad (5)$$

This is the case of fast axis vertical (FA vertical). Similarly, we can determine the corresponding Jones matrix for a half-wave plate (HWP) or eighth-wave plate (EWP) or arbitrary phase of retarded. Jones matrices derived for various wave plates are summarized in Table 1.

We desire now to create a new simulation presenting a means of producing polarizing modes from a Jones calculation corresponding to the physical arrangement of optical elements in the *Mathematica* computing environment. In a plane wave, the electric field vector $\vec{E}$ always oscillates parallel to the fixed direction in space. Light of such character is said to be linearly polarized. If the linearly polarized light passes through a quarter-wave plate, elliptically polarized light emerges. The same can be said of the magnetic field vector $\vec{B}$, which maintains an orientation perpendicular to the electric field vector such that the direction of $\vec{E} \times \vec{B}$ is everywhere the direction of wave propagation. Thus, the possibility of polarizing light is essentially due to its transverse character. Therefore, the *Mathematica* simulation should show the transverse character of the $\vec{E} \times \vec{B}$ with its vectorial behaviors dynamically satisfying Maxwell's wave equations. In addition to the transverse character of the polarizing wave, the helicity of a polarized wave is a critical factor in quantum cryptography communication technology or cosmology in modern physics.

Table 1. Summary of Jones vectors in the most common and Jones matrices of the wave plates and phase retarders (Pedrotti & Pedrotti 1987).

Polarized light	Jones vectors	helicity
linearly polarized	$\begin{pmatrix} \text{Cos}\,\alpha \\ \text{Sin}\,\alpha \end{pmatrix}$	0
linearly polarized	$1/\sqrt{2}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}$	±1
right(left)-handed elliptically	$1/\sqrt{A^2+B^2+C^2}\begin{pmatrix} A \\ B+i\,C \end{pmatrix}$	±1
Polarized light	**Jones matrices**	
general phase retarder	$M=\begin{pmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{pmatrix}$	
linearly polarizer(β,TA)	$M=\begin{pmatrix} \text{Cos}^2\beta & \text{Cos}\beta\,\text{Sin}\beta \\ \text{Cos}\beta\,\text{Sin}\beta & \text{Sin}^2\beta \end{pmatrix}$	
quarter-wave plate(V/H)	$M=e^{\mp i\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & \pm i \end{pmatrix}$	
half-wave plate(V/H)	$M=e^{\mp i\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	
eighth-wave plate(V/H)	$M=\begin{pmatrix} 1 & 0 \\ 0 & e^{\pm i\pi/4} \end{pmatrix}$	

† Wave plate (V/H) stand for the fast transmission axis of vertical/horizontal.

2.2 Polarization modes *Mathematica* simulation

We have used *Mathematica* to implement interactive Jones matrix calculations and animations for the generation and propagation of the polarization modes in the solid state. First, we desire to confirm that the complex vector field $\hat{E}$ of Eq. (3) with Jones vectors in the matter satisfy the Maxwell's vector equations Eqs. (6) and (7) in both numeric and graphic simulations.

$$\hat{\mathcal{K}} \cdot \hat{E} = 0, \hat{\mathcal{K}} \cdot \hat{E} = 0, \hat{\mathcal{K}} \times \hat{E} = \omega\,\hat{B} \quad (6)$$

$$\vec{S} = \vec{E} \times \vec{H} \quad (7)$$

where $\hat{\mathcal{K}}$, $\hat{E}$, and $\hat{B}$ are both complex vectors and $\vec{E}$, $\vec{B}$ and $\vec{H}$ are real vectors with $\vec{B} = \mu\vec{H}$ where $\vec{H}$ is a magnetic intensity vector in the matter. We examine the process of generating polarization modes with the normalized Jones vectors calculations in the *Mathematica* simulation as below. The *Mathematica* input code is shown below:

Mathematica code #1

```
In[11]:= E0 = 1/√2{1, 0} + 1/√2{0, 1};
In[12]:= LP = {Cos[α], Sin[β]};
In[13]:= LW = {{Cos²β, Cosβ Sinβ}, {Cosβ Sinβ, Sin²β}};
In[14]:= QWV = {{1, 0}, {0, I}}; QWH = {{1, 0},{0, -I}}};
In[15]:= HW = {{1, 0}, {0, -1}};
In[16]:= EWV = {{1, 0}, {0, Exp[ I 1/4π}};
         EWH = {{1, 0}, {0, Exp[ -I 1/4π}};
In[17]:= l1 = E0*LP; l2=LW ;

In[22]:= E0*LP/. α ->1/4π
In[23]:= E0*LP·QWV/. α ->1/4π
In[24]:= E0* LP·QWH/. α ->1/4π
In[25]:= E0* LP·HW /. α ->1/4π
In[26]:= E0* LP·EWV /. α ->1/4π //N
In[27]:= E0* LP·EWH /. α ->1/4π //N
In[28]:= l1.l2/.{α ->1/4π, β ->1/3π} //N
In[29]:= l1.l2/.{α ->1/4π, β ->3/4π} //N
```

Mathematica returns calculations;

Out[22]:= {I/2, I/2}
Out[23]:= {I/2, I/2} (Fig. 2, Fig. 3(a))
Out[24]:= {I/2, -I/2} (Fig. 3(b))
Out[25]:= {I/2, -1/2} (Fig. 3(c))
Out[26]:= {0.5, 0.353553 + 0.353553 I } (Fig. 3(e))
Out[27]:= {0.5, 0.353553 - 0.353553 I } (Fig. 3(f))
Out[28]:= {0.341506,0.591506}} (Fig. 3(g))
Out[29]:= {0,0} (Fig. 3(h))

The *Mathematica* calculations show the emerging polarization mode through the physical arrangement of optic elements, which promptly confirm those Jones vectors from the animating platform. For instance, in In[23]:= E0*LP·QWV/. α ->1/4 π produces Out[23]:= {I/2, I/2}, that is the right-handed circularly polarized light (RHCP) as shown in Fig. 2. From the arrangement of linearly polarizers in right angle no wave emerging shown as Out[29]:= {0, 0} while the Out[28]:= {0.341506, 0.591506} which show the linear polarization (see Figs. 3(g) and 3(h)). This enables us to switch on or off the polarization by the combination of arrangements of polarizers and wave plates.

2.3 Helicity of the elliptically polarized wave

The handedness of an elementary particle depends on the correlation between its spin and its momentum (Goldhaver et al. 1958). If the spin and momentum are parallel, the particle can be said to be right-handed or have a helicity of 1. If they are anti parallel, the particle is left-handed or have a helicity of -1. We may also adopt this definition to modern optics, since the circularly polarized electromagnetic wave is just a helical motion with helicity. The helicity of the polarized electromagnetic wave is a critical factor in modern communication technology and photonics (Aad et al. 2012; Goldhaver et al. 1958; Rubenhok et al. 2013). However, determination of the helicity is a perplex issue because we need to aware of the vectorial behaviors accurately unless we may observe propagating polarized wave (Goldhaver et al. 1958). Therefore, it is very helpful to simulate advanced polarization modes in the medium, and further, it would be more helpful to evaluate the helicity operator in a physics state.

Here we simply estimate the helicity with a calculation of phase shift in the process in the *Mathematica* (Wolfram 2015a). We can determine the helicity based on a calculation of the phase shift in the block respectively; phase shifts of the E_1=E0*Jv after passing through the polarizer and of E_2=Jm·E_1 after the wave plate respectively. For evaluation of the helicity of the polarized waves in progression, we have provided a *Mathematica* module, helicity [JvC_,JmC_]:= Module[{E1, E2, dp11, dp12, dp13, dp21, dp22, dp23}]. If you type in two numbers assigned to the polarizer JvC and wave plate JmC in the module, then helicity [JvC, JmC] will return helicity with a list of phase shifts. The outputs of the helicity [JvC, JmC] are shown as below:

Mathematica code #2

```
In[81]:=    helicity[1,2]                            (Fig.3 (a))
Out[81]:=   {0., 0., 0.}}
            helicity1 = 0
            {1.5708, 1.5708, 1.5708}
            helicity2 = +1

In[82]:=    helicity[1,3]                            (Fig.3 (b))
Out[81]:=   {0., 0., 0.}}
            helicity1 = 0
            {-1.5708, -1.5708, -1.5708}
            helicity2 = -1

In[83]:=    helicity[2,7]                            ( E0*LCP.EWH)
Out[83]:=   {-1.5708, -1.5708, -1.5708}
            helicity1 = -1
            {-2.35619, -2.35619, -2.35619}
            helicity2 = -1

In[84]:=    helicity[2, 5] ]                         ( E0*LCP.HWH)
Out[84]:=   {-1.5708, -1.5708, -1.5708}
            helicity1 = -1
            {- 4.71239, - 4.71239, - 4.71239,}
            helicity2 = +1
```

The helicity[1, 2] returns two helicities with two lists of three phase differences in the two divisions for the optical device arrangement of linearly polarizer (JvC=1) and quarter-wave plate (JmC=2). The helicity[2, 7] returns the helicities of the optical arrangement of left-circularly polarizer (JvC=2) and eighth-wave plate (JmC=7). It needs to be noticed that the three phase differences are all 1.5708 (π/2) of the helicity[1, 2] which interprets the vertical component E_{2z} lead continuously the horizontal component E_{2y} with a constant phase shift (π/2) on the yz plane of $\vec{E}_2$ field vector propagating in x_1 direction perpendicular to this plane with velocity ω/k. This results in $\vec{E}_2$ vector rotating in a counter clockwise direction (right-handed, ↺) around the advancing x_1 direction. That is, if we grasp our finger along the spin direction with right hand, then thumb directs the advancing direction of propagation wave, hence the wave is right-handed polarized wave regardless of viewer . This is very helpful in confirming the helicity of the polarized wave on the end block of the platform as shown in

Figs. 1 and 2. We can correctly determine the helicity of the polarized wave in the E2 block by seeing the spin direction of the wave; either the spin direction is parallel (helicity=+1, ↺) or anti parallel (helicity=-1, ↻) to the advancing x_1 direction (momentum direction). Helicity of E_2 can be changed simply by the choice of JmC optical element like as In[83] and In[84] codes. The half-wave plate (HWH) shifts phase $-\pi$ in helicity[2, 5] while eighth-wave plate (EWH) shift phase $-\pi/4$ in helicity[2,7]. Furthermore we may easily calculate the helicities of the wave train on the helicity [JvC, JmC] module for any combination of optical elements in *Mathematica*, so that we can predict exactly helicity of the producing polarization modes. For more information for the helicity [JvC, JmC] module refer to citation (Yun 2015).

3. DYNAMIC POLARIZATION PLATFORM WITH JONES VECTORS IN MATHEMATICA

We have provided a dynamic polarization platform with Jones vectors (jdpmp) in the **Graphics3D** in *Mathematica*, which simulates the polarizing modes dynamically while presenting the helicity of a running polarization mode. The platform manipulates three zones graphically using

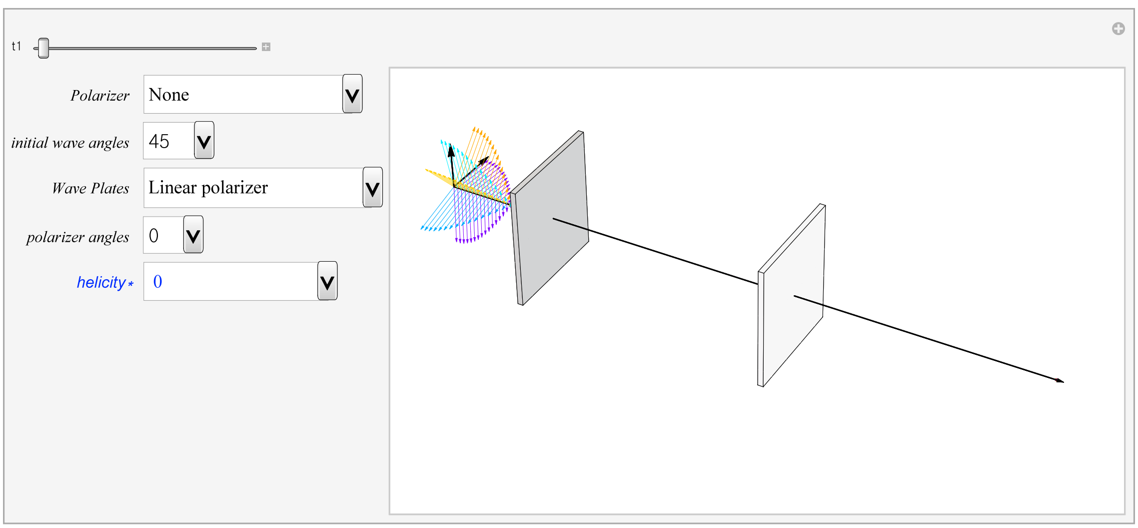


Fig. 1. The starting platform of the dynamic polarization modes platform with Jones vectors (jdpmp). Unpolarized EM waves $\vec{E}_0$ injected to the polarizer (LP) have not passed the polarizer yet. The simulation will run when you click the ▶ appearing while you spread the ⊕ of *t1* panel of the platform.

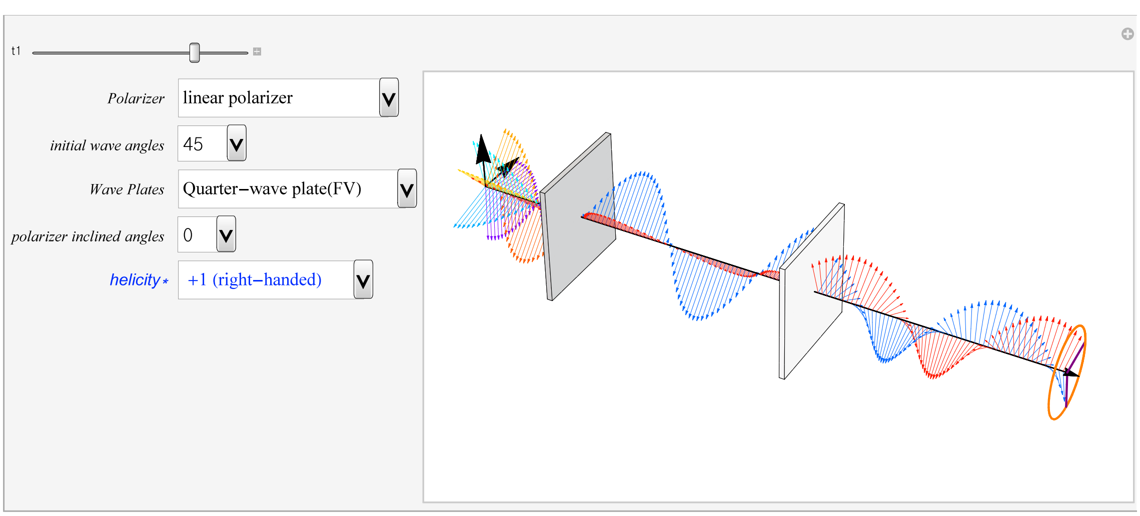


Fig. 2. Dynamic polarization modes platform (jdpmp). The picture shown is a snapshot of the right-handed circular polarized (RHCP) propagation EM wave train. Unpolarized EM waves $\vec{E}_0$ injected into the linear polarizer (LP) with 45° to the horizontal transmission axis come from the vacuum and pass through the quarter-wave plate (QWPV), which result in the right-handed circular polarized EM wave train. This picture shows the E0*LP·QWPV/.*a*->1/4*π* process in the Jones vectors manipulation with a helicity of +1.

the Piecewise function of *Mathematica* depending on the polarizer and wave plate, with the Manipulate function of *Mathematica*. To present the transverse characteristic property satisfying Eq. (1) together with Eqs. (6) and (7), we used the Arrow function in *Mathematica* for drawing the vector array: Table[Arrow [{ {x1, 0, 0}, {x1, Ey, Ez} }]] in the orthogonal {x1, Ey, Ez} coordinate system. In addition, the same was used for the magnetic field vector $\vec{B}$ calculated from the relation $\vec{B}=1/u\ \vec{k} \times \vec{E}$ of Eq. (6) also in solid. **Graphics3D** can draw the vector array of the $\vec{E}$ (blue) and $\vec{B}$ (red) fields of every point in the block as real polarization modes propagating in the advancing $\vec{k}$ direction, in our case, advancing in the x_1 direction. This simulation scheme differs basically from that of an animation representing the envelope of the polarization propagation of $\vec{E}$ field only using the Animate function of graphic tools (Harrison 2015; Mooleskamp & Stokes 2015). Also, in this work, we simulated polarization process with vector fields satisfying wave equation and Maxwell's vector equations at every point during the process. Similar numerical method has been applied once to solve the Lagrange's equation in non-inertial frame (Kim & Yun 2014).

The platform starts by clicking the button ► on pop up ⊕ of the *t1* panel on the platform in Fig. 1. We can choose an incoming angle to the polarizer and rotate the polarizer to compose an arrangement of devices for the polarization mode indent shown as Fig. 2. While the program is running, we will change the polarizer or wave plate, with the changed animation running continuously. Even if the animation is stopped, we can change the configuration of the platform and observe the polarizing mode of the changed state.

Fig. 3 shows snapshots of three kinds of polarization modes. Figs. 3a and 3b shows a circularly polarization mode, Figs. 3c and 3d linearly polarization, Figs. 3e and 3f elliptically polarization according to the arrangement of optical devices. If we utilize linear polarizer JmLinear [β] instead of a wave plate, then either it passes a linearly polarized wave or block in the angle β. While making right angles between two polarizers no wave passes through the polarizer plate as shown in Figs. 3g and 3h. Thus, controlling of phase of polarized light waves is available for the phase encoding in the quantum key distribution (QKD) of the unconditional security in quantum communication technology (Rubenhok et al. 2013).

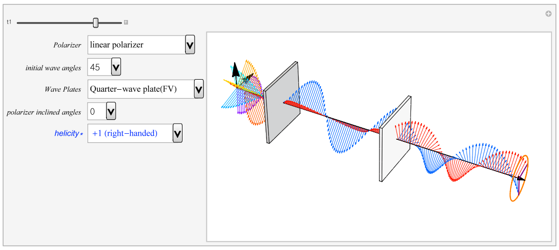
Fig. 4 presents snapshots of three kinds of polarization modes viewed in the ViewPoint (200, 0, 0) against the propagation direction for confirmation of the transverse behavior and progression of the Poynting vector. It shows the orthogonal transverse of the propagating process of the waves and confirms the flow of Poynting vector. This is the visualization of the graphic version of $\vec{S}=\vec{E}\times\vec{H}$ satisfying Maxwell's vector equations Eqs. (6) and (7). The traces of both $\vec{E}$ and $\vec{H}$ are dynamically shown in the polarization modes at the edge of the *jdpmp* platform.

The platform will run faster or slower and stop and restart again by clicking the pop-up menu of the t1 panel for more analytical observations while the polarization mode is running. During the platform stop, you will change to another mode only if you click a panel, then the changed mode will be presented automatically. At that time you can save the presenting mode or print out the mode status. While the platform is running, you can confirm the helicity by observing the spin direction of the field vectors along the orange trace of the helical propagation of the $\vec{E}$ and $\vec{B}$ vector fields at the edge of the platform. The helicities on the *helicity* panel are correct in the case of E1=E0*LP(α) manipulation on the *jdpmp* platform. The helicities of other arrangements of optical devices are calculated promptly on the helicity [JvC, JmC] module in *Mathematica*.

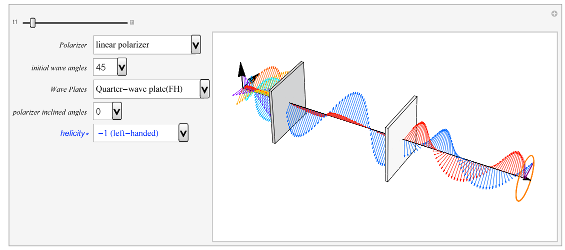
As far as we know, other than *jdpmp*, there is no instructive platform that simulates a transverse EM wave satisfying Maxwell's equation in the vectorial version of polarization modes (Harrison 2015; Mooleskamp & Stokes 2005; Tamm 1997). The complete version of jdpmp.nb program including helicity [JvC, JmC] module is available from the site (Yun 2015). If your PC has not installed *Mathematica*, you can run the jdpmp.cdf instead on the Wolfram CDF Player (Wolfram 2015).

4. SUMMARY

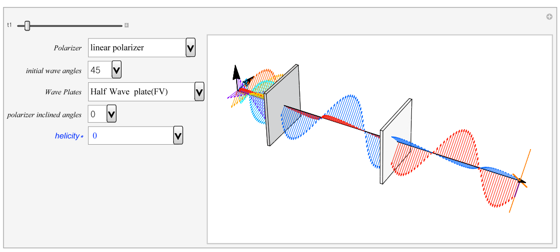
We have provided a dynamic polarization mode platform, *jdpmp*, for simulating and producing polarization modes with the Jones calculations, corresponding to the physical arrangement of optical elements, in *Mathematica*. The platform animates the polarization process of the $\vec{E}$ vector field together with $\vec{B}$ using the Arrow vectors, so that the transverse EM wave advance in the propagation direction, satisfying Maxwell's wave equation at every point on the advancing axes of the platform. Consequently, the vectors $\vec{E}$, $\vec{B}$, $\vec{k}$ (in that order) form a right-hand orthogonal set. *jdpmp* can be accomplished graphically in **Graphics3D** of *Mathematica* based on the numerical vector calculations in *Mathematica*. The platform can be manipulated dynamically and interactively by advancing the polarized mode while clicking the panels of the polarizer and wave plate using the Manipulate function in *Mathematica*, so that the program will continuously simulate the changed mode. While the program is running the helicity of the polarized



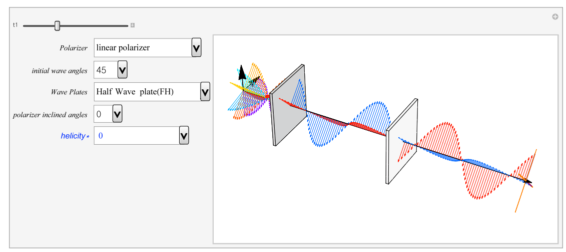
(a) Producing right-handed circularly polarized EM wave in the $E0 * LP \cdot QWPV$ arrangement



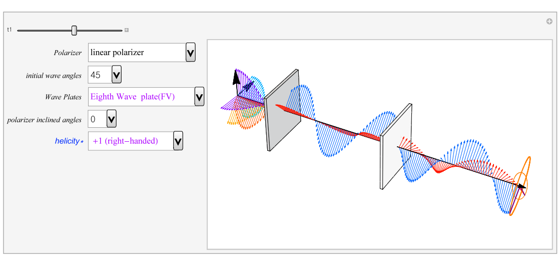
(b) Producing left-handed circularly polarized EM wave in the $E0 * LP \cdot QWPH$ arrangement



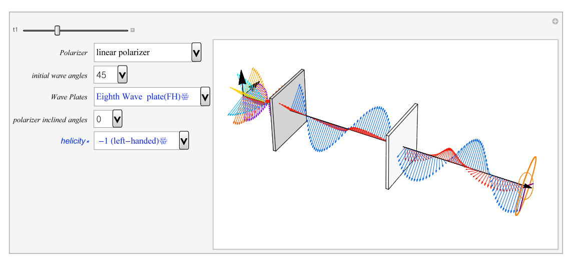
(c) Producing linearly polarized EM wave in the $E0 * LP \cdot HWPV$ arrangement



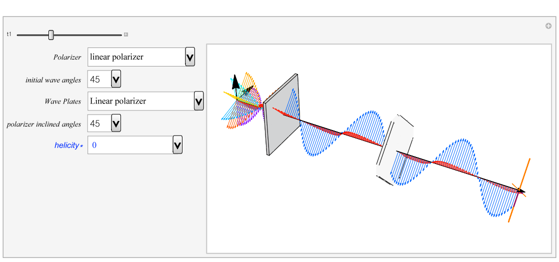
(d) Producing linearly polarized EM wave in the $E0 * LP \cdot HWPH$ arrangement



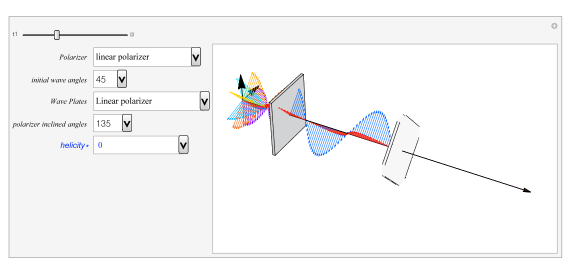
(e) Producing right-handed elliptically polarized EM wave in the $E0 * LP \cdot EWPV$ arrangement



(f) Producing left-handed elliptically polarized EM wave in the $E0 * LP \cdot EWPH$ arrangement

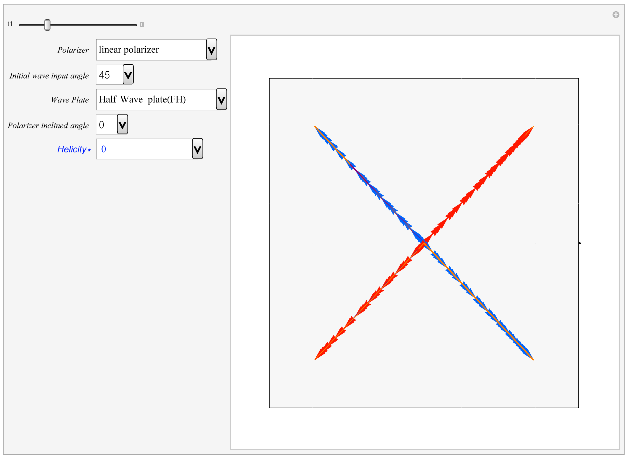


(g) Producing linearly polarized EM wave by the arrangement of two linear polarizers

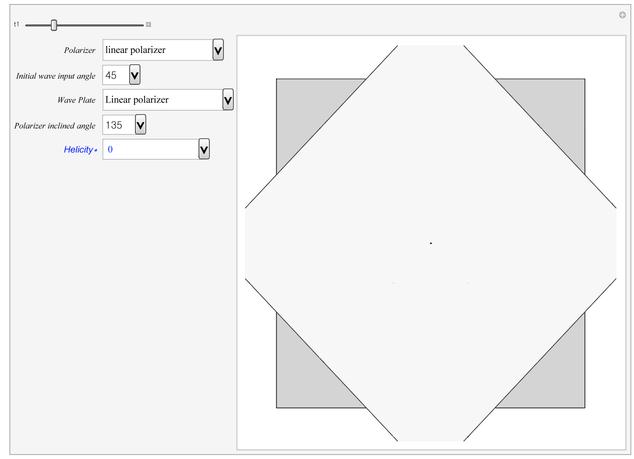


(h) No producing any polarized EM wave at the right angle between two polarizers

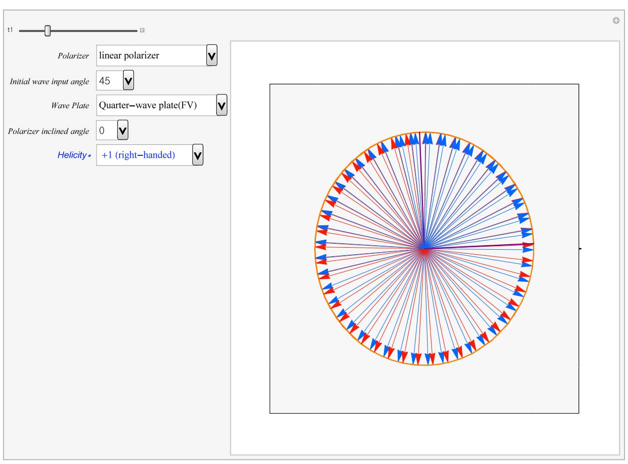
Fig. 3. Producing different polarized modes by the different physical arrangements of polarizer and wave plate: (a) RHCP of helicity +1, (b) LHCP of helicity -1, (c) LP, (d) LP, (e) RHEP of helicity +1, (f) LHEP of helicity -1, (g) LP, (h) No wave.



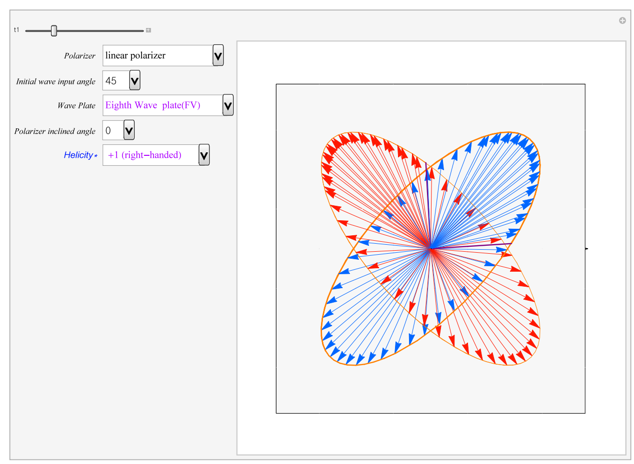
(a) LP by the E0*LP.HWV($\alpha- > \pi/4$) array



(b) By the E0*LP.LW($\alpha- > \pi/4, \beta- > 3/4\pi$)array



(c) RHCP by the E0*LP.QWV($\alpha- > \pi/4$) array



(d) RHEP by the E0*LP.EWV($\alpha- > \pi/4$) array

Fig. 4. Snapshots of the polarized modes at the ViewPoint (200, 0, 0); (a) LP of Fig. 3c, (b) No wave of Fig.3h, (c) RHCP of Fig. 3a, (d) RHEP of Fig. 3e. It presents $\vec{E}$ fields by the blue arrows and $\vec{H}$ fields by the red arrows.

mode is displayed on the panel. The module helicity [JvC, JmC] will return the helicity of the process on *Notebook* of *Mathematica* when you're typing in a polarizer and a wave plate. The module is helpful for students or researchers to inspect the phase difference or helicity of a polarized mode for various kinds of physics arrangements. We expect the platform *jdpmp* will be a useful starting platform for physics students and researchers to explore polarizations in science.

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REFERENCES

Aad G, Abbott B, Abdallah J, Khalek SA, Abdelalim AA et al., Search for the Standard Model Higgs Boson in the Diphoton Decay Channel with 4.9 fd^{-1} of pp Collision Data at $\sqrt{s}$=7 TeV with ATLAS, Phys. Rev. Lett. 108, 111803 (2012). http://dx.doi.org/10.1103/PhysRevLett.108.111803

Ade PAR, Aikin RW, Bento SJ, Bischoff CA, Bock JJ et al., Detection of B-Model Polarization at Degree Angular Scales by BICEP, Phys. Rev. Lett. 112, 241101 (2014). http://dx.doi.org/10.1103/PhysRevLett.112.241101

Born M, Wolf E, Principle of Optics 7th ed. (Cambridge University Press, Cambridge, 1999), 795.

Boyle LA, Steinhardt PJ, Turok N, Inflationary predictions for scalar and tensor fluctuations reconsidered, Phys. Rev. Lett. 96, 111301 (2006). http://dx.doi.org/10.1103/PhysRevLett.96.111301

Harrison DM, Optics Applets [Internet], cited 2015 Feb 15, Available from: http://www.cabrillo.edu/~jmccullough/Applets/Flash/Optics/CircPol.swf.

Calvin W, NASA Technology Views Birth of the Universe [Internet], cited 2014 Mar 17, Available from: http://jpl.nasa.gov/news/news.php?release=2014-082.

CfA, First Direct evidence of cosmic inflation [Internet], cited 2014 Mar 17, Available from: http://cfa.harvard.edu.

Elser D, Bartley T, Heim B, Wittmann Ch, Sych D et al., Feasibility of free space quantum key distribution with coherent polarization states, New J. Phys. 11, 045014 (2009). http://dx.doi.org/10.1088/1367-2630/11/4/045014

Fowles GR, Introduction to Modern Optics (Holt, Reinhart and Winston, 1975).

Georgi H, 1982, Lie Algebra in Particle Physics (The Benjamin INC, 1982), 162.

Goldhaver M, Grodzins L, Sunyar AW, Helicity of Neutrinos, Phys. Rev. Lett. 109, 1015 (1958). http://dx.doi.org/10.1103/PhysRev.109.1015

Jackson JD, Classical Electrodynamics, (John Wiley & Sons, 1975), 274.

Jones RC, A New Calculus for the Treatment of Optical Systems, J. Opt. Soc. Am. 31, 488-493 (1941). http://dx.doi.org/ 10.1364/JOSA.31.000488

Kim B, Yun HJ, Pedagogical Mathematica Platform Visualizing the Coriolis Effects in 3-Cell Atmospheric Circulation Model, J. Astron. Space Sci. 31, 91-99 (2014). http://dx.doi.org/ 10.5140/JASS.2014.31.1.91

Leitch EM, Kovac JM, Pryke C, Carlstrom JE, Halverson NW et al., Measurement of Polarization with the Degree Angular Scale Interferometer, Nature 420, 673-771 (2002). http://dx.doi.org/10.1038/nature01271

Masayoshi T, Cutting-edge terahertz technology, Nat. Photonics 1, 97 (2007). http://dx.doi.org/10.1038/nphoton.2007.3

Mathematica, Wolfram *Mathematica* [Internet], cited 2015 Feb 15, Available from: http://www.wolfram.com/mathematica

Mooleskamp FE, Stokes KL, Polarization of an Optical Wave through Polarizers and Wave Plates [Internet], cited 2015 Feb 15, Available from: http://demonstrations.wolfram.com/PolarizationOfAnOptiionsaturlWaveThroughPolarizersAndWavePlates

Pedrotti FL, Pedrotti LS, Introduction to Optics (Prentice-Hall, 1987).

Reitz R, Milford FJ, Christy W, Foundation of Electromagnetic Theory, 4th ed. (Addison Wesley, Reading, Massachusetts, 1993).

Rubenhok A, Slater JA, Chan P, Lucio-Martinez I, Tittel W, Real-World Two-Photon Interference and Proof-of-Principle Quantum Key Distribution Immune to Detector Attacks, Phys. Rev. Lett. 111, 130501 (2013). http://dx.doi.org/10.1103/PhysRevLett.111.130501

Tamm P, A Physicist's Guide to *Mathematica* (Academic Press, San Diego 1997), 291-293.

Tang X, Ghassemlooy Z, Rajbhandari S, Popoola WO, Lee CG et al., Free-space optical communication employing polarization shift keying coherent modulation in atmospheric turbulence channel, in 7th International Symposium, Newcastle, England, 21-23 July 2010.

Vallone G, D'Ambrosio V, Sponselli A, Slussarenko S, Lorenzo Marrucci L et al., Free-Space Quantum Key Distribution by Rotation-Invariant Twisted Photons, Phys. Rev. Lett. 113, 060503 (2014). http://dx.doi.org/10.1103/PhysRevLett.113.060503

Wolfram, Wolfram CDF Player for Interactive Computable Document Format [Internet], cited 2015 Feb 15, Available from: http://www.wolfram.com/cdf-player

Yao XS, Yan LS, Zhang B, Willer AE, Jiang J, All-optic scheme for automatic polarization division demultiplexing, Opt. Express 15, 7407-7414 (2007). http://dx.doi.org/10.1364/OE.15.007407

Yun HJ, Choi YD, Dynamic Polarization Mathematica Platform Realized with Poynting Vectors, New Phys.: Sae Mulli 63, 1118-1127 (2013). http://dx.doi.org/10.3938/NPSM.63.1118

Yun HJ, jdpmp programs [Internet], cited 2015 Feb 15, Available from: http://home.mokwon.ac.kr/~heejy/program.htm