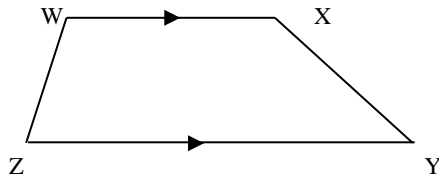**LESSON 8.5**   **Trapezoids**

A TRAPEZOID HAS EXACTLY ONE PAIR OF PARALLEL SIDES (Page 536) *(The parallel sides are called the bases and the other two sides are called the legs.)*
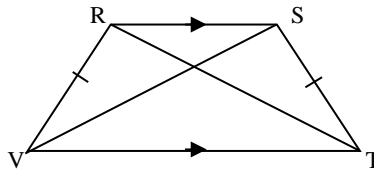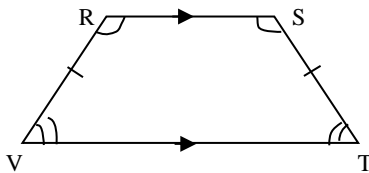

In trapezoid WXYZ:
$\overline{WX} \parallel \overline{ZY}$; $\overline{WZ} \nparallel \overline{XY}$

$\overline{WX}$ and $\overline{ZY}$ are the bases.

$\overline{WZ}$ and $\overline{XY}$ are the legs.

Because $\overline{WX} \parallel \overline{ZY}$ and $\angle$ W and $\angle$ Z are consecutive interior angles, $\angle$ W and $\angle$ Z are supplementary; likewise $\angle$ X and $\angle$ Y are supplementary.

ISOSCELES TRAPEZOID: Properties:
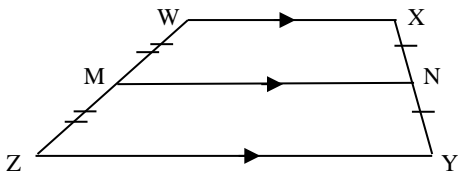   **1. It has $\cong$ legs.** *(Definition-page 537)*
    2. The base angles are $\cong$. *(Theorem-page 537)*
    3. The diagonals are $\cong$. *(Theorem-page 537)*

In isosceles trapezoid RSTV,
1. $\overline{RV} \cong \overline{ST}$
2. $\angle R \cong \angle S$; $\angle V \cong \angle T$
3. $\overline{RT} \cong \overline{SV}$

MIDSEGMENT OF A TRAPEZOID is the segment that connects the midpoints of the legs. The midsegment is parallel to the bases and its length is one half the sum of the legs. *(Theorem-page 538)*

In trapezoid WXYZ, $\overline{MN}$ is the midsegment
(Points M and N and the midpoints of the legs.)

Therefore, $\overline{WX} \parallel \overline{MN} \parallel \overline{ZY}$ and

$$MN = \frac{1}{2}(WX + ZY) \quad OR \quad 2MN = WX + ZY$$

Vocabulary
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases.** For each of the bases of a trapezoid, there is a pair of **base angles,** which are the two angles that have that base as a side.

The nonparallel sides of a trapezoid are the **legs** of the trapezoid. If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid.** The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.

Theorem 8.14: If a trapezoid is isosceles, then each pair of base angles is congruent.

Theorem 8.15: If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

Theorem 8.16: A trapezoid is isosceles if and only if its diagonals are congruent.

Theorem 8.17 Midsegment Theorem for Trapezoids:
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

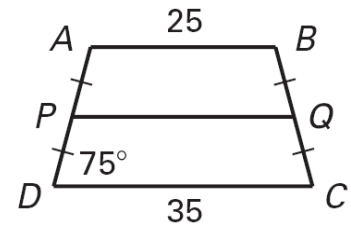**EXAMPLE 1**   Use properties of trapezoids

In the diagram, *ABCD* is an isosceles trapezoid and $\overline{PQ}$ is the midsegment.

    a. Find $m\angle B$.

    b. Find PQ.

    Solution

    a. Because $\angle D$ and $\angle A$ are consecutive interior angles formed by intersecting two parallel lines, they are supplementary. So, $m\angle A = 180° - 75° = 105°$. By Theorem 8.14, $\angle A \cong \angle B$. So, $m\angle B = 105°$.
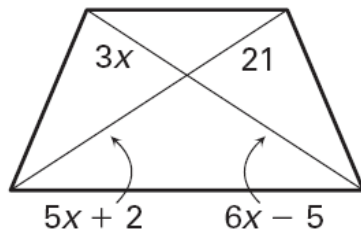
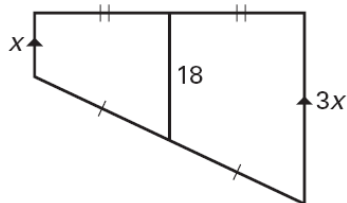   b. By Theorem 8.17, $PQ = \dfrac{1}{2}(AB + CD) = \dfrac{1}{2}(25 + 35) = \dfrac{1}{2}(60) = 30$.

Exercises for Examples 1 and 2

Find the value of *x*.

    1.

    2.

Answers:

    1. 7

    2. 9