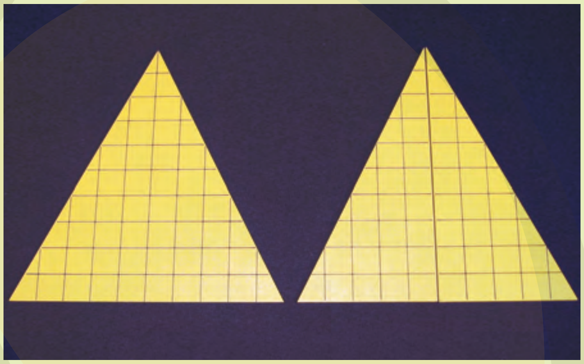
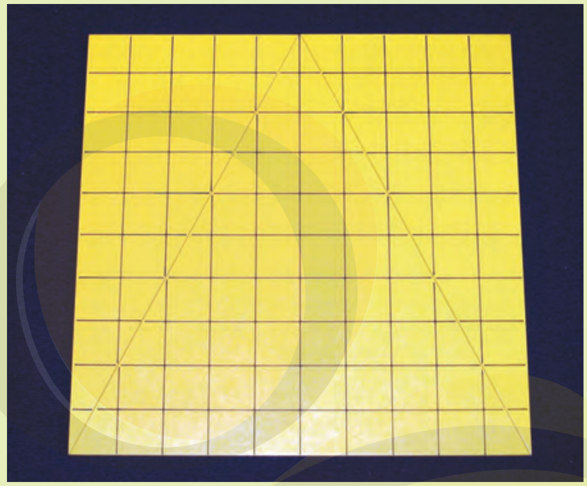
ACTIVITY 5

Calculating the Area of an Acute Triangle



These two acute triangles are congruent



The large acute triangle has one-half the area of this square

Purpose

To learn to calculate the area of an acute triangle.

Material

Yellow Triangles for Area.

Equivalent Figure Material (triangle tray).

Whiteboard and whiteboard markers.

Math journals and pencils.

Presentation

- Most Montessori teachers present this concept in Year 5 and Year 6.
- The students should have completed the Year 5 activity about computing the area of rectangles before doing this activity.
- Invite a small group of students to a mat or table to learn how to calculate the area of an acute triangle.

YEAR 5: FINDING THE AREA OF AN ACUTE TRIANGLE USING A SQUARE

- Place the Yellow Triangles for Area in the work area. Select the acute triangle and discuss whether the students can calculate its area. (Not easily. Some squares are not whole.)
- Select the two tall right triangles and use them to form an acute triangle congruent to the first one. State that this triangle is congruent to the whole acute triangle. Invite a student to demonstrate its congruency by laying the two-piece acute triangle over the whole acute triangle.
- Place the two tall right triangles alongside the acute triangle so that the three pieces form a ten-unit by ten-unit square. Ask the students if they can calculate the area of the square. (Yes. Ten rows x ten squares = 100 squares, or 100 square units.)

- Remind the students that the acute triangle is exactly one-half as large as the square. Encourage the students to state the area of the acute triangle. (One-half of 100 square units = 50 square units.) Set aside the two tall right triangles, leaving the acute triangle in the work area.

- Ask the students to make a drawing of the acute triangle in their journals, then record the calculations for area.

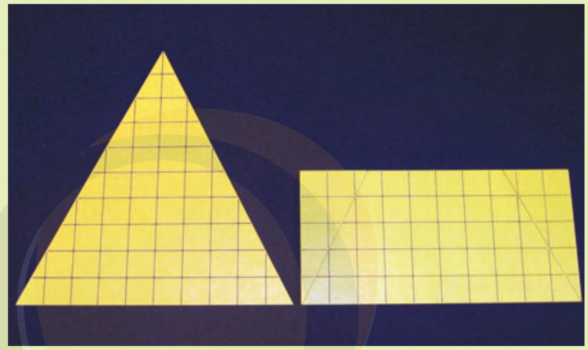
- Repeat the process with other acute triangles, again asking the students to make a congruent triangle, form a square, calculate the area of the square, and calculate the area of the triangle. For each new triangle, ask the students to make a drawing of the triangle in their journals, then record the calculations for area.

YEAR 5: FINDING THE AREA OF AN ACUTE TRIANGLE BY USING A RECTANGLE

- Explain that an acute triangle's area can also be found by making equivalent rectangles.

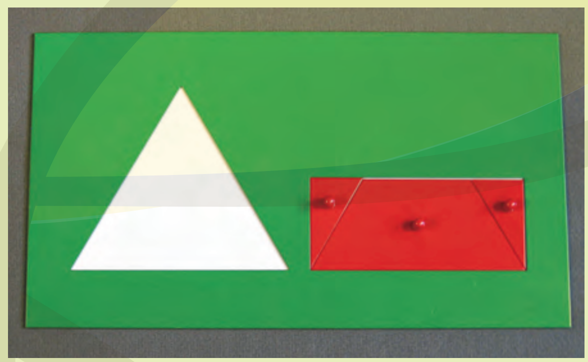
- Make a triangle congruent to the acute triangle using the isosceles trapezoid and the two small right triangles. Invite a student to demonstrate the three-piece triangle's congruency by placing it on top of the whole acute triangle.

- Slide and rotate the small right triangles down to the ends of the trapezoid to form a rectangle. State that the triangle has been rearranged into an equivalent rectangle. Encourage a student to state the area of the rectangle. (Ten squares per row x five rows = 50 square units.) Set aside the three-piece equivalent rectangle.



The acute triangle is equivalent to a long rectangle formed by rearranging

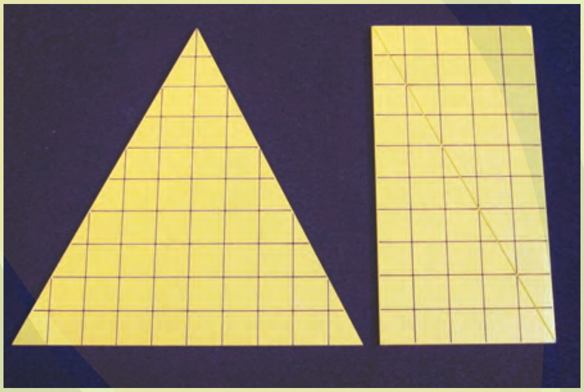
- Use the triangle tray from the Equivalent Figure Material to illustrate again how a triangle can be rearranged into an equivalent rectangle.



The area of an acute triangle illustrated by the Equivalent Figure Material

- Explain that the acute triangle's area can be found one more way. Make a triangle congruent to the acute triangle using the two tall right triangles. Invite a student to demonstrate the two-piece triangle's congruency by placing it on top of the whole triangle.

- Flip one of the tall right triangles and rotate it so the triangles meet along their longest sides. This results in a tall rectangle. State that the triangle has been rearranged into an equivalent rectangle. Invite a student to state the area of the tall rectangle. (Five squares per row x ten rows = 50 square units.)



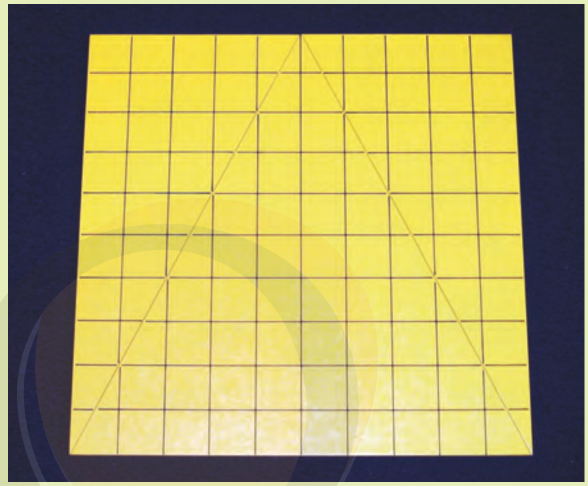
The triangle is equivalent to a tall rectangle formed from its halves

- Confirm that the methods discussed show that the area of the acute triangle is 50 square units.

- Ask the students to make a drawing of two ways for finding the area of a triangle by using a rectangle.

YEAR 6: USING A SQUARE TO DISCOVER THE FORMULA FOR AREA OF AN ACUTE TRIANGLE

- Place the rectangle from the Yellow Triangles for Area in the work area and review the concept that a rectangle's area equals its length times its width. Ask the students to record the formula for a rectangle's area in their journals: $A = lw$. Remind the students that this formula is true even for a square, where $l = w$. Set aside the rectangle.



The large acute triangle has one-half the area of this square

- Create a square using the acute triangle and the tall right triangles, as in Year 5. Review the Year 5 finding that this acute triangle is equivalent to one-half of a square. Invite the students to sketch in their journals (not to scale) the arrangement of triangles made into a square.

- Point to the base of the acute triangle and name it. Point to the height of the acute triangle and name it. Emphasize that the height of the triangle is perpendicular to the base and is different from the length of the side.

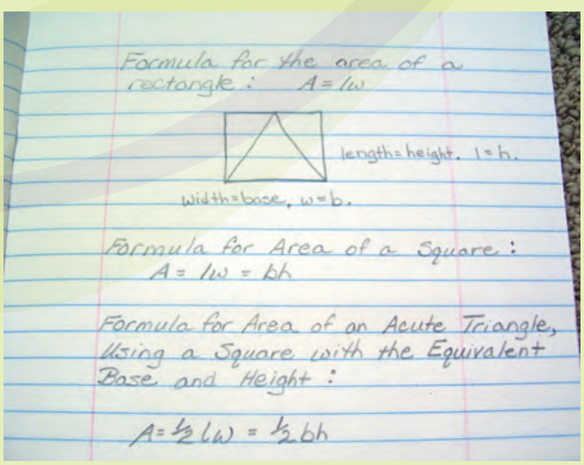
- Invite the students to examine the square. Show that its width is equal to the base of the triangle and its length is equal to the height of the triangle. Ask the students to label the bottom of their sketch as follows: width = base. $w = b$. Ask the students to label one side of their sketch as follows: length = height. $l = h$.

- Invite the students to determine a different way of stating the formula for calculating the area of a square, since length equals base and width equals height.

- Confirm that the formula is $A = lw = bh$. Write the formula on the whiteboard and ask the students to copy it into their journals under this heading: Formula for the Area of a Square.

- Review that the materials showed that the area of the acute triangle is one-half the area of the square with the same base and height. Then invite the students to determine the formula for calculating the area of the acute triangle.

- Confirm that since the acute triangle is one-half the area of the square, the formula for calculating the area of the acute triangle is $A = 1/2lw = 1/2bh$. Write the formula on the whiteboard and ask the students to record it in their journals under this heading: Formula for Area of an Acute Triangle, Using a Square with the Equivalent Base and Height.



Area of a triangle = 1/2bh

- Ask the students what two steps they could carry out, using the formulas just discussed, to confirm that the area of the acute triangle with a base of 10 units and a height of 10 units is equivalent to one-half the area of the square with a base of 10 units and a height of 10 units. Encourage the students to refer to the yellow material as needed.

- Confirm the two steps and write them on the whiteboard:

  1. Find the area of the square with a base of 10 units and a height of 10 units.

     $A = lw = bh$

     $= 10$ units x 10 units

     $= 100$ square units

  2. Find the area of the acute triangle with a base of 10 units and a height of 10 units.

     $A = 1/2lw = 1/2bh$

     $= 1/2$ (10 units x 10 units)

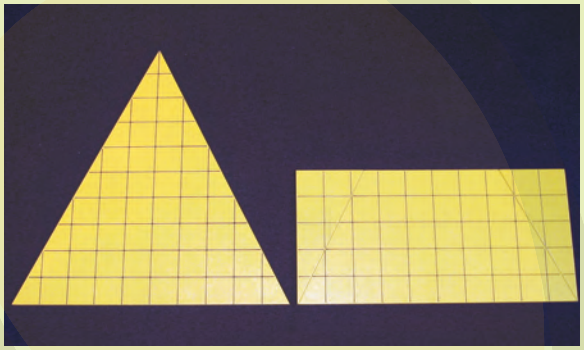
     $= 1/2$ (100 square units)

     $= 50$ square units

- Ask the students to record the steps in their journals.

YEAR 6: USING RECTANGLES TO DISCOVER THE FORMULA FOR AREA OF AN ACUTE TRIANGLE

- Place the acute triangle from the Yellow Triangles for Area in the work area. Make a three-piece triangle congruent to it using the isosceles trapezoid and the two small right triangles. Review and demonstrate the concept that the three-piece triangle can be rearranged to form an equivalent long rectangle, one with an equivalent base.



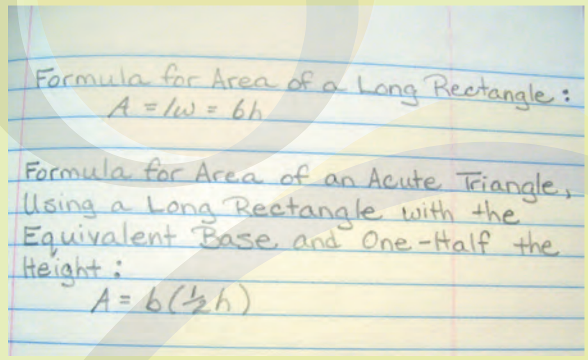
An acute triangle is equivalent to the long rectangle

- Referring to the material as needed, review with the students the following important points: (1) the bases of the long rectangle and the acute triangle are the same; (2) the height of the long rectangle is one-half the height of the acute triangle; and (3) the material shows that the areas of the long rectangle and the acute triangle are equivalent.

- Ask the students to state the formula they discovered for calculating the area of the long rectangle. Confirm that the formula is $A = lw = bh$. Write the formula on the whiteboard and ask the students to record it in their journals under this heading: Formula for Area of a Long Rectangle.

- Invite the students to determine the formula for calculating the area of the acute triangle, using the long rectangle with the same base and one-half the height.

- Confirm that since the acute triangle is the same area as the long rectangle and has the same base, and the long rectangle is one-half the height of the acute triangle, the formula for calculating the area of the acute triangle is $A = b(1/2h)$. Write the formula on the whiteboard and ask the students to record it in their journals under this heading: Formula for Area of an Acute Triangle, Using a Long Rectangle with the Equivalent Base and One-Half the Height.



Area of an acute triangle = b(1/2h)

- Ask the students what two steps they could carry out, using the formulas just discussed, to confirm that the area of the acute triangle with a base of 10 units and a height of 10 units is equivalent to the area of the long rectangle with a base of 10 units and a height of 5 units. Encourage the students to refer to the yellow material as needed.

- Confirm the two steps and write them on the whiteboard:

  1. Find the area of the long rectangle with a base of 10 units and a height of 5 units.

     $A = lw = bh$

     $= 10$ units x 5 units

     $= 50$ square units

2. Find the area of the acute triangle with a base of 10 units and a height of 10 units.

$A = b(1/2h)$

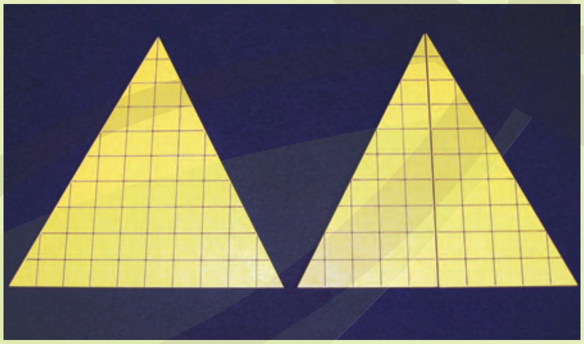
$= 10$ units x (1/2 x 10 units)

$= 10$ units x 5 units

$= 50$ square units

- Ask the students to record the steps in their journals.

- Set aside all but the whole acute triangle. Tell the students there is one more way to find the formula for area of an acute triangle. Make a two-piece acute triangle congruent to the acute triangle using the two tall right triangles. Review and demonstrate the concept that the two-piece acute triangle can be rearranged to form an equivalent tall rectangle.



The acute triangle is equivalent to two right angle triangles

- Referring to the material as needed, review with the students the following important points: (1) the heights of the tall rectangle and the acute triangle are the same; (2) the base of the tall rectangle is one-half the base of the acute triangle; and (3) the material shows that the areas of the tall rectangle and the acute triangle are equivalent.

- Follow the same steps as for finding the area of the acute triangle using the long rectangle: ask the students to state the formula they discovered for calculating the area of a rectangle; invite the students to determine the formula for calculating the area of the acute triangle, using the tall rectangle with the equivalent height and one-half the base; confirm the formula for calculating the area of the acute triangle $A = (1/2b)h$, and record it in their journals under this heading: Formula for Area of an Acute Triangle, Using a Tall Rectangle with the Equivalent Height and One-Half the Base.

- Ask the students what two steps they could carry out, using the formulas just discussed, to confirm that the area of the acute triangle with a base of 10 units and a height of 10 units is equivalent to the area of the tall rectangle with a base of 5 units and a height of 10 units. Encourage the students to refer to the yellow material as needed.

- Confirm the two steps and write them on the whiteboard:

1. Find the area of the tall rectangle with a base of 5 units and a height of 10 units.

$A = lw = bh$

$= 5$ units x 10 units

$= 50$ square units

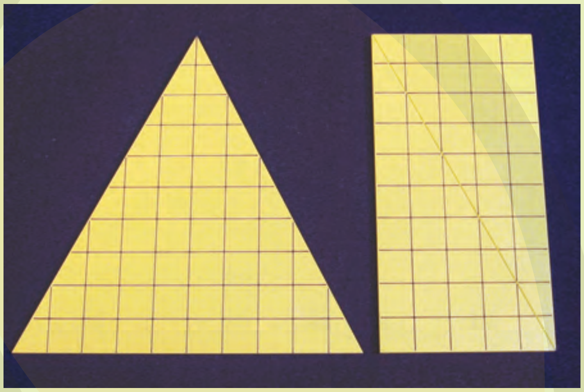
2. Find the area of the acute triangle with a base of 10 units and a height of 10 units.

$A = (1/2b)h$

$= (1/2$ x 10 units) x 10 units

= 5 units x 10 units

= 50 square units



The acute triangle is equivalent to the tall rectangle

- Ask the students to record the steps in their journals.
- Review that the students have seen the formula for the area of the acute triangle expressed three ways. Ask the students to list the three expressions, as follows:

  1. Using the square, we found $A = 1/2(bh)$.
  2. Using the long rectangle, we found $A = b(1/2h)$.
  3. Using the tall rectangle, we found $A = (1/2b)h$.

- As the students summarize the three expressions, write them on the whiteboard, then ask the students to record the summary in their journals.
- Review with the students that the commutative property of multiplication means the order in which terms are multiplied does not matter, so all these expressions are equivalent.
- Explain that the formula most often used for the area of an acute triangle is $A = 1/2bh$. Write the formula on the whiteboard and ask the students to record it in their journals under this heading: The Formula Most Often Used for Area of an Acute Triangle.
- Invite the students to look closely at this formula, as well as at the three expressions just recorded, and find yet another way of expressing the formula — one that has not yet been discussed. Confirm that another way of expressing the formula is $A = (bh)/2$.

Extensions

- Manipulate the material as in the activity, creating squares and rectangles equivalent to the large acute triangle. Record the formula for area that goes with each arrangement of the material.
- Use a rectangular sheet of paper made from one-half or one-third of a sheet of ordinary paper. Locate the middle by folding the paper in half. Draw an isosceles triangle with one entire edge of the paper as its base and with its apex at the fold line on the opposite side of the paper. Cut out the isosceles triangle. Use the triangle and the leftover paper to demonstrate that the area of the isosceles triangle is equivalent to one-half the area of the rectangular sheet of paper used.
- The formula $A = 1/2bh$ works even if the acute triangle is not isosceles. Demonstrate this as follows: Use a square or rectangular sheet of paper.

Record the formula for its area in the journal. Select one entire edge of the paper to form the base of a triangle. Using a ruler, draw a straight line from one end of the base obliquely across the paper to any point on the other edge of the paper, but not the middle of the edge. Using the ruler, draw a straight line from this point to the other end of the base. The result will be a large triangle in the middle of the page and a smaller triangle at each end. Cut out the large triangle. Now arrange the two smaller triangles so they exactly cover the large triangle. Answer the following questions: Are the two small triangles together equivalent to the large triangle? What is the formula for the area of the large triangle? Repeat using rectangles with different proportions.

